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INFLUENCE OF PORE-SIZE DISTRIBUTION ON POROUS ABSORBER PROPERTIES

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1. INTRODUCTION

The pores in naturally-occurring granular materials are difficult to specify in length, shape or connectivity. Nevertheless a plausible approach to understanding their acoustical characteristics is to start from some idealised pore microstructure.

Previous publications [1, 2] have introduced a modified classical approach to the description of the acoustical properties of air filled granular materials based upon a model microstructure of identical tortuous capillary pores of arbitrary shape. The relationship of the resulting theory with the rigid frame limit and high frequency regime of the widely-used Biot theory [3] has been discussed also [4]. Useful approximations have been derived and applied to results of acoustical measurements. In its final form the modified classical theory [5, 6] predicts that the acoustical properties of porous media depend on porosity, flow resistivity, tortuosity and a pore shape factor ratio. The first three of these may be measured non-acoustically by gravimetric, air-flow and electrical conduction methods respectively. However, the basis for introduction of the pore shape factor ratio is phenomenological and it has been used as an adjustable parameter relating the effective or acoustic flow resistivity to that defined for steady flow and measured non-acoustically. For cylindrical pores the pore shape factor ratio has a value of 0.5 and acoustical and steady flow resistivities are identical. Exact expressions may be obtained for the propagation constant and wave or characteristic impedance in a rigid medium containing either identical parallel cylindrical pores or identical, parallel sided slits, [1-3, 7-8]. Norris [9] has provided a rigorous derivation of the acoustical properties of a medium containing identical arbitrarily-shaped pores. His general results for the low frequency approximation are expressed in terms of the steady flow permeability and an inertial factor defined as product of porosity and the real part of the complex density of fluid in the pores divided by the fluid density. It should be noted that the inertial factor introduced by Norris[9] may be distinguished clearly from tortuosity in having a non-unitary value even for straight pores. His result for the low frequency approximation of complex density suggests that effective and actual flow resistivities are equal and independent of pore shape. Norris[9] considers also the effects of pore size distribution and pore interconnections.

In this paper we consider the effect of a log-normal distribution of pore characteristic dimension (e.g. radii or slit semi-widths) on the acoustical parameters of a rigid porous material modelled as an array of arbitrarily-shaped tortuous pores.

2. THEORY

2.1 Identical straight cylindrical and slit pores.

The complex density of fluid in a medium containing identical parallel cylindrical pores of radius r with their axes along the direction of propagation is given by [1,2,7,9].

$$\tilde{\rho}_c(\omega) = (\rho_f/\Omega) \{ [- J_0(\lambda\sqrt{i})]/[J_2 (\lambda\sqrt{i})] \} \tag{2}$$

where ρ_f represents the equilibrium fluid density and

$$\lambda = r \left(\frac{\omega}{\upsilon}\right)^{\frac{1}{2}} \tag{3}$$

$\omega = 2\pi f$, f being the frequency, $\upsilon = \mu/\rho_f$, and μ is the dynamic coefficient of viscosity of the fluid.

The complex stiffness of fluid in this ideal medium is given by [1,2,7]

$$\tilde{K}_c(\omega) = (\gamma\rho_0)\left\{1 + \frac{2(\lambda - 1)J_1\left(\lambda\sqrt{(iN_{pr})}\right)}{N_{pr}^{\frac{1}{2}}\lambda\sqrt{i}\,J_0\left(\lambda\sqrt{(iN_{pr})}\right)}\right\}^{-1} \quad (4)$$

where $\gamma\rho_0$ is the isothermal bulk modulus, γ is the ratio of specific heats and N_{pr} is the Prandtl number.

The governing equations for the bulk medium are an equation of state,

$$\left[\tilde{K}_c(\omega)\right]^{-1} = (1/\rho_f)(d\rho/dp) \quad (5)$$

where ρ_f represents the equilibrium fluid density and p is pressure, an equation of motion

$$-(\partial p/\partial x) = \tilde{\rho}_c(\omega)(\partial u/\partial t) \quad (6)$$

where u represents the average fluid particle velocity, and an equation of continuity

$$-(\partial u/\partial x) = (\Omega/\rho_f)(\partial\rho/\partial p)(\partial p/\partial t) \quad (7).$$

Combination of these leads to

$$\frac{d^2p}{dx^2} = \Omega\left[\tilde{\rho}_c(\omega) / \tilde{K}_c(\omega)\right]\frac{d^2p}{dt^2} \quad (8)$$

from which a propagation constant k_c may be defined as

$$k_c^2 = \Omega\omega^2\tilde{\rho}_c(\omega) / \tilde{K}_c(\omega) \quad (9)$$

It should be emphasized that equation (2) is an exact and rigorous result for an ideal rigid porous medium containing identical parallel circular cylindrical pores and in which thermal effects are ignored. It has been derived also by Norris [9] as a particular result of a general rigorous homogenisation process for a medium containing pores of arbitrary shape. Similarly equation (4) is an exact result for an ideal cylindrical pore medium containing a conducting non-viscous fluid. Strictly the equation of motion for such a rigid porous medium containing a viscous, conducting fluid should be solved. However Zwikker and Kosten [7] have shown that the inaccuracy resulting from separate treatment of viscous and thermal effects is small. Consequently equations (9), (2) and (4) together with $\gamma p_0 = \rho_f c_p^2$ may be considered to constitute a result that is close to exact for an ideal rigid porous medium containing parallel identical cylindrical pores.

Similar results for an ideal medium containing identical slit-like pores of semi-width b with their axes along the direction of propagation are

$$\tilde{\rho}_s(\omega) = (\rho_f/\Omega)\left[1 - \left(\lambda_s\sqrt{(-i)}\right)^{-1}\tanh\left(\lambda_s\sqrt{(-i)}\right)\right]^{-1} \quad (10)$$

and

$$\tilde{K}_s(\omega) = (\lambda\rho_0)\left[1 + \frac{(\lambda - 1)}{\lambda_s\sqrt{(-i)}N_{pr}}\tanh\left(\lambda_s\sqrt{(-i\,N_{pr})}\right)\right]^{-1} \quad (11)$$

where

$$\lambda_s = b\sqrt{(\omega/\upsilon)}$$

Again equation (10) has been derived as a particular case of a general homogenisation theory[9].

2.2 Size distributions of cylindrical and slit pores

Following Yamamoto and Turgut [10], we consider first a medium containing circular cylindrical pores and we assume a log-normal distribution of pore radii given by

$$e(r) = F(\phi) = (1/\sigma)\,(2\pi)^{-0.5} \exp\left[-\,(\phi - \bar{\phi})^2/2\sigma^2 \right]$$

where $\phi = -\log_2 r$ (r in mm), σ is the standard deviation and $\bar{\phi}$ is the mean value of ϕ.

For a medium containing circular capillary pores, aligned with the (x -) direction of flow, and subject to an oscillatory pressure gradient $\partial p/\partial x$ where time dependence exp $(-i\omega t)$ is understood, the fluid velocity, averaged over the cross sectional area of a single pore, is given by

$$U_{Ave} = \frac{\partial p}{\partial x} \frac{1}{-i\omega\tilde{\rho}_c(\omega)} \tag{12}$$

The average velocity in the medium containing a log-normal distribution of pore radii, is then given by

$$\bar{U}_{Ave} = \int_0^\infty e(r) U_{Ave}(\omega, r)\, dr \tag{13}$$

The bulk complex density may be defined by

$$\tilde{\rho}_{Ave}(\omega) = -\frac{\partial p}{\partial x} / i\,\omega\,\bar{U}_{Ave} = \int_0^\infty e(r)\,\tilde{\rho}_c(\omega)\, dr$$

$$= \int_0^\infty e(r)\;\frac{\rho_f}{\Omega}\left\{\frac{-J_0(\lambda\sqrt{i})}{J_2(\lambda\sqrt{i})}\right\}dr \tag{14}$$

Yamamoto and Turgut [10] derive an equivalent expression for the complex viscosity correction factor used originally by Biot [3] and evaluate this for three values of the standard deviation σ in the assumed log-normal pore radius distribution. They do not offer any detail of how the evaluation is carried out. Probably a numerical method was used since it is unlikely that the integral in (14) or their equivalent has a straight forward analytical solution. However, a situation of interest in assessing the acoustical properties of air filled soil at frequencies in the range 100 Hz to 5000 Hz is that of $\lambda << 1$ corresponding to low frequency and small pore radius (or width). By expanding the Bessel functions to O (λ^2) it is possible to show that [1,2,4,7] equation (2) gives,

$$\tilde{\rho}_c(\omega) \sim \frac{4}{3}\frac{\rho_f}{\Omega} + \frac{8i\mu}{\Omega\omega r^2} \tag{15}$$

Hence

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$$\bar{\rho}_{Ave}(\omega) \sim \frac{4}{3}\frac{\rho f}{\Omega} + i\int_{-\infty}^{\infty}\frac{8\mu\sqrt{(1/2\pi)}}{\sigma\pi(\Omega\omega)}\exp\left[1.3863\phi - \frac{(\phi-\bar{\phi})^2}{2\sigma^2}\right]d\phi \tag{16}$$

where use has been made of $r^{-2} = 2^{2\phi} = 4^{\phi} = e^{1.3863\phi}$.

and $\int_0^{\infty} f(\phi)d\phi = 1$.

The remaining integral may be rewritten as

$$I = \frac{8\mu\sqrt{(1/2\pi)}}{\sigma(\Omega\omega)}\frac{1}{\bar{r}^2}\int_{-\infty}^{\infty}\exp\left[1.3863\,x - x^2/2\sigma^2\right]dx \tag{17}$$

Where $x = \phi - \bar{\phi}$ and $\bar{r}$, the mean pore radius $= 2^{-\bar{\phi}}$

A standard integral of this form is evaluated straight forwardly, [12]

$$\int_{-\infty}^{\infty}\exp(-p^2x^2 \pm qx)\,dx = \frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right) \tag{18}$$

If $p = \frac{1}{\sigma\sqrt{2}}$ and $q = 1.3863$, then

$$I = (\sigma_b/\omega)\ \exp\ [4\,(\sigma\ln 2)^2] \tag{19}$$

where for a log-normal pore size distribution, the bulk flow resistivity (=μ/permeability) is given by [10]

$$\sigma_b = \frac{8\mu}{\Omega\bar{r}^2})\ \exp\left[[-2\,(\sigma\ln 2)^2\right] \tag{20}$$

Substitution of (19) into (16) and comparison with an equation of the form

$$\tilde{\rho}_{Ave}(\omega) = \frac{4}{3}\frac{\rho f}{\Omega} + \frac{i4S_d{}^2\sigma_b}{\omega} \tag{21}$$

leads to an explicit relationship between S_d and υ for $\lambda \ll 1$,

$$S_d = \frac{1}{2}\exp\left[2\,(\sigma\ln 2)^2\right] \tag{22}$$

In equation (21) $4\,S_d{}^2\sigma_b$ (= σ_e) may be termed an acoustic or <u>effective</u> flow resistivity and S_d represents a <u>size distribution factor</u>.

For slit-like pores [1,2,8]

$$\bar{\rho}_s(\omega) \sim \frac{\rho f}{\Omega} + \frac{3i\mu}{\Omega\omega b^2} \tag{23}$$

Assumption of a log-normal distribution of slit widths with mean value $\bar{b}$, standard deviation σ and a bulk flow resistivity σ_b given by

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$$\sigma_b = \frac{3\mu}{\Omega \bar{b}^2} \exp\left[-2(\sigma \ln 2)^2\right] \tag{24}$$

leads again to the definition (22) for S_d given above.

As alternatives to (15) and (23), it is possible to consider approximations for $\lambda >> 1$ corresponding to high frequency and large pore radius (or slit width). These are relevant to the acoustical properties of low flow resistivity sands, snow, artificial air-filled media consisting of glass beads or lead shot and pervious road surfaces in the frequency range 1000 Hz and above.

The relevant approximation for an ideal medium containing identical cylindrical pores is [1,2,7]

$$\tilde{\rho}_c(\omega) \sim (\rho_f/\Omega)(1 + 2\sqrt{i}/\lambda_c)$$

$$= (\rho_f/\Omega)\left[1 + \frac{2\sqrt{i}}{r}\left(\frac{\mu}{\omega\rho_f}\right)^{\frac{1}{2}}\right] \tag{25}$$

$$\text{where } r = \sqrt{(8\mu/\Omega\sigma_b)} \tag{26}$$

and σ_b is given by equation (24) with $\sigma = 0$.

After substituting in equation (14), again making use of equation (18) and comparing with an equation of the form

$$\tilde{\rho}_{Ave}(\omega) = (\rho_f/\Omega)\left[1 + \frac{S_d\sqrt{i}}{\mu}(2\omega\rho_f\,\Omega\sigma_b)^{\frac{1}{2}}\right] \tag{27}$$

a relationship between S_d and s may be derived as

$$S_d = \frac{1}{2}\exp\left[\frac{3}{2}(\sigma \ln 2)^2\right] \tag{28}$$

Figure 1 shows a plot of S_d for $0 < \sigma < 1$. Both the low and high frequency deductions in Figure 1 indicate a range $0.5 < S_d < 1.3$, for this range of standard deviation. For $\sigma < 0.3$, the value of S_d is approximately constant and similar to the value (0.5) for identical pores.

The similarity between the pairs of equations (2) and (4) and (10) and (11), and consequently of their approximations suggest, without rigorous derivation, that the influence of pore size distribution on the complex stiffness of the fluid in the pores, will also be indicated by S_d evaluated from equations (21) and (22) or (27) and (28). These results are consistent with domination of viscous and thermal processes within the bulk medium by smaller pores where the size distribution is sufficiently broad.

As remarked earlier, the expressions for S_d are independent of pore shape, consequently, as long as the pores in the medium are assumed to have identical shapes, the results in equations (24) and (28) may be extended to media containing arbitrarily-shaped pores.

2.3 Approximations for propagation constant

For a medium containing identical circular cylindrical pores of radius r aligned with the driving pressure gradient, approximation (15) may be combined with the equivalent approximation to order λ^2 of (4), through equation (9), to yield a low frequency approximation for the bulk propagation constant.

Hence

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$$k_c^2 \sim \gamma\left(\frac{\omega}{c_0}\right)^2\left[\left(\frac{4}{3} - \frac{\gamma - 1}{\gamma}N_{pr}\right) + \frac{i\Omega\sigma_b}{\rho f\omega}\right] \quad (31)$$

where $\sigma_b = 8\mu/\Omega r^2$

A similar expression with 4/3 replaced by 5/4, follows from approximation (23) and the equivalent approximation to order λ^2 of (11), where for slits $\sigma_b = 3\mu/\Omega b^2$. For media containing aligned straight arbitrarily shaped pores with log-normal distributions of characteristic dimension having standard deviation σ,

$$k_b^2 \sim \gamma\left(\frac{\omega}{c_0}\right)^2\left[\left(I_0 - \frac{\gamma - 1}{\gamma}N_{pr}\right) + \frac{4iS_d{}^2\sigma_b\Omega}{\rho f\omega}\right] \quad (32)$$

Note that (32) may also be written

$$k_b \sim \frac{1}{C_0}\left[\frac{i\gamma\Omega\sigma_e\,\omega}{\rho f}\right]^{\frac{1}{2}}\left[1 - \frac{ia\rho f\omega}{2\Omega\sigma_e}\right] \quad (33)$$

to satisfy causality, and with $a = I_0 - \frac{\gamma - 1}{\gamma}N_{pr}$ for arbitrarily shaped pores.

Note that I_0 is the inertial factor evaluated by Norris[9] for several specific pore shapes. If the pores are tortuous then σ_e may be modified to include a factor q^2, which also appears explicitly as a factor multiplying a.

At high frequencies or for $\lambda >> 1$, approximation (26) together with the equivalent approximation of (11) lead to

$$k_b \sim q\,(\omega/C_0)\left[\frac{1 + \frac{2S_d}{q}\left(\frac{i\Omega\sigma_b}{2\rho f\omega}\right)^{\frac{1}{2}}}{1 - \frac{2\,(\gamma - 1)S_d}{\sqrt{N_{pr}}\,q}\left(\frac{i\Omega\sigma_b}{2\rho f\omega}\right)^{\frac{1}{2}}}\right] \quad (36)$$

for a size distribution of tortuous circular cylindrical pores.

It is not difficult to show that equation (36) applies also to a medium containing a size distribution of tortuous slit pores. This suggests that (36) is a general result and is independent of pore shape. Although Norris [9] did not extend his analysis to high frequencies it is likely nevertheless, that the high frequency approximation for arbitrarily-shaped tortuous pores will also be independent of pore shape. Physically this can be understood as a consequence of plug flow at high frequencies. It should be noted that, as remarked earlier, further approximation of (36) leads to the result that the ratio of sound speeds inside and outside the rigid air saturated porous medium at high frequencies is related simply to tortuosity (strictly the square root of tortuosity). This result has been used extensively in the literature [14].

Approximations (34) and (36) enable deduction of best fit parameters (aq^2 and $S_d{}^2$) from measured data of propagation constant and surface normal impedance at low and high frequencies (defined in terms of λ). For example, representing k_b by c + id where c is the phase constant and d is the attenuation constant it is possible to deduce from (34) that at low frequencies

$$aq^2 \sim \frac{C_0{}^2}{4\pi^2\gamma f^2}\,(c^2 - d^2) \quad (37)$$

and that $$S_d{}^2 \sim \frac{\rho f c_0{}^2 cd}{4\pi\gamma\Omega\sigma f} \quad (38)$$

For fluid constant values appropriate to air at sea level and 20^oC $\gamma = 1.4$, $\rho_f = 1.2$ Kgm^{-3}, and $C_0 = 343$ $m\ s^{-1}$, these approximations reduce to

$$aq^2 \sim 2128\,(c^2 - d^2)/f^2 \tag{39}$$

and

$$S_d^2 \sim 8024.8\,cd/(f\Omega\sigma) \tag{40}$$

At high frequencies it is possible to deduce from (36) that

$$q \sim \frac{C_0(c-d)}{2\pi f} = 54.6\,(c-d)/f \tag{41}$$

and that $$S_p^2 \sim 2\rho_f b^2 d^2/(\pi f\Omega\sigma) = 42247\,d^2/(f\Omega\sigma) \tag{42}$$

using the same fluid constant values as above, and where $b = 1 + \frac{\gamma - 1}{\sqrt{N}}$.

3. COMPARISON WITH MEASURED DATA

3.1 Low frequency approximations

While Zwikker and Kosten argue that the degree of approximation given in (31) and hence in (34) is valid only for $\lambda < 1$, numerical calculations carried out by the author and others [15] indicate that it is valid for $\lambda < 4$. Consequently (34) has been used together with approximations (38) to (42) to fit data from in situ probe microphone measurements over a frequency range 100 Hz to 5000 Hz in soil and sand. The theoretical basis for the deduction of propagation constant from these data and the experimental technique are described in detail elsewhere [15-17]. An example result is shown in Figure 2 where the real and imaginary parts of the propagation constant are termed phase constant and attenuation constant respectively. Although there is little difference in the theoretical predictions for slow wave propagation constant in the high flow resistivity sand and in the dry loam, the introduction of an adjustable pore size distribution factor and an adjustable structural factor enables improved agreement with data in the moist sandy loam and in a low flow resistivity sand. Use of the Bruggeman relationship for soils ($q^2 = \Omega^{-1}$) without allowance for a pore-shape dependent inertial factor and without allowance for pore size distribution fails to achieve reasonable predictions for these cases.

3.2 High frequency approximations

Data obtained by high frequency pulse measurements on spherical glass bead packings in an impedance tube[20] have been fitted by equation (36) after use of approximations (41) and (42) together with the measured flow resistivity and porosity. For these media the acoustically fitted tortuosities are close to the values expected for stacked spheres according to the self similar model.

4. DISCUSSION AND CONCLUSIONS

By making use of low- and high- frequency approximations the classical theory for the acoustical characteristics of rigid porous media containing identical circular capillary pores or parallel-sided slits has been extended to allow for a log-normal size distribution of such pores. Reference to recent rigorous results for the low frequency behaviour of media containing arbitrarily-shaped pores, has enabled further but non-rigorous extension to media containing log-normal size distributions of arbitrarily-shaped tortuous pores. The acoustical properties of such media may then be described in terms of porosity, flow resistivity, a structural factor depending on pore shape and tortuosity (aq^2) and a pore size distribution factor (S_d) which is related to the standard deviation of the (log-normal) size distribution.

A value of S_d greater than 0.5 implies an acoustical or effective flow resistivity which exceeds that measured for steady flow at low velocities. Comparison of predictions of the extended classical model with data for air-filled media, whose flow resistivity and porosity have been determined non-acoustically, reveals the necessity for an adjustable structural factor and pore size distribution factor. For soils and sands with relatively high flow resistivities values of S_d less than 0.5 are required. However, for low flow resistivity packings of spherical particles, higher values of S_d (greater than 0.5) are required. It should be remarked that our earlier suggestion [2,4], based on fitting impedance tube data [21], of $S_d = 0.26$ for stacked spheres, was mistaken. It was the result of using values for the flow resistivity of the stacked spheres and sands featured in those data that were a factor of 10 too large. Reanalysis of these data yields values for S_d which are consistent with those deduced for the glass beads in section 3.2. Clearly, a model with a tortuous log-normal distribution of arbitrarily-shaped pores may not be an adequate representation of the interstices between stacked spheres. On the other hand, the consistency of S_d values found for the various stacked sphere media suggest that the representation and interpretation in terms of a pore size distribution is plausible and that the acoustic properties are dominated by the narrower cross-sections in the interstices.

It is also clear that interpretation of the acoustic data on soils and sands in terms of a log-normal pore size distribution, at present, can only be qualitative without any independent (non-acoustical) information on this distribution. However, it is possible to measure pore size distribution non-acoustically[22]. Consequently future work will be devoted to study of acoustical data for media where porosity, flow resistivity and pore size distribution have been obtained non-acoustically. Further study will concern the influence of distributions other than log-normal and variation of pore cross sections along their lengths.

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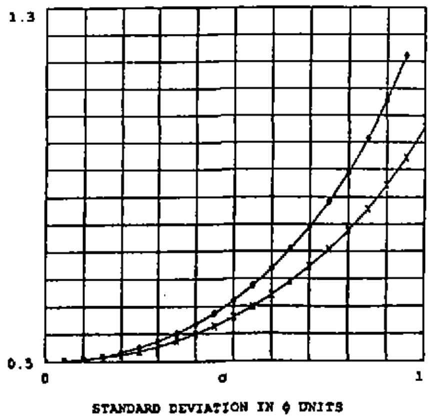
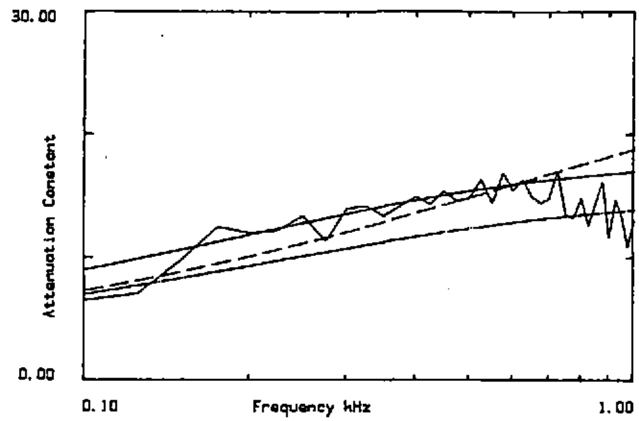
FIGURE 1

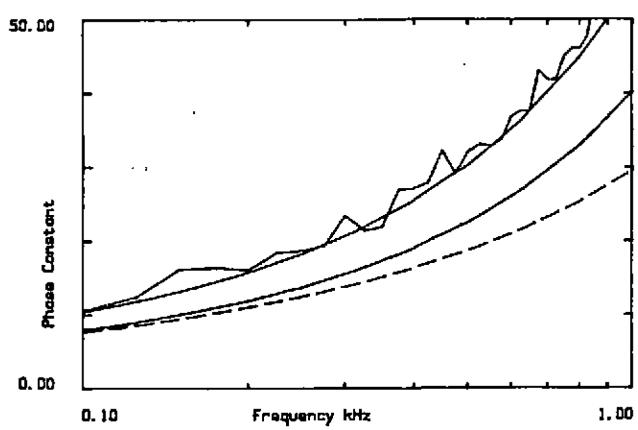


joined diamonds = predictions of small λ approximation
joined crosses = predictions of large λ approximation

FIGURE 2



Measured attenuation constant for moist sandy loam of flow resistivity 54000 mks units and porosity 0.33. The predictions use these values with pore size distribution factor 0.647 and tortuosity 5.2 or inverse of porosity as appropriate.



Measured phase constant for moist sandy loam of flow resistivity 54000 mks units and porosity 0.33. The predictions which are of three low frequency approximations (continuous and dash-dot lines) use these values with pore size distribution factor 0.647 and tortuosity equal to 5.2 or to the inverse of porosity as appropriate.