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SOUND VELOCITY GRADIENTS AND PROPAGATION OVER AN IMPEDANCE BOUNDARY

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INTRODUCTION

Sound propagation over long ranges in an inhomogeneous medium has been studied widely, for example, ref [1], [2], [3] and [4] etc, give detailed descriptions of the problem. Atmospheric inhomogeneity in the form of stratification is of particular interest and is caused by temperature and/or wind gradients. The presence of these gradients implies variations of sound speed with height which causes a continuous refraction of the sound rays. It is useful to construct a simple model of the interaction between continuing refraction and ground effects [4].

THEORY

A Linear Sound Velocity Profile

A major difference between the effect of temperature and wind gradients is that the reciprocity law holds for the former case but not for the latter. In other words, the presence of a wind gradient causes the sound field to be anisotropic such that the sound pressure is lower in the upwind direction. On the other hand, the field is exactly the same for a temperature stratified medium if we interchange the positions of the source and the receiver. However, a wind gradient is equivalent to a temperature gradient as far as the curvature of the sound ray is concerned. The effect due to the temperature and wind gradients are additive. One may simplify the analysis by assuming the atmosphere is steady and vertically stratified.

The model of a vertically stratified medium allows one to use a linear sound velocity profile,

$$C(Z) = C_0(1 + aZ) \tag{1}$$

where C is the velocity of sound, a is the normalised sound velocity gradient, Z is the height above ground level and subscript o denotes the condition at Z = 0.

The assumption of a linear sound velocity profile leads to a circular ray path. It is then straightforward to calculate the path of direct and indirect waves [4].

Sound Pressure Near to an Impedance Boundary

The field potential above an impedance boundary due to a point source can be calculated by Weyl-van der Pol formula [5] as

$$\phi_{tot} = \frac{e^{ikR_1}}{4\pi R_1} + [R_p + (1 - R_p)F]\,\frac{e^{ikR_2}}{4\pi R_2} \tag{2}$$

where R_1 and R_2 are the distances from the source and its mirror image to the receiver respectively, θ is the grazing incidence, k is the wave number in air, R_p and F are the plane wave reflection coefficient and the boundary loss factor respectively. The geometry of the problem is shown in Figure 1. Both R_p and F are functions of the grazing incidence and normal impedance for a locally reacting surface.

The use of a linear sound velocity gradient gives an improved evaluation of the path lengths, R_1 and R_2 respectively, and the grazing angle θ. This, in turn, will give a better estimation of the total sound field due to a point source, over an impedance surface.

Ground Impedance Model

There are various simplified models for the acoustical properties of the ground [6]. The variable porosity model is used in the present work. This means that the normal impedance, Z, can be expressed as

$$Z = 0.218\left(\frac{\sigma}{f}\right)^{0.5} + i\left[0.218\left(\frac{\sigma}{f}\right)^{0.5} + 9.74\left(\frac{\alpha}{f}\right)\right] \qquad (3)$$

where α and σ are the effective rate of change of porosity with depth and effective flow resistivity at the surface, respectively and f is the frequency.

Level Difference Spectrum

The level difference between two vertically separated microphones is defined as the total sound pressure of the upper receiver subtracted from that of the lower receiver. It is calculated by the following equation

$$\text{Level difference (dB)} = 20\log_{10}\left(\frac{\text{total sound pressure of upper receiver}}{\text{total sound pressure of lower receiver}}\right) \qquad (4)$$

The level difference spectrum has a distinctive property that it is independent of the source spectrum.

Determination of Ground Parameters and Normalised Sound Velocity Gradient

It has been suggested that a reasonable procedure for obtaining the acoustical properties of a given ground surface is to invert a short range measurement of excess attenuation using the theory given in equation (2) and an appropriate impedance model [7]. In principle this procedure may be extended to longer ranges and to include a meteorological parameter.

Consequently to deduce the ground parameters and the normalised sound velocity gradient from the experimental result one calculates the value of chi-squared (x^2) which minimises the expression

$$x^2(\alpha,\sigma,a) = \sum_{i=1}^{N} \left[(L.D._p(f_i) - L.D._e(f_i)\right]^2 \qquad (5)$$

where N is the total number of data points and suffixes p and e denote predicted and experimental values respectively. It should be noted that the magnitude of the sound pressure is used in the level difference calculation because of the great sensitivity of the phase to the atmospheric conditions.

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EXPERIMENTAL RESULTS AND DISCUSSION

Experiments were carried out (a) at short range in an anechoic chamber and (b) over a sports ground. The anechoic chamber measurements were intended to be, as near as possible, independent of meteorological conditions.

The short range experiments were carried out in a small anechoic chamber of size 3.9 m x 3.95 m x 2 m and the chamber was fully anechoic only above 2 kHz. A sand tray measuring 1.8 m x 1.22 m and 0.3 m deep was used to simulate a flat ground surface. A Tanoy driver unit with a cylindrical tube extension was used as a point source. The results of fitting level differences for these short range indoor measurements are summarised in Figures 2(a) and (b). It can be seen that the predicted level differences with or without sound velocity gradient are identical. One would expect this characteristics for short ranges and indoor experiments where the effect of inhomogeneity in the atmosphere is unimportant.

Outdoor experiments were performed on a calm and sunny day in a sports field. The source/receiver separation varied from 50 m to 100 m. The receiver array consisted of vertically-separated microphones at 0 m, 1 m and 2 m. Typical results for each range are summarised in Figures 3(a), (b), (c) and (d). It is apparent that there is a substantial improvement in fitting experimental data with the inclusion of the linear sound velocity gradient for a known source/receiver geometry. For a given range and the various source heights the effective flow resistivity (σ) values are remarkably constant. The second ground parameter (α) values fluctuate. However it is known from best fits to these data without including a linear velocity gradient parameter, that α is able to compensate for meteorological effects to some extent by taking large negative values. Consequently through the nature of the best fit procedure, even with a velocity gradient parameter included some variation in α is expected. The ground parameter values tend to vary with range, but those at 50 m and 100 m are consistent with values obtained from short range (~ 2 m) measurements obtained near the corresponding areas of specular reflection. Finally it should be remarked that for a given range the velocity gradients yielded by the best fit procedure for the 0 and 1 m level difference tend to be higher than those for the 0 and 2 m level difference. A typical set of the 'best-fit' ground parameters and the normalized sound velocity gradient is shown in Table 1.

CONCLUDING REMARKS

By assuming a linear sound velocity gradient in the atmosphere, a ray-based modification to the Weyl-van der Pol formula is then used to calculate the sound field over a surface of finite impedance. Experiments were performed and the results were compared with theoretical predictions by means of a least squares procedure. Tolerable agreement is obtained for a variety of surfaces, ranges and source heights.

ACKNOWLEDGEMENTS

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(a) 0 and 1 m Level Difference

(i) Fitting without linear sound velocity gradient

Hs / m	α / m^{-1}	σ / $\times10^{5}$ mks unit	R.M.S. Error
10.33	-1415	4.278	2.24
9.74	-1510	1.905	2.55
9.38	-1694	2.746	1.84
8.97	-1587	2.627	2.66
8.14	-832.1	4.256	1.76
8.20	-810.0	3.990	1.35
7.97	-424.2	2.898	1.82
7.43	-420.7	2.933	1.50
7.20	-803.1	4.075	1.11
6.43	-647.3	2.709	0.95

(ii) Fitting with linear sound velocity gradient

Hs / m	α / m^{-1}	σ / $\times10^{5}$ mks unit	a / s^{-1}	R.M.S. Error
10.33	-2.27	1.745	-0.198	0.90
9.74	-28.7	1.961	-0.333	0.85
9.38	-309.7	3.110	-0.278	0.82
8.97	-133.2	2.319	-0.321	0.97
8.74	-314.9	3.074	-0.195	0.96
8.20	-499.3	3.585	-0.158	0.91
7.97	-115.7	2.322	-0.210	0.84
7.43	-144.8	2.560	-0.20	0.74
7.20	-640.0	4.167	-0.122	0.97
6.43	-642.6	3.301	-0.142	0.73

(b) 0 and 2 m Level Difference

(i) Fitting without linear sound velocity gradient

Hs / m	α / m^{-1}	σ / $\times10^{5}$ mks unit	R.M.S. Error
10.33	-1140	4.111	2.38
9.94	-1595	2.636	2.48
9.38	-1004	2.351	2.47
8.97	-1453	3.141	2.89
8.74	-881.3	5.421	1.97
8.20	-2312	1.670	2.50
7.97	-738.1	5.281	2.05
7.43	-1496	8.511	1.94
7.20	-578.1	3.973	1.46
6.43	-1181	3.062	2.55

(ii) Fitting with linear sound velocity gradient

Hs / m	α / m^{-1}	σ / $\times10^{5}$ mks unit	a / s^{-1}	R.M.S. Error
10.33	100.4	1.00	-0.126	1.19
9.74	3.66	1.718	-0.202	1.39
9.38	-164.3	1.474	-0.166	1.10
8.97	66.4	1.321	-0.175	1.22
8.74	-86.7	2.365	-8.14×10^{-2}	1.27
8.20	-820.0	9.732	-7.38×10^{-2}	1.82
7.97	7.48	1.979	-0.102	1.28
7.43	-247.5	3.607	-0.104	1.30
7.20	-364.1	3.093	-3.50×10^{-2}	1.38
6.43	-572.6	2.790	-0.112	1.79

Table 1: Predicted ground parameters and linear sound velocity gradient for a known source/receiver geometry with 87.5 m range.

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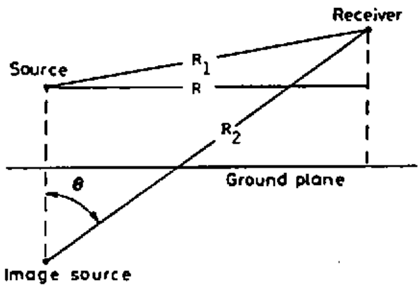

Figure 1 : Geometry of a point source and a receiver.

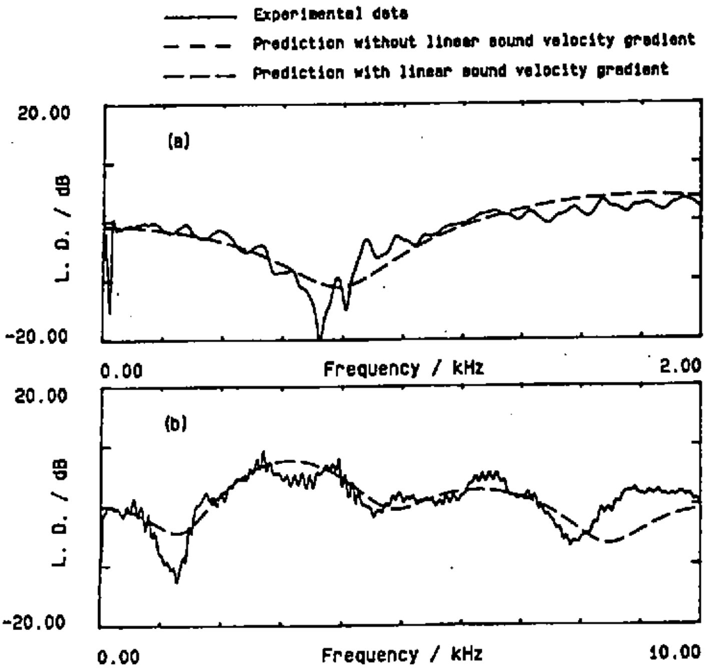

Figure 2 : Comparisons of experimental data with two prediction methods. The variable porosity model is used to calculate the impedance of the ground. Predictions based on the geometry of
(a) source height at 0.21m, two receivers at 0.05m and 0.39m, respectively and range at 0.76m;
(b) source height at 0.24m, two receivers at 0.05m and 0.25m, respectively and range at 1.3m.

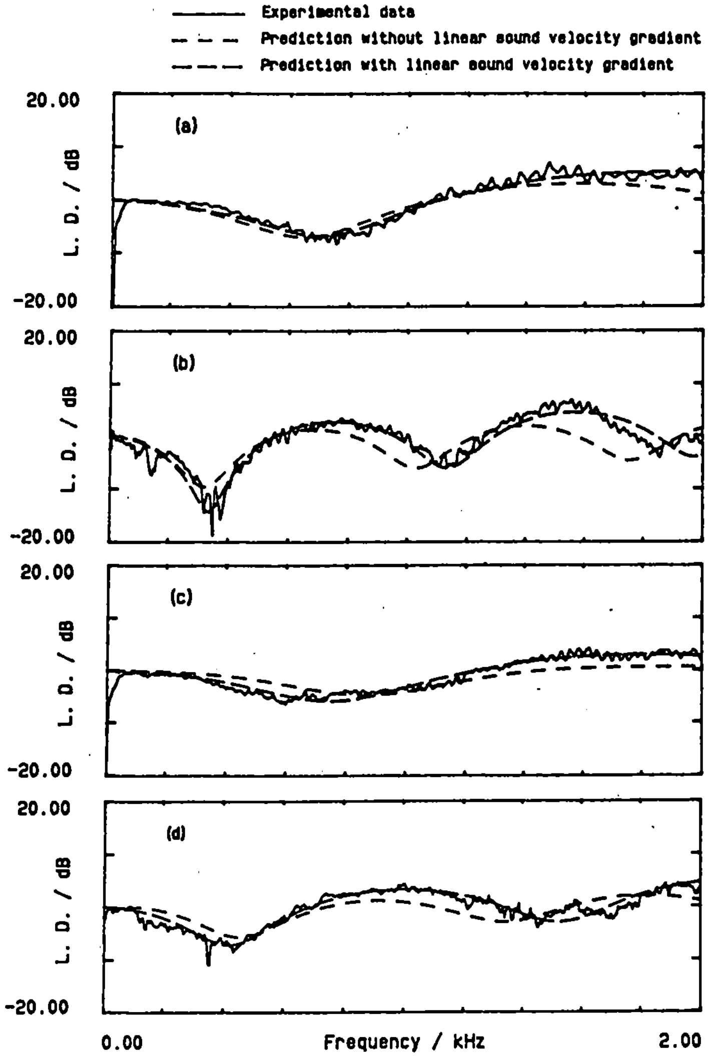

Figure 3 : Comparisons of experimental data with two prediction methods. The variable porosity model is used to calculate the impedance of the ground. Predictions based on the geometry of
(a) source at 4.26m, receivers at 0&1m and range at 50m;
(b) source at 7.13m, receivers at 0&2m and range at 62.5m;
(c) source at 9.38m, receivers at 0&1m and range at 87.5m;
(d) source at 8.63m, receivers at 0&2m and range at 100m.