BRITISH ACOUSTICAL SOCIETY: Meeting on "ABSORPTION OF SOUND": 17th September 1971 at the University of Salford.

OBLIQUE INCIDENCE BEHAVIOUR OF POROUS FIBROUS ABSORBENTS.

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Introduction

A scattering analysis has previously been applied to a simple model consisting of an array of parallel fibres which are freely suspended in air (1). At oblique incidence the theory predicts considerable extended reaction in low density Rocksil materials (2).

At normal incidence the theory when based upon an identical fibre model gives good agreement only when a value for the fibre radius of twice the measured arithmetic mean is assumed. The discrepancy between theory and measurement is otherwise partially removed by the adoption of a more realistic model which includes a distribution of fibre radii and an approximation to the shot (i.e. globules of unfiberised glass) content (3). In addition the boundary conditions assumed for the individual fibres and their orientation are found to greatly effect the normal-incidence behaviour. It is therefore of interest to examine the influence of these factors upon the predicted behaviour at oblique incidence.

Anistropy

A model of parallel fibres with the fibres lying parallel to the direction is highly anisotropic. Moreover the properties for incidence in the xy plane are rather different from those in the yz plane. For incidence in the xy plane the model is essentially isotropic. The random orientation case is chosen in two dimensions only so that the fibres lie in the xz plane (one could choose the yz plane equally well). For this case the model is effectively isotropic for incidence in xz plane but anisotropic for incidence in the xy plane.

Fibre flexibility

Calculations can be made assuming that the fibres
(i) are completely free to move and flex;
(ii) are rigidly fixed in space;
(iii) are rigid but free to move.

All of these cases exhibit extended reaction of a similar form for oblique incidence on 25 and 50mm layers.

Distribution of radii and shot content

The effect is to diminish slightly the extent of the extended reaction in the imaginary part but increase the variation in the real part of surface impedance for incidence in the xy plane. Although the behaviour at normal incidence is similar for both a model of identical fibres of 5mm radius and a model containing the actual radius distribution, the behaviour at oblique incidence is clearly different for these models.

Random orientation

This produces a very different form of extended reaction to that for parallel fibre models. The extended reaction persists throughout the frequency range for oblique incidence in the xz plane. The calculations are based upon the premise that the randomly orientated set of fibres are equivalent to a set of parallel fibres with scattering amplitudes for the individual fibres which are averaged over all angles.

The predictions for the parallel fibre model suggest that the degree of extended reaction is dependent upon frequency, being least for frequencies of the order of 2kHz.

Homogeneous or mixture behaviour

The predictions of extended reaction, even for oblique incidence cases such that either the parallel or the randomly orientated fibre models are effectively isotropic, suggest that a 'modified fluid' analysis of fibrous absorbents is not sufficient . The term 'modified fluid' applies to those theories which reduce to single-wave (isotropic) analyses in the limit of high porosity and frame flexibility, and to those which allow some anisotropy.

A drawback of the suspension model is that only a crude allowance can be made for frame waves. If a continuous frame is assumed then it is possible to predict extended reaction by the incusion of a bulk shear wave (4). However the scattering analysis predicts that extended reaction can occur in materials with rigid frames where shear waves could not propagate. In these cases the behaviour can only be attributed to inhomogeneity.

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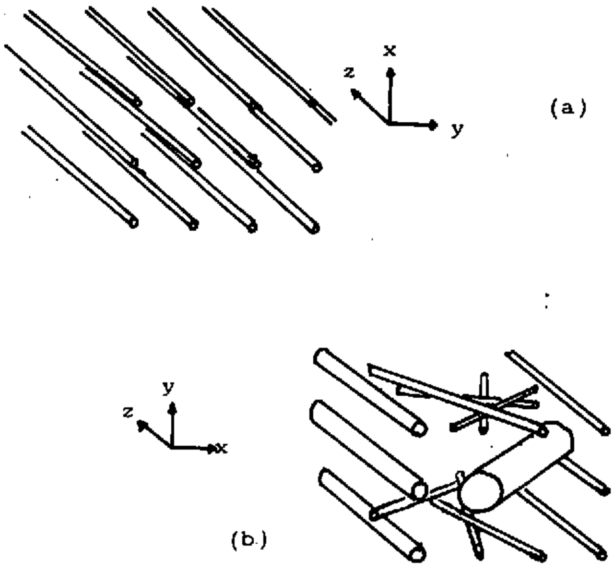
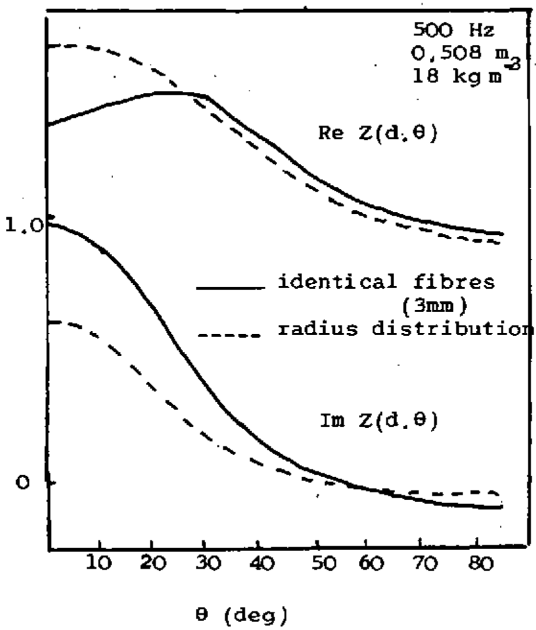


Fig. 1 (a) Parallel identical fibre model

(b) Model containing random orientation and radius distribution.


500 Hz
0,508 m
18 kg m-3
Re Z(d,θ)
1,0
identical fibres
(3mm)
radius distribution
Im Z(d,θ)
0
10
20
30
40
50
60
70
80
θ (deg)

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A STACKED CYLINDER MODEL FOR THE PREDICTION OF THE ACOUSTIC PARAMETERS OF FIBROUS MATERIALS

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This paper describes how a generalised acoustic propagation theory in porous media[1] has been applied to a specific model of stacked cylinders which resembles "Rocksil" fibrous materials.

The generalised theory is formulated from stress-strain relations of the porous medium and a dissipation coefficient. In all, five unknowns exist, four elastic moduli and the "effective" pore radius. These are derived theoretically from the model shown in figure 1. A conceptual compression of the model in the x direction yields a bulk Youngs modulus and Poissons ratio. Furthermore there is a net flow of fluid from the aggregate. These three quantities along with the stress-strain relationships allow the inter-relation of the elastic moduli to be established. Further consideration determines one of the elastic moduli and hence all four are known. It is possible to express the dissipation coefficient as a function of the specific flow resistance. The flow resistance is determined after Iberall[2], by summing the viscous drag forces on the fibres and fibre bonding points.

The equation of motion for the aggregate is then[3]

$$(Q^2-PR)k^4 + (w^2(P\rho_2 + R\rho_1) + jwb(P+R+2Q)F(\mu))k^2$$
$$- (w^4\rho_1\rho_2 + jw^3b(\rho_1 + \rho_2)F(\mu)) = 0$$

Where P, Q and R are elastic constants, $\rho_1 = (1-\beta)\rho_s$, $\rho_2 = \beta\rho_f$, β is the porosity, ρ_s and ρ_f the densities of the solid and fluid portions of the aggregate. $F(\mu)$ is a dissipation correction function depending on the angular frequency ω, β, ρ_f and the flow resistance (σ_f), via the parameter $\mu = \sqrt{\frac{8\omega\rho_f}{\beta\sigma_f}}$

b is the dissipation coefficient denoted by σ_f/β^2.

The above equation is usually solved numerically on a digital computer, however an approximate solution can be shown to be,

$$k' = \pm\left[\frac{w^2\rho_2}{R} + jwbF(\mu)\left(\frac{P+R}{PR}\right)\right]^{\frac{1}{2}} \text{ and } k'' = \pm w\sqrt{\frac{\rho_1}{P}}$$

Hence two wave types can exist in the aggregate, one being approximately real (very small attenuation) and the other having comparable real and imaginary parts.

Solution of the boundary conditions at the front and back of a hard backed layer (thickness d) allows the face impedance to be to be expressed as

$$Z(d) = Z_c \coth(-jk'd)$$

where $$Z_c = \frac{R}{\omega\beta}\left\{\frac{\tau' - \tau''}{\frac{1-\beta+\beta\tau'}{k'} - \frac{1-\beta+\beta\tau''}{k''}\left(\frac{\coth(-jk'd)}{\coth(-jk''d)}\right)}\right\}$$

where τ' and τ'' are constants denoting the ratio of the strain amplitudes in the solid and fluid components for the ' or " wave type. It can be shown that $\tau' \simeq -P/R$ ($\sim$-80) while $\tau'' \simeq 1$.

The ratio of the two "coth" terms is approximately unity as $|\tau'| >> |\tau''|$, the characteristic impedance is expressible as

$$Z_c = \frac{K_f}{\omega\beta}\left\{\frac{\tau' - \tau''}{\frac{\tau'}{k'} - \frac{\tau''}{k''}}\right\}$$

K_f is the fluids bulk modulus, Po for isothermal processes and γPo for an adiabatic process. The thermal dependance of K_f may be expressed in terms of a function of the parameter μ.

In figures 2 and 3 predicted curves are compared with some measured values for a Rocksil material of 18 Kg/m^3 density.

A more complete solution of the equations of motion allows the face impedance of a hard backed layer to be expressed for any angle of incidence in terms of the variables above plus a rotational wave number (or propagation constant) K^r and an equivalent amplitude ratio τ^r.

$$Z(d,\theta) = \frac{K_f}{\omega}\left\{\frac{\tau' + \frac{1-\beta}{\beta} + \Gamma\left(\tau'' + \frac{1-\beta}{\beta}\right)}{\left[\left(\frac{1-\beta+\beta\tau'}{k'}\right)\cos\theta' + \left(\frac{1-\beta+\beta\tau^r}{k^r}\right)\frac{\sin\theta^r}{\cos 2\theta^r}\sin 2\theta'\right]\tanh(-jk'd\cos\theta') + \Gamma\left[\left(\frac{1-\beta+\beta\tau''}{k''}\right)\cos\theta'' + \left(\frac{1-\beta+\beta\tau^r}{k^r}\right)\frac{\sin\theta^r}{\cos 2\theta^r}\sin 2\theta''\right]\tanh(-jk''d\cos\theta'')}\right\}$$

where the refracted angles θ', θ'', and θ^r are related to the angle of incidence θ, by Snell's law, viz

$$k_o \sin\theta = k'\sin\theta' = k''\sin\theta'' = k^r\sin\theta^r$$

k_o is the free air propagation constant and Γ is a function of the three wave types and the elastic constants.

In figure 4 predicted angular variations are compared with measured values[4].

CONCLUSIONS

This approach has been found to give good agreement over a large range of aggregate densities. Variations due to fibre rigidity and fibre radius substantiate the work of other researchers.

The agreement obtained at oblique incidence although not as good as the normal incidence results, does suggest that the mechanism of propagation under such circumstances depend strongly on the "frame" waves and hence any inherent anisotropy.

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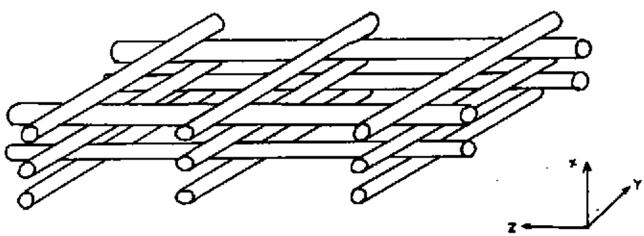

Fig. 1 Idealised model of a fibrous medium.

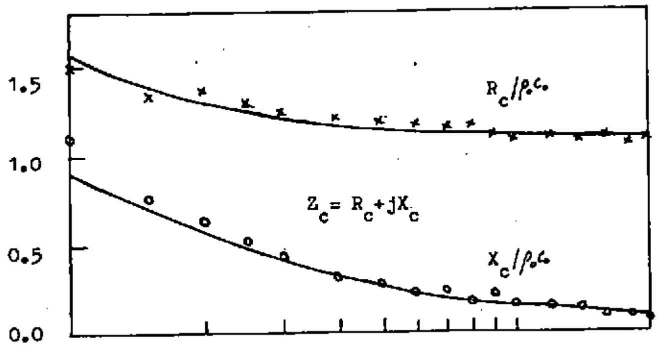

Fig. 2 Characteristic impedance versus frequency.

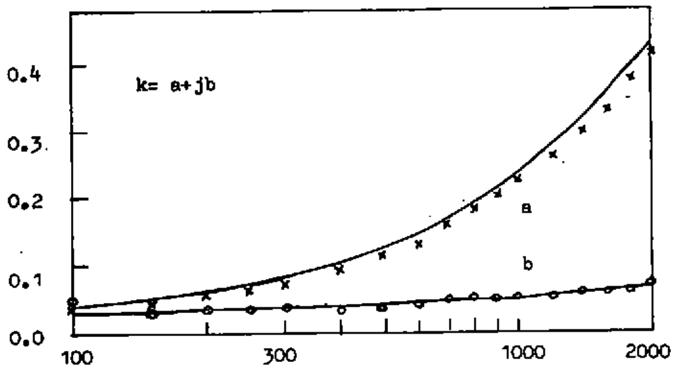

Fig. 3 Propagation constant versus frequency. Density 18 Kg/m^3.

—— Predicted curves. × , o measured real and imaginary parts.

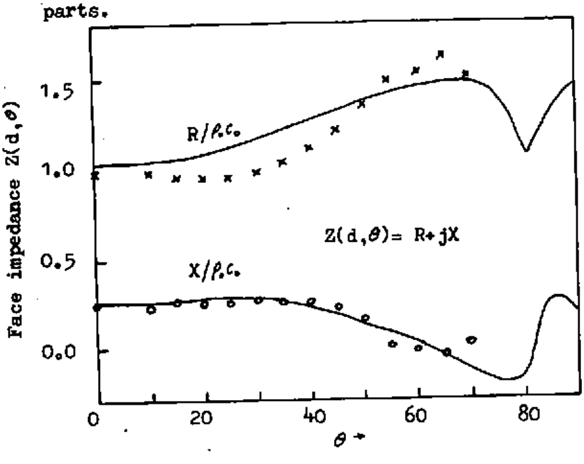

Fig. 4 Face impedance of a 50mm hard backed layer versus angle of incidence . Density 76Kg/m^3. 4000 Hz.