BRITISH ACOUSTICAL SOCIETY

71/02

SCATTERING PHENOMENA IN ACOUSTICS

26TH FEBRUARY 1971

SCATTERING THEORY FOR POROUS ABSORBERS

BY

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INTRODUCTION

A useful object in the study of wave propagation through porous media has been to establish an analytic relationship between the microstructure and bulk acoustic parameters i.e. the propagation constants and characteristic impedence. A particular problem which arises in this connection is the choice of suitable characteristic parameters.

Previous investigators have often assumed such media to be homogeneous and isotropic, whereas each pore or aggregate particle necessarily constitutes an inhomogeneity within the material, their distribution and orientation affecting the macroscopic isotropy. Many of the theories start from a conceptual model of a continuous solid matrix through which run parallel cylindrical pores. The departure from this ideal pore structure to be found in most materials of the porous absorbent type inevitably requires the introduction of "structural factors" which are phenomenological.

Alternative theories are based on models which specify the form of the solid frame rather than that of the fluid "pores" and thus in the case of granular and fibrous materials correspond more closely to their unconsolidated nature. A more detailed review and classification of these theories is to be found elsewhere(1). One such model of parallel fibres in air has the inherent disadvantage that it does not include fibre contact and bonding. However it is possible to make some allowance for this in a scattering approach. Furthermore scattering theory gives analytic expressions for the bulk acoustical parameters in terms of fibre radius, concentration and flexibility. Simply observable aspects of microstructure in real materials such as distribution of fibre radii, shot content and fibre orientation can also be included and their effects on acoustic propagation explained.

SCATTERING THEORY

Consider first an isolated freely-suspended, cylindrical fibre. Wave motion inside both fibre solid and imbedding fluid can be analysed into dilatational, thermal and viscous (shear) components. A plane dilatational wave striking the fibre therefore may be considered to cause three scattered waves in the fluid and three waves induced in the fibre. Introducing potential notation and cylindrical harmonic expansions the eight boundary conditions available at oblique incidence are necessary to evaluate the required scattering coefficients. Since a typical fibre has a radius between one and thirty microns in the audio frequency range we are in the Rayleigh scattering regime and it is not difficult to show that only the first two orders of scattering need be considered. The energy dissipation due to a single fibre per unit incident energy (normal incidence) can then be written as

$$(4/k_D^f)\ \mathrm{Re}\ (A_o+A_1)$$

where k_D^f is the propagation constant of dilatational waves in air, Re denotes the real part and A_o, A_1 are the zero and first order coefficients in the expansion of the scattered dilatational wave. $\mathrm{Re}(A_o)$ and $\mathrm{Re}(A_1)$ may be regarded separately as indications of the thermal and viscous dissipations respectively. Both increase with fibre radius and frequency and are considerably angle dependent (Figures 1 and 2).

When an array of fibres with a concentration N per unit volume sufficient to correspond to that of a fibrous absorbent is considered, secondary scattering and rescattering of waves must be taken into account. A theory by Twersky(2) gives the field at any point within the array as an integral equation. This states that the field at any scatterer at any point x consists of contributions from

(i) the (normally) incident plane wave

(ii) the backward scattered waves from fibres beyond x

and (iii) the forward scattered waves from fibres in front of x.

A solution of this equation enables the derivation of a bulk propagation constant as a function of fibre concentration and the scattering coefficients. Consideration of the impedance tube situation enables calculation of characteristic and surface impedances.

For a typical commercially available material and using the average fibre radius (R) quoted by the manufacturers (three microns) the agreement between theory and measurement is poor. However considerable improvement is possible by increasing the value of R used (Figures 3 and 4).

A SCATTERING PHENOMENON?

The forms given to the external field and the bulk acoustic parameters by the scattering approach are not dissimilar to those of the conventional "modified fluid" analysis. The internal and reflected waves are macroscopically plane. However, the scattering theory suggests that the internal field is always the sum of forward and backward components even for a layer of infinite thickness. Furthermore at oblique incidence the theory predicts that the propagation constant and characteristic impedance become functions of angle[3]. This implies considerable departure from continuum behaviour and allows extended reaction even for an isotropic material with high attenuation.

FIBRE FLEXIBILITY

Scattering coefficients may be calculated quite readily for fibres which are (i) freely suspended
(ii) rigidly fixed in space
(iii) rigid but free to move.

In an actual material any fibre is subject to complex boundary conditions of contact with other fibres in addition to being coated and bonded with a thin layer of resin which is sprayed on during manufacture. Cases (i) and (ii) may be considered to be extremes and case (iii) an intermediate situation. By plotting bulk parameters calculated according to these three types of fibre constraint and comparing with measured values it is possible to suggest the possible effects of "frame" action and also Zwikker and Kosten's "decoupling"[4].

DISTRIBUTION OF RADII

Typically in Rocksil materials the fibre radius varies between one and thirty microns. The scattering coefficients have an involved dependence on R therefore it is necessary to modify the multiple scattering approach in order to include the effect of radius distribution. This is done by replacing the products of fibre concentration and scattering amplitudes as they appear in the relevant expressions by appropriate summations. The effect of so doing is to improve the correspondence between theory and measurement.

SHOT CONTENT

Materials which are made from mined rock may contain up to 30% by weight of unfiberised glass globules or "shot". This clearly affects the calculation of the "pure" fibre concentrations. In the theoretical analysis the simplest assumption to make is that any "shot" particle approximates to a large, long cylinder of appropriate radius. This assumption results in further improvement to the correspondence between theory and measurement. The influence of "shot" content and the presence of the larger fibres on the predicted acoustic performance of the bulk material can be physically explained by reference to the scattering coefficients.

FIBRE ORIENTATION

A close examination of Rocksil microstructure reveals that

(i) the fibres tend to make up a layered structure with the layers lying roughly parallel to the surface and visible from any cross section, and

(ii) the fibres have a fairly random orientation within these layers.

By restricting consideration to a two dimensional problem and assuming both "infinite" length and uniform distribution of the fibres over all angles in the plane it is possible to deduce the effect of random orientation from the scattering approach. Again the effect can be explained in terms of the variation of the scattering coefficients with angle.

CONCLUSION

It has been suggested that theories based on the capillary pore model and using continuum techniques are unable to adequately relate the acoustic behaviour of highly porous, fibrous materials to their microstructure. For this purpose a scattering approach has been proposed as an alternative. A major defect of the approach i.e. disregard of fibre contact and bonding, can possibly be overcome. The analysis allows a number of conclusions concerning the influence of certain aspects of microstructure on fibrous material absorption.

REFERENCES

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2. V. TWERSKY 1962 Journal of Mathematical Physics 3, 700-715 On scattering of waves by random distributions I. Free space scatterer formalism.

3. K. ATTENBOROUGH 1971 J. Sound Vib 14 1-9 The prediction of oblique incidence behaviour of fibrous absorbents.

4. C. ZWIKKER and C.W. KOSTEN 1949 Sound Absorbing Materials Elsevier, Amsterdam

Fig. 1

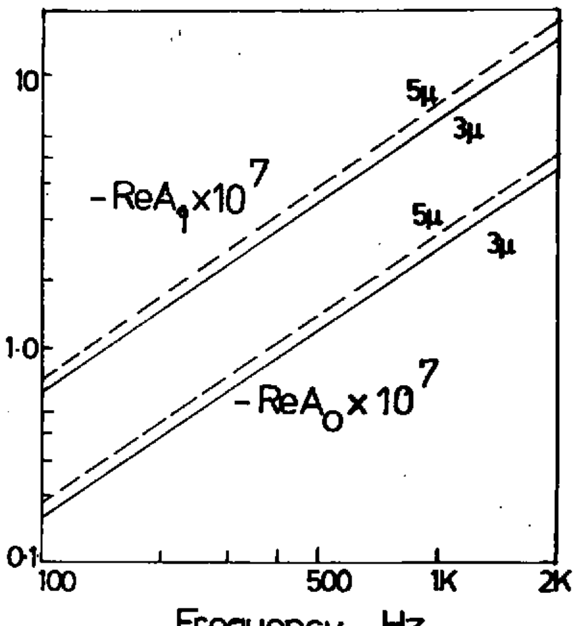
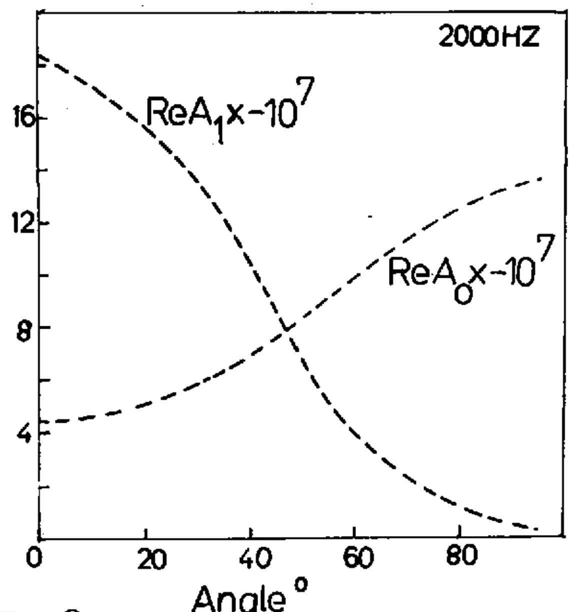
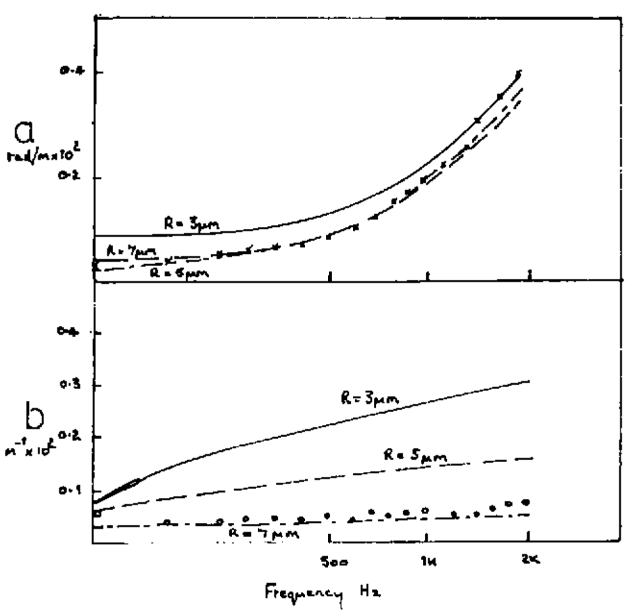


Fig. 2



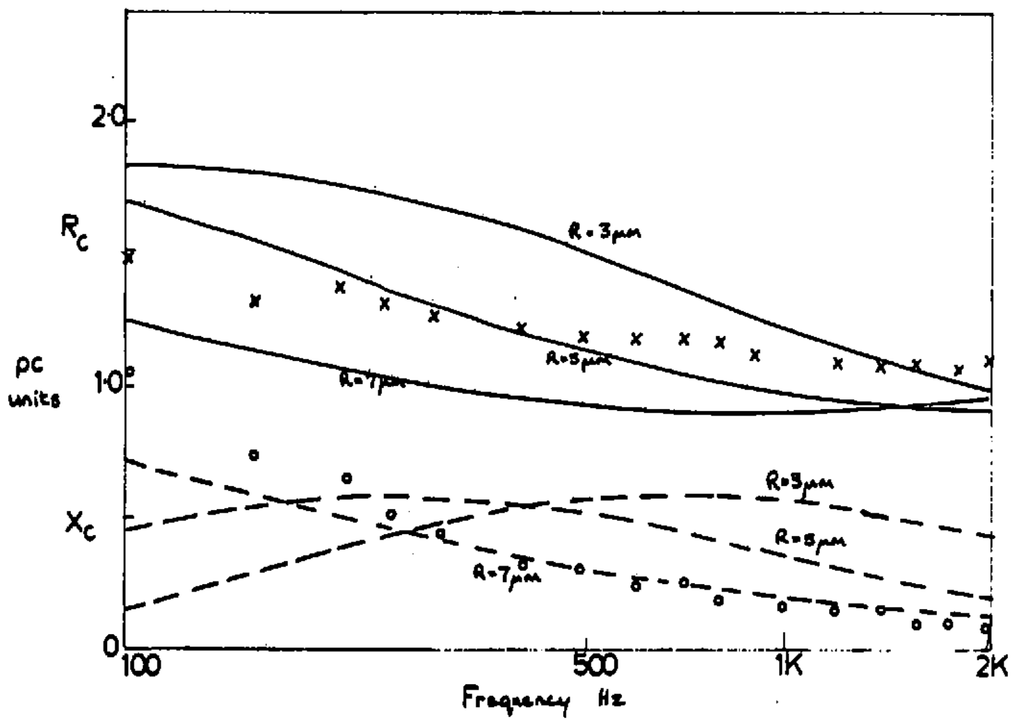
2000HZ
ReA1x-10^7
ReA0x-10^7
16
12
8
4
0
20
40
60
80
Angle °

Fig. 3



a
rad/m x 10^2
0.4
0.2
R = 3μm
R = 7μm
R = 5μm
b
m^-1 x 10^2
0.4
0.3
0.2
0.1
R = 3μm
R = 5μm
R = 7μm
500
1K
2K
Frequency Hz

Fig. 4



2·0
1·0
0
R_c
ρc units
X_c
R = 3μm
R = 5μm
R = 7μm
R = 3μm
R = 5μm
R = 7μm
100
500
1K
2K
Frequency Hz