THE ELUCIDATION OF ANCIENT GREEK HARMONIA MODE – HETERODYNE RESONANCE

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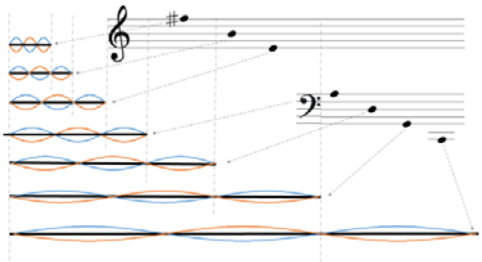
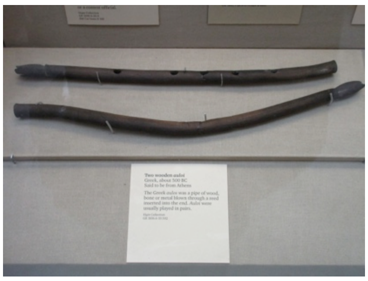
ABSTRACT

The ancient acoustic structures of “enharmonic” based on quarter tones and micro intervals are theoretically and mathematically modelled on the basis of heterodyne resonance and linear and non-linear resonances.

1. Ancient Harmonia Modes with Architecture of Micro Tones

It is known that “Music” was one of the essential arts for liberal citizens in ancient Greek Police Societies [?]. Music Mode Theories in those days are generally consisted by two systems [?]: 1. Considering number ratio of Pythagoras school important, 2. Considering the “midpoint” (i.e. Difference according to Aristotle/Aristoxenos) important. Pythagoras Tuning centered on Perfect 5th. seems to be rational in contemporary view, but it is impossible to realize harmonic tones on A STRING, and problematic on tuning (Fig.1).





LEFT: **Figure.1.** Pythagoras Tuning is hard to be harmonic on A STRING.
RIGHT: **Figure.2.** “Aulos”, B.C. 4-5 Century (British Musium).

We also show “Aulos” in Fig.2, as a wind instrument of Greece in B.C. 4-5 Century. Let’s note that it is “midpoint” taken by empirically with Aristotle=Aristoxenos concepts, rather than “harmonic ratios”, to drill the finger holes into the hollow tube of a tree. In ancient Greece, tunings are defined on length ratio of string (The idea “Frequency” had appeared after 19 C. with establishment of Fourier Analysis). The “length ratio” tuning is possible for only “octave”, “4th”, “5th”, and “2nd” between 4th and 5th. For more detailed tones, those are tuned basically by “ears”, and the discussion between Pythagoras-like numeric ratios and Aristoxenos-like differences was controversial for long years.

To show the geometrical relationships among “octave”, “4th”and “5th”, we reconstructed “HELIKON” [3] (Fig.3, 4)

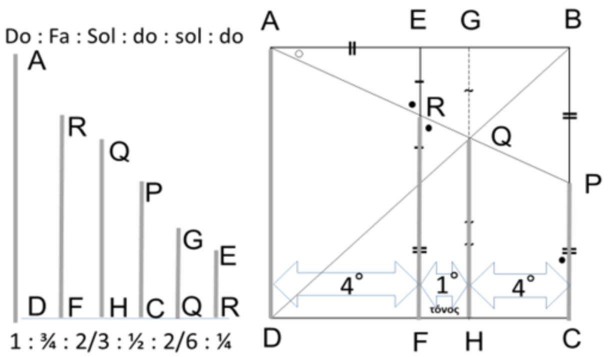


Figure.3. "Helikon" by Ptolemy, **Figure.4.** Reconstructed "Helikon"

Plotting like Fig.3, we can get both Perfect 4th, one is for the ratio between line AD and RF (e.g. "do" and "fa"), the another for the ratio between line QH and PC (e.g. "sol" and "do (octave above)"). The difference between 4th and 5th constructed above is defined as "one tone"— Tonos, and this "one tone" is used to divide the 4th. This structure for 4th is called as "Tetrachord". The Tonos has 8:9 ratio in frequency by simple calculation from contemporary view.

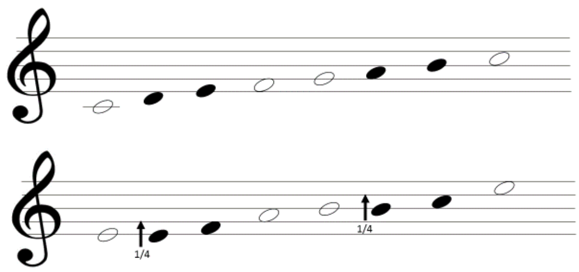


Figure.5. Tetrachords and an example of harmonia mode (Dorian)

The Upper side of Fig.5 shows a example of Aristoxenos-like Tetrachord. It places Tonos on "do" --- that is "re", a Tonos placed on "re" --- that is "mi", a "Half Tone" placed on "mi" --- that is "fa". The Half Tone is considered as "a half of Tonos" qualitatively. It is obvious by simple calculation, this residual is NOT the "HALF" of "Tonos" both rationally and differentially. Therefore, Aristoxenos had been criticized later by Neo-Platonism school, etc. However it is clear by the example of "Aulos" pipe above, in musical practice, the framework of Aritsoxenos is very effective and it has influenced long time.

2. Harmonia Tuning And Quarter Tone

These "Tetrachord" structure, i.e. using 4 strings for Perfect 4th are the "Harmonia Modes" widely spread in ancient Greek world, which divide the half tone divided more (It seems to be from an idea of "Moderation" of Aristotle).

For example, let's try to make a Harmonia Mode built in Tetrachord structure like "Mi-La/Si-Mi". The tone between "Mi" and "Fa" is a half tone in this context above, let's insert a moderate string "Mi↑" between Mi and Fa. It is also the half-tone relationship for "Si" and "Do", let's insert a moderate string "Si↑". The resulting mode is shown as lower side of Fig.5.

These tones construct the several kinds of "Harmonia Modes" with specific sound behaviour = "Mode".

Through the vast literature research, Western Classical Philology interpreted that such temperament of modes with “quarter tones” is "Harmonia" = octave relationship. But the real sound effects of those are not shared well among the Western Classical Philologists.

In general, we can observe “beat” when we sound the proximate pitch strings at the same time. This is a physical sound waves interference, if we suppose the vibration of string as a sine wave, it is able to indicate the mechanism with a simple calculation. It is not only the case for a proximate waves to observe the beat between them as physical entity vibration of difference between two frequencies. But also in several Engineering fields, mixing of two waves has been widely applied in such Heterodyne amplification. According to the ancient Greek scale above, it is 8:9 ratio in basic frequencies corresponding to two sounds as “Tonos”= whole tone, at the same time, then the base tone can sound corresponding to 1/8, i.e. 3 octaves below. In the same fashion, the differential fundamentals on diatonic scale are shown below. We can find most of constituting sounds for diatonic scale is “differential-harmonic”.

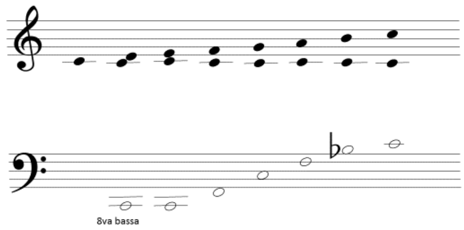


Figure.6. Differential fundamentals on diatonic scale

In the same fashion, by defining two tones which are half-tone or more narrow-pitch tone, it can sound the base tone like 1/16 to the original tone (4 octaves below), 1/32 (5 octaves below), or 1/64 (6 octaves below), at the same time.

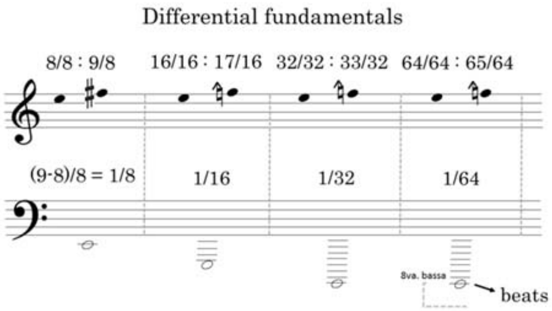


Figure.7. Differential fundamentals in Harmonia Mode [1]

What is the so-called “Discovery of Harmonia Modes” in ancient Greek world does not described well in detail in the literatures. But we can easily check the base tone by micro-tone tunings, with a simple calculation and reconstruction of the ancient instruments. The acoustic sounds of ancient Greek Harmonia Modes can be found as the use of the base tone compliant to Heterodyne phenomenon. For example, Dorian mode described by Aristides Quintilianus in A.D. 3 C., and Phrigian mode also described by him are shown below. Those 2 modes are from 6 modes described in Plato’s “The Republic” in B.C. 4 C. [1].

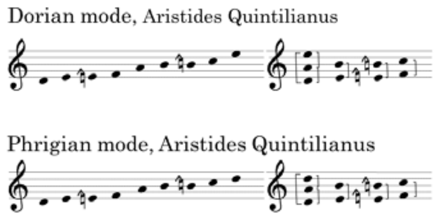


Figure.8. Examples of Harmonia Modes [1]

3. Mathematical Model with sine waves: Heterodyne effects

Let us examine the “difference tone” characteristics in the fundamental of Harmonia modes, by mathematical model based on sine waves. This is the approximation of physical sound phenomena.

Suppose there are two sine waves s_1 and s_2, corresponding to different frequencies f_1 and f_2 ($f_1 > f_2$), where the amplitude is the same A, and each initial phases for both s_1 and s_2 are φ_1 and φ_2, t is time parameter.

$$s_1 = A \cdot sin(2\pi f_1 t + \varphi_1) \ \dots \ (1)$$

$$s_2 = A \cdot sin(2\pi f_2 t + \varphi_2) \ \dots \ (2)$$

Let us consider about the SUM of sine waves $s_1 + s_2$. Suppose $\varphi_1 = \varphi_2 = 0$, $2\pi f_1 t = \alpha$, $2\pi f_2 t = \beta$ to ease the problem, we can find the well-known formula,

$$s_1 + s_2 = A \cdot \{\sin\alpha + \sin\beta\} = 2A \cdot \cos\frac{\alpha - \beta}{2} \cdot cos\frac{\alpha + \beta}{2} \ \dots \ (3)$$

Thus, we can get the other two sine waves generated by sinusoidal synthesis,

$$\frac{\alpha - \beta}{2} = 2\pi\frac{(f_1 - f_2)}{2}t \ \dots (4), \qquad \frac{\alpha + \beta}{2} = 2\pi\frac{(f_1 + f_2)}{2}t \dots (5)$$

for frequencies $\Delta_1 = \frac{f_1 - f_2}{2}$ and $\Delta_2 = \frac{f_1 + f_2}{2}$. Heterodyne Modulation is using this mechanism to modulate the signal wave. And this Δ_1 is exactly the base tone in Harmonia tuning. Actually, we can hear not the sound waves themselves but the work of them, we have to square the both sides, to consider those phenomena.

$$\{\sin\alpha + \sin\beta\}^2 = \sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta \ \dots (6)$$

4. Reconstruction of ancient instruments and difference base tone

Ancient Greek strings harp “Kithara” have been transmitted from B.C. 5-6 century to contemporary, and its resonance body is made of a shell of a turtle. It seems there was a mythical, magical or divine revelation sense. These “Kitharas” have over four strings, and tuned with several tetrachords, as estimated. Reconstructing the instrument like “Kithara” by contemporary resources, we can reproduce the difference base tone by Heterodyne effects.

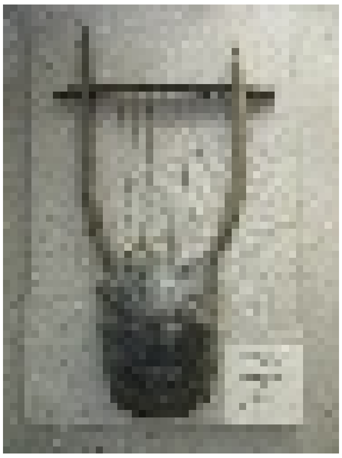


Figure.9. Kithara (British Museum)

Figure.10. Reconstructed Kithara with contemporary material

Figure.11. Reconstructed Lyre Especially for base notes

However it is difficult to examine quantitatively for the real sound instrument with strings for vibration and resonance for a wood-box or a turtle shell. Thus, we made a choice to observe the differential base tone by the sound source with generated sine waves by computer and amplified the sound by Acrylic cavity. For these, we will report later.

5. Fractional harmonic micro-intervals

Ptolemy had introduced various “chromatic” harmonic modes totally different from present system of well-tempered chromaticism (Fig.12).

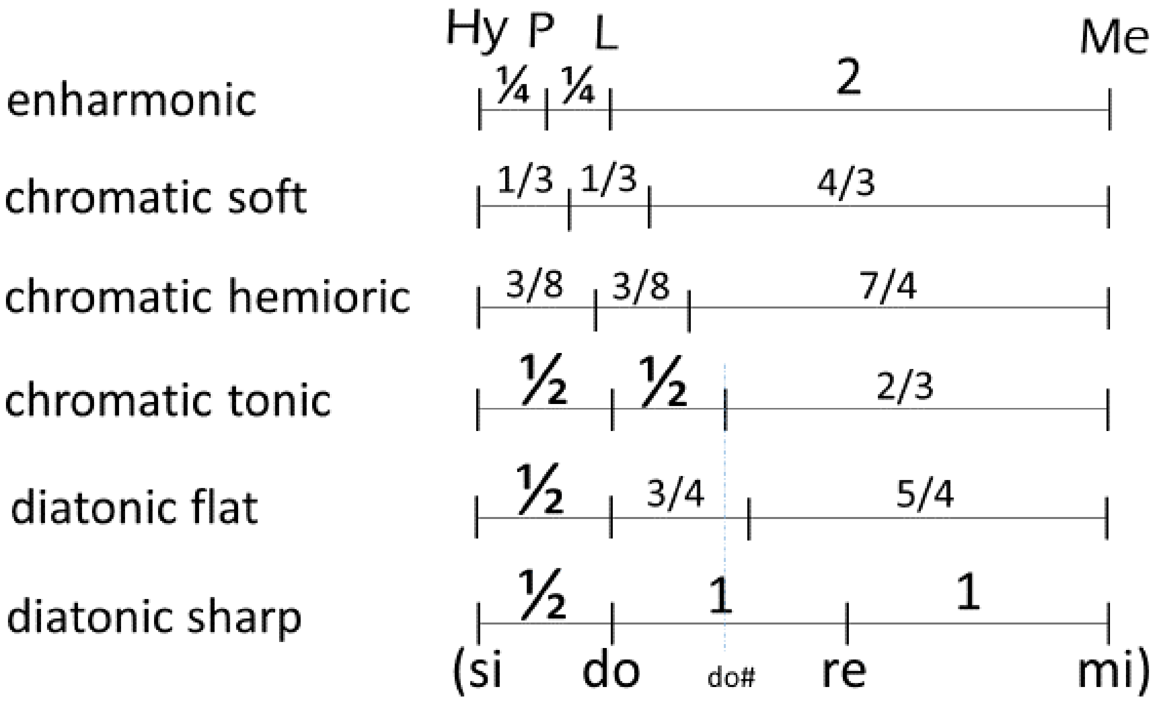


Figure.8. Various “chromatic” modes of classical enharmonic system [2]

There is no detailed explanation about the reality of those tunings and modes. It is apparent that musicians those days had thought over them with analogies from geometry, and they are totally false from contemporary point of view.

However, they had kept this tradition quite long time --- more than 1000 years and it values several times longer than modern well-tempered system.

From this fact there could be some physical reality of acoustic resonance about these fractional micro intervals of ancient enharmonic-chromatic tuning systems.

Now try to analyse the case of 1/3 tone. Tonos = 1 full tone is described, from modern point of view, as a frequency ratio in 8:9 = 1.125 times frequent as the original fundamental tone would be listened as " 1 full note higher in pitch".

Simultaneously, a half tone would be defined, though in several different ways, but now dare roughly sum up as frequency ratio in 16:17 =1.0626.

If we multiply this (half tone ratio) 17/16 two times, it would yield

$$17\times 17 / 16\times 16 = 289/256 = 1.128 \fallingdotseq 1.125 = \text{tonos}$$

and ancient people would understand from theoretical harmonia's point of view they are "rational"; in deed those ancient musicians have the final judgement for the correctness in their auditory senses and this filled the gap between the "theory" of those days and the real musical acoustics.

Here in those ratio in 8:9 and 16:17, differential harmonics would be heard.

$$8:9 \mid \Delta = 1/8 \rightarrow \text{3 octaves lower heterodyne resonance}$$
$$16:17 \mid \Delta = 1/16 \rightarrow \text{4 octaves lower heterodyne resonance}$$

Now try to define a micro interval ratio as 12:13.

$$12:13 \mid \Delta = 1/12 = 1/3 \times 1/4 \rightarrow \text{3 octaves \& 5}^{\text{th}}\text{ lower heterodyne resonance}$$

The value of the ratio 13/12 = 1.0833. $13\times 13/12\times 12 = 169/144 = 1.1736\ldots$. The former is higher than "semi tone" and lower than "full tone", and the latter is higher than a full tone. This interval could be resonate well in possible physical condition and was able to be interpreted as 3/4 tone or other possible fractional micro interval.

Thinking over the problem similarly, we can obtain

$$24:25 \mid \Delta = 1/24 = 1/3 \times 1/8 \rightarrow \text{4 octaves \& 5}^{\text{th}}\text{ lower heterodyne resonance}$$

If we difine $25/24 = 1.0416\ldots \equiv \phi$,

$$\phi \times \phi = 1.0850\ldots \text{ and}$$

$$\phi \times \phi \times \phi = 1.1302\ldots \fallingdotseq 1.125 = \text{tonos}$$

Thus, this interval ϕ could be interpreted as "1/3 tone" for the "chromatic soft" mode in ancient harmonic tuning system. We can also get

$$18:19 \mid \Delta = 1/18 = 1/3\times 1/3\times 1/2 \rightarrow \text{4 octaves \& 2}^{\text{th}}\text{ lower heterodyne resonance}$$

If we difine $25/24 = 1.05555\ldots \equiv \Psi$,

$$\Psi \times \Psi = 1.1141\ldots < 1.125 = \text{tonos} \quad \text{and}$$

$$\Psi \times \Psi \times \Psi = 1.1302\ldots > 1.125 = \text{tonos}$$

Thus Ψ could be interpreted as 3/8 fractional ancient micro interval in chromatic hemioric mode.

All these are hypothetical and from historian's point of view there is no way to proof whether they are true or not.
However, we can perform computational research for these micro intervals and also make comparative study to those existing modes in middle eastern area between Balkans and India; namely Turkey, Syria, Iran, Kurdish, Iraq, Kirghiz, Uzbekistan, Azerbaijan, Afghanistan, Pakistan and any other culture between west and east. All of them could be offspring both of Hellenism and Byzantine, as long as Muslim, and we can find hereditary traces of those ancient Greek Harmonia and Vhromatism.
Quite sorry to point out, those areas above are almost all in the midst of conflict. Since, we would like to make a musical bridge between those from the common birth place of ancient Harmonia

6. Concluding Remarks

To examine these phenomenon above, we had selected the conditions below. For micro-difference tone, it is hard to tune precisely, we decided to generate it by PC digitally. And for its resonance, we decided to use a Cavity since it is easier to input the test sound source and record its output. It results a difference base tone and its harmonics, and the resulting graph is as below. We are continuing such experiment carefully and this is a working report in the midst of it.

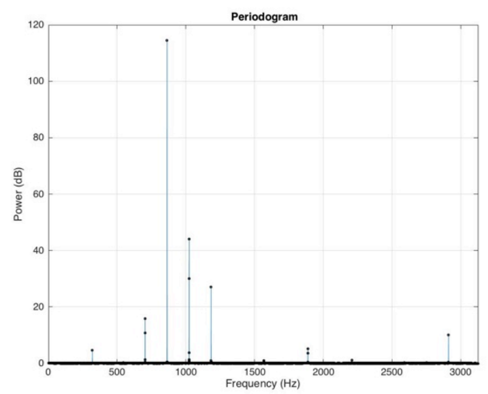


Figure.12.

Figure.13.

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