24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

A STUDY ON VIBRATION TRANSMISSION CHARACTERISTIC OF AN AUDIO-BASE BY USING FOUR-TERMINAL CONSTANTS OF ELECTRIC NETWORK

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Many kinds of audio-bases are used to adjust or modify the reproduced music sound of stereophonic equipment by just propping the equipment on them. Their mechanism how to modify the sound of stereophonic equipment was not clarified in spite of popular usage. In this study, to clarify the working mechanism of audio-base theoretically, we derive an analytical expression of vibration transmission characteristics. The relationship between input and output is described by four-terminal constants by introducing four-terminal network model for input-output of audio-base. The four-terminal constants are easily given from solutions of wave equation solved under two extreme boundary conditions at end, that is “open” and “close” corresponding with infinite and zero impedances. The factors on “form” and “material of the audio-base” are reflected in the four-terminal constants as “flair constant”, “cross-section”, “Young’s modulus” and “wave velocity” used in the wave equation. Here, to evaluate the transmission characteristic of vibration energy from loaded equipment into audio-base, a driving impedance at input is most convenient and it is easily evaluated by using the four-terminal constants with following impedance of audio-base. Finally, we carried out numerical evaluation of driving impedance of audio-bases formed into exponential and conical horn and compared with experimental results. In conclusion, we could find out an alternating frequency point of vibration transmission characteristic of audio-base.

Keywords: audio-base, vibration transmission, four-terminal network, four-terminal constants

1. Introduction

Stereophonic equipment is used to realize a past stage performance in present place beyond time and space by precise representation of not only music or performance sound but also the high order acoustic and sensational information on surrounding environment [1-3]. For real representation, the stereophonic equipment is required to furnish an ability to represent the minute acoustic state of recording cite like as reverberation of hall, sound besides music, breathing of player *etc*. Though, the music signal has enough signal level to represent precisely against induced noise in stereophonic equipment, this very faint sound information at recording cite is too minor to represent precisely against the noise. Most origins of this noise are related with electrical causes but some origins seem to be related with mechanical vibration. When the vibration is induced on the equipment and it becomes an origin of noise induced by electro-magnetic effect, vibration suppression become an effective method to reduce the noise.

Usually, many kinds of audio-bases are used to adjust or modify the reproduced music sound of stereophonic equipment just putting the equipment on them. Considering this usage, their working should be limited in mechanical action just changing the vibration state of loaded equipment. In actually, stereophonic equipment has built-in power transformer and/or CD driver as vibration origin and furthermore airborne sound own reproduced music sound will be vibration origin. Then, the audio-bases are excited by loaded equipment on their top surfaces, and the vibration transmits to another end. This vibration transmission characteristic seems important to reproduced sound quality. In previous studies [4,5], we clarified experimentally that the audio-base decreases vibration on the stereophonic equipment and as a result, it reduces the noise included in output signal.

In this study, we derive vibration transmission characteristic of audio-base by using four-terminal constants common in electric network analysis [6]. To compare the difference in effect of audio-bases related with their form, we employed 3 types of audio-bases formed into exponential horn, conical horn and cylindrical ones. To evaluate the vibration transmission characteristics depends on the driving point "throat" or "mouth" of audio-base, we evaluated numerically the driving impedance by using the four-terminal constants under an assumption with following impedance made by same material. Furthermore, it is convenient to evaluate the four-terminal constants for multiplex cascade connection of elements. That is just given as products of each fundamental matrix consist of four-terminal constants. On the other hand, based on electric circuit theory, the amount of transmitted vibration energy from oscillating source to loaded impedance can be evaluated by ratio of loaded one to internal one of source [6]. Consequently, comparing the driving impedances between driving points at "throat" and "mouth", we could find an alternate frequency on amount of transmitted vibration energy related with driving impedance depends on exciting point. This alternate frequency in vibration transmission is correspond with experimental result of difference in vibration velocity between input and output of audio-base which illustrate vibration reduction level.

2. Four-terminal network expression of audio-base

An audio-base has "throat" and "mouth" as input or output. Usually, the "throat" is set on the floor and the "mouth" is attached to loaded equipment. Considering the previous results [4,5], that the audio-base decreases vibration on the loaded stereophonic equipment, in this study, to clarify the work of audio-base, we carried out theoretical analysis of vibration transmission characteristic by using four-terminal electric circuit model. The forced longitudinal vibration at input causes stress and oscillatory velocity in an audio-base and they transmit to another end of audio-base. Let an audio-base be excited by input vibration P_i, v_i and cause vibration P_t, v_t at output, the audio-base can be illustrated by the four-terminal constants as shown in Fig. 1 with internal impedance Z_a and following load impedance Z_L.

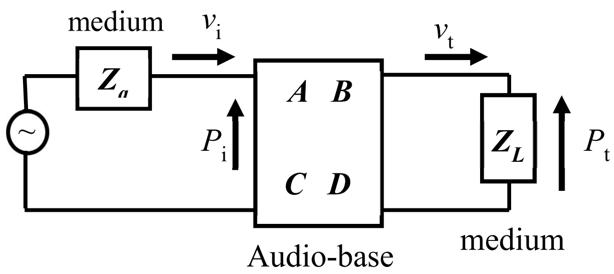


Figure 1: Four-terminal electrical network expression for input and output of an audio-base.

Here, let the input and output of audio-base at x=0 and x=h respectively. Then, based on the mechano-electric transformation, we can describe the relationship between input and output by a four-terminal electrical network model as follows;

$$P(0) = A\,P(h) + B\,v(h)\,, \tag{1}$$

$$v(0) = C\,P(h) + D\,v(h)\,, \tag{2}$$

where $P(0)$ and $v(0)$ are the stress and velocity of vibration at input and $P(h)$ and $v(h)$ are those at output with four-terminal constants A, B, C and D. They are defined as

$$A = P(0)/P(h)|_{v(h)=0}, \qquad C = v(0)/P(h)|_{v(h)=0}, \tag{3-1}$$

$$B = P(0)/v(h)|_{P(h)=0}, \qquad D = v(0)/v(h)|_{P(h)=0}. \tag{3-2}$$

Here, $v(h)$=0 means the "fixed" condition with infinity impedance and $P(h)$=0 means the "free" condition at the output. When the audio-base is followed by impedance Z_L at output, the driving impedance is given as

$$Z_0 = \frac{P(0)}{v(0)} = \frac{A\,Z_L + B}{C\,Z_L + D}\,. \tag{4}$$

2.1 Vibration transmission characteristic of audio-base formed into exponential horn

First, we treat an exponential horn formed audio-base. Detailed analysis of this horn was developed in physics of instruments or speaker system as shown in several literatures [7-8] as acoustical transmission characteristics in a horn. As the audio-base is made by solid matter like as metal (brass, iron, magnesium *et.*), wood and polymeric materials and the height of audio-base h is very short comparing with wave length in audible frequency, we can suppose the wave travel in the audio-base is plane wave and travel along the axis of the horn. When a horn is strained, elastic forces are produced. Considering the balance of stress and inertia at cross section $S(x)$, we have equation of motion as following one-dimensional wave equation,

$$1/S(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(S(x)\frac{\mathrm{dP}}{\mathrm{dx}}\right)=1/c^2\,\frac{\mathrm{d^2P}}{\mathrm{dt^2}}, \tag{5}$$

with a velocity $c =\sqrt{E/\rho}$, where E is Young's modulus and ρ is density. The cross section of the horn at x becomes as

$$S(x) = S_0 \exp(-mx)\,. \tag{6}$$

Where S_0 is cross section at x=0 and m is flare constant. By substituting Eq. (6) into Eq. (5), we have

$$\frac{\mathrm{d^2P}}{\mathrm{dx^2}} + m\,\frac{\mathrm{dP}}{\mathrm{dx}} = 1/c^2\frac{\mathrm{d^2P}}{\mathrm{dt^2}}. \tag{7}$$

2.1.1 Four-terminal constants in normal direction

When "throat" is excited, we had well known solution as

$$P(x) = \exp(-\tfrac{m}{2}x)\{A_1 \exp(\tfrac{m}{2}\alpha x)+B_1 \exp(-\tfrac{m}{2}\alpha x)\}\,. \tag{8}$$

Here, $\alpha = \sqrt{1-\left(\frac{2k}{m}\right)^2}/2$. The velocity is given as

$$v(x) = \frac{m}{j2k\rho c}\exp(-\frac{m}{2}x)\{\, A_1\,(1+\alpha)\exp(\frac{m}{2}\alpha x)+B_1\,(1-\alpha\,)\exp(-\frac{m}{2}\alpha x)\,\}. \tag{9}$$

Under sinusoidal excitation at $x = 0$ with amplitude v_0, the boundary condition becomes as

$$v(0) = v_0 \exp(j\omega t). \tag{10}$$

Other boundary conditions at the end of audio-base are given as 1) $v(h) = 0$ and 2) $P(h) = 0$. By using these boundary conditions, the constants A_1 and B_1 are decided as follows:

1) Under the condition $v(h) = 0$, we have

$$v(h) = \frac{m}{j2k\rho c}\exp(-\frac{m}{2}h)\{\, A_1\,(1+\alpha)\exp(\frac{m}{2}\alpha h)+B_1\,(1-\alpha)\exp(-\frac{m}{2}\alpha h)\,\}= 0. \tag{11}$$

By solving Eqs. (10) and (11), we have

$$A_e = \exp(\frac{m}{2}h)\{(1+\alpha)\exp(-\frac{m}{2}\alpha h) - (1-\alpha)\exp(\frac{m}{2}\alpha h)\,\}/2\alpha, \tag{12}$$

$$C_e = \exp(\frac{m}{2}h)\,\frac{m}{j2k\rho c}\,(1-\alpha^2)\,\{\exp(-\frac{m}{2}\alpha h) - \exp(\frac{m}{2}\alpha h)\,\}/2\alpha\,. \tag{13}$$

2) Under another boundary condition $P(h) = 0$, we have

$$P(h) = \exp(-\frac{m}{2}x)\{A_{12}\exp(\frac{m}{2}\sqrt{1-(\frac{2k}{m})^2}\,h)+B_{12}\exp(-\frac{m}{2}\sqrt{1-(\frac{2k}{m})^2}h)\} = 0\,. \tag{14}$$

By solving Eqs. (10) and (14), we have

$$B_e = \exp(\frac{m}{2}h)\,(j2k\rho c\,/\,m\,)\,\{-\exp(-\frac{m}{2}\alpha h) + \exp(\frac{m}{2}\alpha h)\,\}/2\alpha, \tag{15}$$

$$D_e = \exp(\frac{m}{2}h)\{(1+\alpha)\exp(\frac{m}{2}\alpha h) - (1-\alpha)\exp(-\frac{m}{2}\alpha h)\,\}/2\alpha\,. \tag{16}$$

Then we have the fundamental matrix F_e of four-terminal constant as

$$\mathbf{F}_e = \begin{bmatrix} A_e & B_e \\ C_e & D_e \end{bmatrix}. \tag{17}$$

2.1.2 Four-terminal constants in inverse direction

The four-terminal constants for inverse direction from “mouth” to “throat” of audio-base can be easily given as inverse matrix of F_e. Then, we have the four-terminal constants as

$$A_e' = \exp(-\frac{m}{2}h)\{(1+\alpha)\exp(\frac{m}{2}\alpha h) - (1-\alpha)\exp(-\frac{m}{2}\alpha h)\,\}/2\alpha, \tag{18}$$

$$B_e' = \exp(-\frac{m}{2}h)\,(j2k\rho c\,/\,m\,)\,\{-\exp(-\frac{m}{2}\alpha h) + \exp(\frac{m}{2}\alpha h)\,\}/2\alpha, \tag{19}$$

$$C_e' = \exp(-\frac{m}{2}h)\,(m/j2k\rho c\,)\,(1-\alpha^2)\,\{\exp(-\frac{m}{2}\alpha h) - \exp(\frac{m}{2}\alpha h)\,\}/2\alpha, \tag{20}$$

$$D_e' = \exp(-\frac{m}{2}h)\{(1+\alpha)\exp(-\frac{m}{2}\alpha h) - (1-\alpha)\exp(\frac{m}{2}\alpha h)\,\}/2\alpha\,. \tag{21}$$

2.2 Vibration transmission characteristic for conical horn formed audio-base

A conical horn formed audio-base is most popular one. Let “throat” at $x=a$ and “mouth” at $x=h$, then its cross section at x is given by

$$S(x) = \pi\,(m_c\, x)^{2}\ , \tag{22}$$

where $m_c=R/h$, with radius R at $x=h$ and the height of horn h-a. By substituting Eq. (22) into Eq. (5), we have

$$\frac{d^2P}{dx^2} + 2/x\,\frac{dP}{dx} = 1/c^2\,\frac{d^2P}{dt}\ . \tag{23}$$

2.2.1 Four-terminal constants in normal direction

When “throat” is excited with sinusoidal oscillatory input, we have well known solution as

$$P(x) = \frac{A_1}{x}\exp(jkx)+\frac{B_1}{x}\exp(-jkx)\}\ , \tag{24}$$

and

$$v(x) = \frac{1}{j\omega\rho}\frac{1}{x^2}\{\,A_1\,(1+jkx)\exp(-jkx)+B_1\,(1-jkx)\exp(jkx)\,\}. \tag{25}$$

At exciting point $x = a$, the boundary condition becomes as

$$v(a) = v_0\exp(j\omega t). \tag{26}$$

Other boundary conditions at the end of audio-base are given as 1) $v(h) = 0$ and 2) $P(h) = 0$. By using these boundary conditions, the constants A_1 and B_1 are decided as follows:

1) Under the condition $v(h) = 0$, we have the velocity as

$$v(h) = \frac{1}{j\omega\rho}\frac{1}{h^2}\{\,A_1\,(1+jkh)\exp(-jkh)+B_1\,(1-jkh)\exp(jkh)\,\}= 0. \tag{27}$$

By solving Eqs. (26) and (27), we have

$$A_c = -\frac{h}{a}\,\{\,(1-jkh)\exp(jk(h-a))\ -\ (1+jkh)\exp(-jk(h-a))\,\}\ /2jkh, \tag{28}$$

$$C_c = -\frac{h}{a^2}\frac{1}{j\omega\rho}\{\,(1-jkh)\,(1+jka)\exp(jk(h-a)) - (1+jkh)\,(1-jka)\exp(-jk(h-a))\,\}/2jkh. \tag{29}$$

2) Under another boundary condition $P(h) = 0$, we have the stress as

$$P(h) = \frac{1}{h}\,\{\,A_1\ \exp(-jkh)+B_1\exp(jkh)\,\} =\ 0\ . \tag{30}$$

By solving Eqs. (26) and (30), we have

$$B_c = \frac{h^2}{a}\,(j\omega\rho)\{\,\exp(jk(h-a))\ -\ \exp(-jk(h-a))\,\}\ /2jkh, \tag{31}$$

$$D_c = \frac{h^2}{a^2}\,\{\,(1+jkh)\exp(jk(h-a))\ -\ (1-jkh)\exp(-jk(h-a))\,\}\ /2jkh\ . \tag{32}$$

2.2.2 Four-terminal constants in inverse direction

The four-terminal constants for inverse direction of conical horn formed audio-base can be easily given as inverse matrix of $\boldsymbol{F}_c$. Then, we have the four-terminal constants as

$$A_c' = \frac{a}{h}\,\{\,(1+jka)\exp(jk(h-a)) \;-\; (1-jka)\exp(-jk(h-a))\,\}\,/2jka, \tag{33}$$

$$B_c' = \frac{a^2}{h}\,(j\omega\rho)\{\,\exp(jk(h-a)) \;-\; \exp(-jk(h-a))\,\}\,/2jka, \tag{34}$$

$$C_c' = -\frac{a}{h^2}\,(1/j\omega\rho)\{\,(1-jkh)(1+jka)\exp(jk(h-a)) - (1+jkh)(1-jka)\exp(-jk(h-a))\,\}/2jka, \tag{35}$$

$$D_c' = -\frac{a^2}{h^2}\,\{\,(1-jkh)\exp(jk(h-a)) \;-\; (1+jkh)\exp(-jk(h-a))\,\}\,/2jka. \tag{36}$$

2.3 Vibration transmission characteristic for cylindrically formed audio-base

The four-terminal constants for cylindrical audio-base with height h are given using the results for finite cylindrical pipe as

$$\boldsymbol{F}_{cyl} = \begin{bmatrix} A_{cyl} & B_{cyl} \\ C_{cyl} & D_{cyl} \end{bmatrix} = \begin{bmatrix} \cos kh & jZ_s \sin kh \\ j\frac{1}{Z_s}\sin kh & \cos kh \end{bmatrix}. \tag{37}$$

2.4 Four-terminal constants for cascade connection of audio-bases

By using the four-terminal constants, the matrix $\boldsymbol{F}_{dc}$ for double or more staged horn audio-base is easily derived by the products of matrices $\boldsymbol{F}$s. For example, the matrix $\boldsymbol{F}_{dc}$ for double staged $\boldsymbol{F}$s is given as

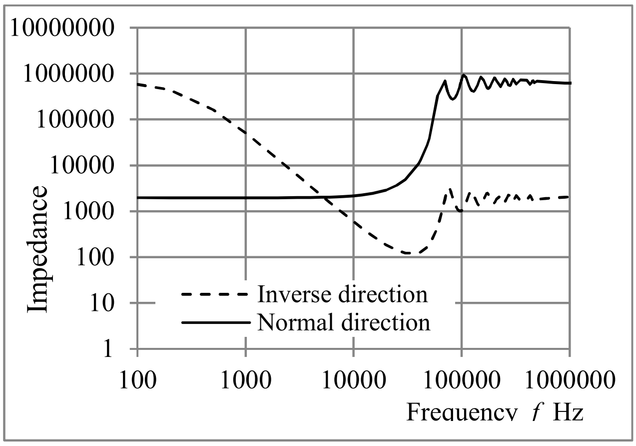
$$\boldsymbol{F}_{dc} = \begin{bmatrix} A_{dc} & B_{dc} \\ C_{dc} & D_{dc} \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix}\begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} = \boldsymbol{FF}. \tag{38}$$

Table 1: Parameters of exponential and conical horn

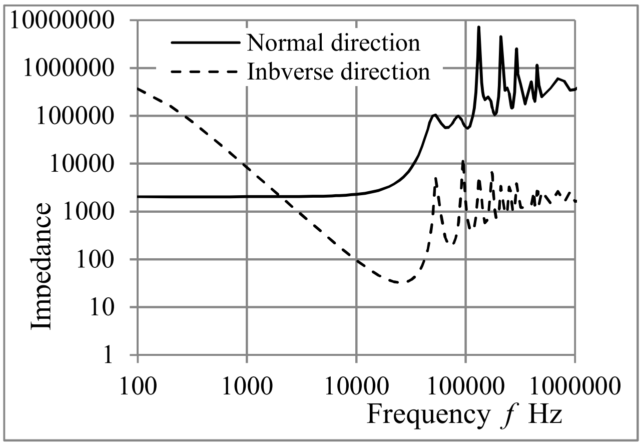
Type	Exponential	conical
material	Nylon	Nylon
density ρ kg/m^3	1110	1110
velocity c m/s	3000	3000
flare constant m	200	0.36
radius of throat r_0 m	0.0007	0.0007
height h m	0.029	0.034

3. Numerical evaluation of vibration transmission characteristics

Next, we carried out a numerical calculation of driving impedance at input of audio-bases formed into exponential and conical horns by using Eq. (4) with fundamental matrix $\boldsymbol{F}$ for each type audio-base. Numerical values and parameters used in calculation are shown in Table 1. Numerical results are shown in Fig. 2 (a) for exponential and (b) for conical ones. In these figures, impedances in "normal" and "inverse" directions alternate each other at specific frequency points at 6000Hz for exponential one and at 2000Hz for conical one. Then, in higher frequency than these alternating points, the vibration energy of loaded equipment is well transmitted to audio-base and decrease the vibration energy on the equipment according to the impedance matching in electric circuit theory [6].



a) Exponential horn type



b) Conical horn type

Figure 2: Comparisons of impedance at exciting point of audio-bases formed into exponential and conical horns made by Nylon.

4. Measurement of vibration transmission characteristics of audio-bases

Next, an experiment was carried out to evaluate the vibration transmission characteristics between input and output for audio-bases formed into exponential and conical horns. As shown in Fig. 3, the bottom (throat side) or top (mouth side) surface was excited by a piezo exciting unit with several frequencies of sinusoidal signal and vibration velocity amplitudes on top and bottom surfaces were measured by laser Doppler vibrographs (Onosokki, LV-1610 and LV-1720) and an FFT analyser (Onosokki, CF5220). As shown in Fig.4, we defined the transmission direction as “normal” and “inverse”. Vibration transmission characteristics were evaluated by subtracting the vibration level on transmitted surface from that of exciting surface.

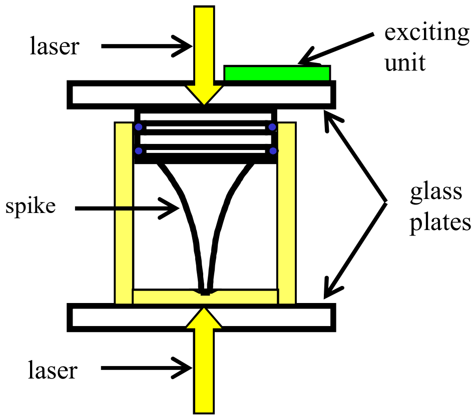


Figure 3: Measuring method of vibration on input and output surfaces of audio-base.

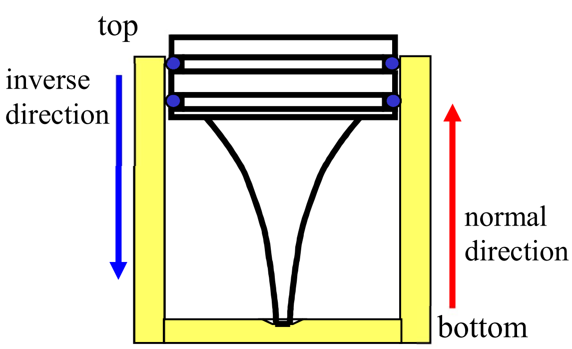
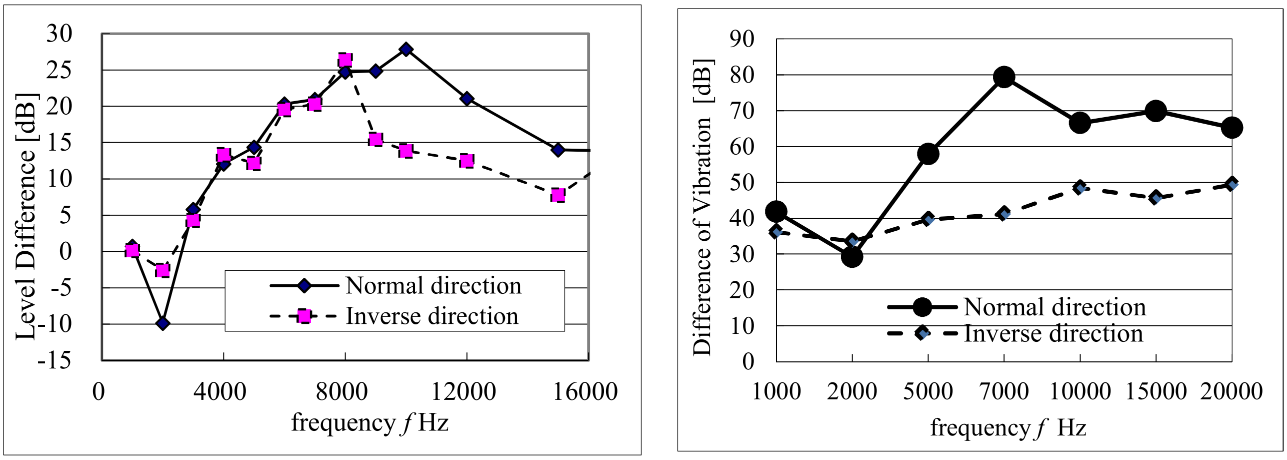


Figure 4: Structure of exponential horn audio-base and direction of vibration transmission.

Figure 5 shows the experimentally observed values of difference in vibration velocity amplitude level between input and output surfaces under transmission directions “normal” and “inverse”. Figure 5 (a) shows the result for exponential horn formed audio-base and (b) for conical one made by Nylon. In these figures, the less difference in vibration level between input and output means well vibration transmission of audio-base. The alternate frequencies are about 8000Hz for exponential one and 2000Hz for conical one. These results roughly correspond with numerical results given in Fig. 2.



a) exponential horn form b) conical horn form

Figure 5: Vibration level difference between top and bottom surface of exponentially and conically formed audio-bases made by Nylon.

5. Conclusions

In this study, we derived four-terminal constants for audio-base to evaluate how the audio-base works to a loaded equipment. The four-terminal constants were derived for several types of audio-bases based on the solutions of wave equation in acoustic horn developed precisely in physics of musical instruments. Based on the numerical results on driving impedance at input of audio-bases, the transmitted vibration energy into audio-base is increase in higher frequency region than alternate frequency point in “inverse” direction. This fact suggests that the audio-base decreases the vibration caused on loaded equipment in higher frequency region than alternate point. And this fact also corresponded to experimental results on vibration transmission characteristics for two kinds of audio-bases.

Finally, we would like to express our cordial thanks to Mr. Kenji Manpuku for his valuable discussion and support.

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