inter•noise 83

A PROBABILITY EVALUATION METHOD FOR THE OUTPUT FLUCTUATION OF NOISE INSULATION SYSTEM IN CONNECTION WITH THE INSTRUMENT CHARACTER AND INPUT FREQUENCY SPECTRUM

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INTRODUCTION

As is well-known, a sound level meter has a proper internal mechanism such as a mean squaring operation and a weighting frequency network. So, the probability characteristic of random fluctuation measured through a sound level meter should be principally influenced by them. While, it is obviously important to evaluate the output sound level fluctuation of sound insulation system with a random noise excitation from the statistical view point, especially in close connection with the deterministic mechanism of system and input character.

In this paper, it is first theoretically considered how the fundamental mechanism of mean squaring operation and insulation system affect the probability distribution form of output level in direct relation to the well-known probabilistic evaluation problem, when a Gaussian random wave passes through the noise insulation system. The effect of an instrument character, the frequency spectrum of input random noise and the noise insulation system are explicitly reflected in each expansion parameters of output probability distribution expression in a series expansion form. The validity of the above theory is confirmed experimentally by applying it to the transmitted noise data of the insulation system with a single-wall observed by use of a usual sound level meter. It is noteworthy that the theoretically derived output level distribution is able to be predicted only with use of information on a time constant of indicating system and the frequency spectra of Gaussian random input, indicating system and sound transmission of insulation system.

THEORETICAL CONSIDERATION

When the input wave $x(t)$ and the output wave $y(t)$ of a sound insulation system (e.g., single-wall, double-wall, etc.) is measured through the sound level meters (see Fig.1), the indicated power outputs, E_x and E, of meters are given as:

$$E_x = \frac{1}{T}\int_0^T x(t)^2\,dt\,, \qquad E = \frac{1}{T}\int_0^T y(t)^2\,dt \tag{1),(2}$$

with an averaging time interval T of sound level meter. Furthermore, the fundamental model for indicating system of usual sound level meter is shown in Fig.2. Let $f(t)$ be the stationary Gaussian random wave.

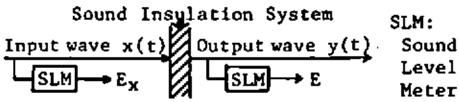

Fig. 1 A model for measurement of noise insulation system.

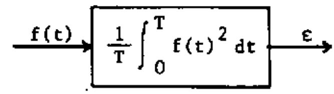

Fig. 2 A model for indicating system of sound level meter.

In this case, by assuming $f(t)$ as a part of the periodic wave with constant period T, the sample process $f(t)$ within a time domain $[0,T]$ of random input can be directly expressed in a form of Fourier expansion series as [1]:

$$f(t) = \sum_{n=N_1}^{N_2} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t) \tag{3}$$

with $\omega_0 \overset{\Delta}{=} 2\pi f_0$ $(f_0 = \frac{1}{T})$ and $a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 t\,dt$, $b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 t\,dt$.

Originally, the DC component, $a_0/2$, of $f(t)$ should be omitted owing to the sound property. Accordingly, the Fourier coeffcients a_n and b_n $(n \neq 0)$ corresponding to AC component are independent each other and become Gaussian type random variable with zero mean $\langle a_n \rangle = \langle b_n \rangle = 0$ and variance S_n $(\langle a_n^2 \rangle = \langle b_n^2 \rangle)$.

Now, the output power fluctuation ε of the mean squaring circuit (see Fig.2) is expressed according to Parceval theorem [1] as follows:

$$\varepsilon \left(\overset{\Delta}{=} \frac{1}{T}\int_0^T f(t)^2\,dt \right) = \sum_{n=N_1}^{N_2} (a_n^2 + b_n^2)/2 = \sum_{n=1}^{N} F_n \tag{4}$$

with $F_n = (a_n^2 + b_n^2)/2$. Hereupon, F_n is a power frequency component of ε at a frequency $f = nf_0$ [Hz] $(f_0 = 1/T)$. The number of sample points N $(= N_2 - N_1 + 1)$ equals to TW (W:frequency band width of $f(t)$) [2]. Thus, the output power fluctuation E of a noise insulation system measured through a sound level meter with input $x(t)$ can be given as:

$$E = \frac{1}{T}\int_0^T y(t)^2\,dt = \sum_{n=1}^{N} \frac{1}{2}(a_{ny}^2 + b_{ny}^2) = \frac{2}{T^2}\sum_{n=1}^{N} |Y(j\omega)|^2_{\omega=2n\pi/T}$$

$$= \frac{2}{T^2}\sum_{n=1}^{N} |H(j\omega)|^2 \cdot |X(j\omega)|^2_{\omega=2n\pi/T} \tag{5}$$

with Fourier coefficients a_{ny} and b_{ny} of $y(t)$ and

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\,e^{-j\omega t}\,dt\,, \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt\,. \tag{6),(7}$$

$H(j\omega)$ is a frequency transfer function of noise insulation system. Similarly, the input power fluctuation E_x of $x(t)$ is given as:

$$E_x = \frac{1}{T}\int_0^T x(t)^2\,dt = \sum_{n=1}^{N} \frac{2}{T^2} |X(j\omega)|^2_{\omega=2n\pi/T} = \sum_{n=1}^{N} \varepsilon_n\,. \tag{8}$$

Substituting Eq.(8) into Eq.(5), the following relation is obtained:

$$E = \sum_{n=1}^{N} |H(j\omega)|^2_{\omega=2n\pi/T} \cdot \varepsilon_n = \sum_{n=1}^{N} \alpha_n \cdot \varepsilon_n \triangleq \sum_{n=1}^{N} F_n \tag{9}$$

with $\alpha_n = |H(j\omega)|^2_{\omega=2n\pi/T}$. From Eq.(9), the output power E can be predicted by use of only information on the frequency amplitude spectrum of Gaussian random input and frequency transfer function of sound insulation system.

In this case, the moment generating function $\psi(\theta)$ in a Laplace transformation form for the output power fluctuation $E = \sum_{n=1}^{N} F_n$ ($N = TW$) is given by:

$$\psi(\theta) = \langle e^{-\theta E} \rangle_E = \langle \exp(- \theta \sum_{n=1}^{N} S_n (F_n / S_n)) \rangle_E = \sum_{n=1}^{N} (1 + S_n)^{-1} \tag{10}$$

with $S_n = \langle F_n \rangle$ ($n = 1, 2, \cdots, N$). In a special case when the spectrum S_n of input wave $x(t)$ takes the same value λ_k ($k = 1, 2, \cdots, M$), Eq.(10) is re-written as:

$$\psi(\theta) = \prod_{k=1}^{M} (1 + \lambda_k \theta)^{-r_k} \tag{11}$$

with $\sum_{k=1}^{M} r_k = N$ and $\sum_{k=1}^{M} r_k \lambda_k = \sum_{n=1}^{N} S_n$.

Thus , the probability density function $P(E)$ of output power can be explicitly derived by use of Heaviside's expansion theorem [3] in an inverse Laplace transform of Eq.(11) as follows:

$$P(E) = \sum_{k=1}^{M} \frac{1}{(r_k - 1)!} \left(\frac{d}{d\theta}\right)^{r_k - 1} \{ e^{\theta E} (\theta + \frac{1}{\lambda_k})^{r_k} \psi(\theta) \} |_{\theta \to -1/\lambda_k}$$

$$= \sum_{k=1}^{M} \sum_{n_1 + \cdots + n_M = r_k - 1} P_\Gamma(E; \lambda_k, n_k + 1) \prod_{m \neq k} g(n_m, r_m, -\lambda_m/\lambda_k) . \tag{12}$$

Where, $P_\Gamma(E; s, m)$ ($\triangleq E^{m-1} e^{-E/s} / s^m \Gamma(m)$) is a gamma distribuion function with two parameters s and m, and

$$g(n_m, r_m, -\lambda_m/\lambda_k) = (- \lambda_m/\lambda_k)^{n_m} \binom{r_m - 1 + n_m}{n_m} (1 - \lambda_m/\lambda_k)^{-r_m - n_m} . \tag{13}$$

Accordingly, the cumulative distribution function, $Q_L(L)$, of output level fluctuation, $L (= 10 \log (E/I_0) = 10 \log (E/(I_0/\lambda_0)))$ is given as:

$$Q_L(L) = 1 - \sum_{k=1}^{M} \sum_{n_1 + \cdots + n_M = r_k - 1} \exp \{ - \frac{I_0}{\lambda_k} 10^{L/10} \} \sum_{\ell=0}^{n_k} \frac{1}{\ell !} \{\frac{I_0}{\lambda_k} 10^{L/10}\}^{\ell}$$

$$\times \prod_{m \neq k} g(n_m, r_m, -\lambda_m/\lambda_k) . \tag{14}$$

EXPERIMENTAL CONSIDERATION

The experimental arrangement is indicated in Fig.3. A single-wall of Aluminium with 1.2mm thickness has been employed. Table 1 shows the values of r_k and λ_k ($k=1,2$) of Eq.(14) in a case with white Gaussian type random input (with frequency band width 224 - 355 [Hz]). The sound transmission factor α_n in Eq.(13) for Aluminium single-wall is obtained in two ways of method [T:based on the mass law, A:experiment]. Figure 4 shows the comparison between theory and experiment for the output cumulative distribution of noise insulation system. Table 2 show the comparison between theory and experiment for evaluation index L_x. Each theoretical results is in a good agreement with experimentally sampled values.

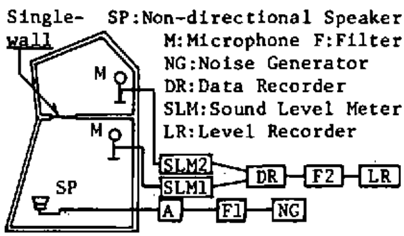

Fig. 3 Experimental arrangement for insulation measurement of single-wall in reverberation rooms.

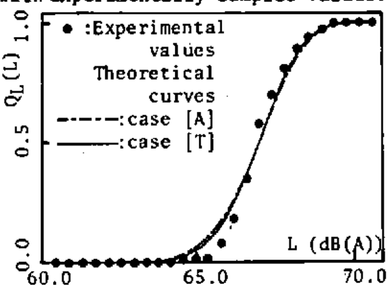

Fig. 4 A comparison between theory and experiment for the output cumulative distribution.

Table 1 Actual evaluation of parameters λ_k and r_k in a case with Aluminium single-wall.

Frequency band width [Hz]	r_k	λ_k ($\times 10^{-7}$) [T]	λ_k ($\times 10^{-7}$) [A]
220 - 280	$r_1=7$	2.15	1.50
280 - 355	$r_2=9$	4.03	4.53

Table 2 A comparison between theory and experiment for evaluation index L_x.

		L_5	L_{25}	L_{50}	L_{75}
Experiment		68.7	67.6	66.9	66.4
Theory	[T]	68.8	67.8	67.0	66.4
	[A]	68.9	67.8	67.0	66.1

(dB(A))

CONCLUSION

A new prediction method for output fluctuation of noise insulation system has been derived in connection with instrument character and input frequency spectrum. The validity and the effectiveness of proposed theory have been experimentally confirmed.

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