

# **ACOUSTIC DISTANCE MEASUREMENT BASED ON THE INTERFERENCE BETWEEN TRANSMITTED AND REFLECTED WAVES USING CROSS-SPECTRAL METHOD BY INTRODUCING ANALYTIC SIGNAL OF LINEAR CHIRP AND HILBERT TRANSFORM FILTER**

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The distance to a target is fundamental information in many engineering applications. Recently, an acoustic distance measurement (ADM) method has been proposed based on the interference between transmitted and reflected waves, but it requires applying twice the Fourier transform. The ADM method in which a linear chirp whose frequency changes linearly with lapse of time is adopted as a transmitted sound has been proposed to reduce the number of applications of the Fourier transform to once. The observed wave obtained by the linear chirp signal has the distance information as a periodicity in time domain. However, due to the influence of the transfer characteristic of measuring system including a loudspeaker and microphone, the ADM method would often estimate the spurious short distance different from true distance. The ADM method by applying the cross-spectral method to observed signals of the adjacent two-channel (2ch) microphones has been also proposed, where the method adopts a linear chirp as transmitted wave and removes the influence of the measuring system. In this method, Hilbert transform is used to obtain analytic signal of linear chirp for the cross-spectrum, but Hilbert transform originally requires applying twice the Fourier transform. This paper describes an advanced study on the ADM method by applying the cross-spectral method to observed signals of the adjacent 2ch microphones, adopting a linear chirp as transmitted wave and removing the influence of the measuring system. More concretely, though the linear chirp signal in a time domain corresponds to the frequency spectrum, it is not a complex signal but a real signal. To apply the cross-spectral method, the analytic signal of linear chirp that is newly obtained using Hilbert transform filter is introduced. We confirmed the validity of the chirp-based ADM method by performing a computer simulation and by applying it to an actual sound field.

**Keywords:** Distance measurement based on phase interference, linear chirp, cross-spectral method, analytic signal, Hilbert transform filter

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## **1. Introduction**

The distance to an object (a target) is fundamental information in many engineering applications, and an acoustical signal, especially an ultrasonic sound, often plays an essential role in measuring such a distance. To measure this distance, it is common to use the time difference between waves reflected and transmitted from the target[1, 2, 3, 4, 5]. However, if the reflected wave is returned before the transmitted wave has been sufficiently attenuated, the reflected wave is buried in the transmitted wave, and it is difficult to measure the distance to the target. A method for removing the influence of

the transmitted wave has been proposed[6], but it requires multiple elements. On the other hand, in the field of microwave radar, there is a technique for short-range measurements that uses the standing wave (i.e., phase interference)[7, 8]. Using standing waves to measure distances is a very practical approach, and simply requires the application of the Fourier transform to the power spectrum of the observed wave. The absolute value of this Fourier transform is called the range spectrum, and the position of its peak is the estimated value  $d$  of the distance from the microphone to the target. We extended the method for measuring short distances by using the standing wave in the audible sound[9, 10]. By combining this approach with knowledge of audible sound, which propagates long distances more easily than does ultrasound, it is possible to make distance measurements over a wide range of lengths. In addition, this method has the advantage that it can be used by anyone, since sound radiation is not regulated by law (unlike the case for microwaves). However, as mentioned above, the conventional acoustic distance measurement (ADM) method requires Fourier transform twice, i.e., from the observed wave to the power spectrum and from the power spectrum to the range spectrum.

The ADM method in which a linear chirp whose frequency changes linearly with lapse of time is adopted as a transmitted sound has been proposed to reduce the number of applications of the Fourier transform to once. The observed wave obtained by the linear chirp signal has the distance information as a periodicity in time domain. However, due to the influence of the transfer characteristic of measuring system including a loudspeaker and microphone, the ADM method would often estimate the spurious short distance different from true distance. The ADM method by applying the cross-spectral method to observed signals of the adjacent two-channel (2ch) microphones has been also proposed, where the method adopts a linear chirp as transmitted wave and removes the influence of the measuring system. In this method, Hilbert transform is used to obtain analytic signal of linear chirp for the cross-spectrum[11], but Hilbert transform originally requires two applications of the Fourier transform.

This paper describes an advanced study on the ADM method by applying the cross-spectral method to observed signals of the adjacent 2ch microphones, adopting a linear chirp as transmitted wave and removing the influence of the measuring system. More concretely, though the linear chirp signal in a time domain corresponds to the frequency spectrum, it is not a complex signal but a real signal. To apply the cross-spectral method, the analytic signal of linear chirp that is newly obtained using Hilbert transform filter is introduced. We confirmed the validity of the chirp-based ADM method by performing a computer simulation and by applying it to an actual sound field.

## 2. ADM using cross-spectral method between adjacent 2ch observations of linear chirp

Though the ADM method is given in a very simple form based on the interference between transmitted and reflected waves, it requires applying twice the Fourier transform. The authors have expanded the ADM method, adopting a linear chirp whose frequency changes linearly with lapse of time as a transmitted sound to reduce the number of applications of the Fourier transform to once. The observed wave obtained by the linear chirp signal has the distance information as a periodicity in time domain. However, due to the influence of the transfer characteristic of measuring system including a loudspeaker and microphone, the ADM method would often estimate the spurious short distance different from true distance.

Figure 1 shows the positions of the microphone, the sound source (loudspeaker), and a target. As shown in Fig. 1, the measuring system with an impulse response  $g(t)$  (consists of the impulse response  $g_L(t)$  of the audio playback system and the impulse responses  $g_{M1}(t)$  and  $g_{M2}(t)$  of the audio recording systems) affects sound observations. Here,  $g_{M1}(t)$  can be assumed to be approximated by  $g_{M2}(t)$  (i.e.,  $g_{M1}(t) \approx g_{M2}(t)$ ). In this research, the ADM is newly proposed using cross-spectral method between adjacent 2ch observations of linear chirp. This method has advantages, such as reducing the number of applications of the Fourier transform to once and eliminating the influence of

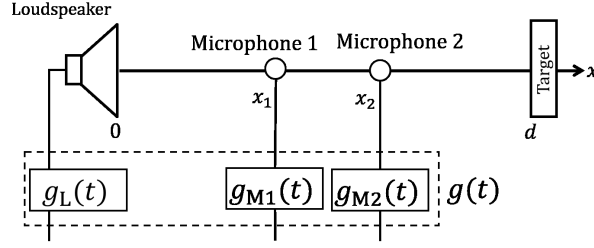


Figure 1: Configuration of ADM with 2ch microphones.

the frequency characteristic of measuring system.

Let a transmitted wave  $v_{TI}(t, x)$ , a linear chirp, be a function of position  $x$  [m] and time  $t$  [s], as follows:

$$v_{TI}(t, x) = A \cos \left( 2\pi \int_0^{t-x/c} f(\tau) d\tau + \theta \right) \quad \text{with} \quad f(t) = \frac{f_W}{T}t + f_1, \quad (1)$$

where  $A$  is the magnitude,  $\theta$  [rad] is the initial phase,  $c$  [m/s] is the velocity of sound,  $f(t)$  [Hz] is the instantaneous frequency,  $T$  [s] is the duration of the transmitted sound, and  $f_W$  [Hz] ( $= f_N - f_1$ ) is the width between the lowest frequency  $f_1$  [Hz] and the highest frequency  $f_N$  [Hz].

Let us consider that the transmitted wave  $v_{TI}(t, x)$  is reflected by  $m$  targets. The wave  $v_{RI_n}(t, x)$  reflected by the  $n$ -th target can be expressed as follows:

$$v_{RI_n}(t, x) = A\gamma_n \cos \left( 2\pi \int_0^{t-(2d_n-x)/c} f(\tau) d\tau + \theta + \phi_n \right), \quad (2)$$

where  $\gamma_n e^{j\phi_n}$  is the reflection coefficient of the  $n$ -th target, and  $d_n$  [m] is the position of the  $n$ -th target.

In this paper, the two microphones, the sound source, and the target are assumed to all be located on  $x$  axis, and the sound source and two microphones are assumed to be located at  $x = 0$  [m],  $x_1$  [m] and  $x_2$  [m], as shown in Fig. 1. Thus, at the  $i$ -th microphone position  $x = x_i$  [m], the composite wave  $v_C(t, x_i)$  ( $i=1, 2$ ), which is the composition of all waves transmitted and reflected, is formulated as:

$$v_{CI}(t, x_i) = v_{TI}(t, x_i) + \sum_{n=1}^m v_{RI_n}(t, x_i). \quad (3)$$

To obtain the analytic signal,  $v_{CQ}(t, x_i)$  orthogonal to  $v_{CI}(t, x_i)$  can be computed by use of Hilbert transform filter as

$$h(n) = \begin{cases} \frac{2}{n\pi} & n : \text{odd}, \\ 0 & n : \text{even}. \end{cases} \quad (4)$$

Thus, analytic signal  $v_C(t, x_i)$  can be obtained by making  $v_{CI}(t, x_i)$  and  $v_{CQ}(t, x_i)$  correspond to real part and imaginary part.

$$v_C(t, x_i) = v_{CI}(t, x_i) + jv_{CQ}(t, x_i). \quad (5)$$

Actually, the impulse response  $g(t)$  of the measuring system caused by the frequency characteristics of the loudspeaker and the microphone affects  $v_C(t, x_i)$ . Since the transmitted wave is a linear chirp, let the frequency response of  $g(t)$  be  $G(f)$ , and from  $f = \frac{f_W}{T}t + f_1$  at the time  $t$ , let  $v_C(t, x_i)$  be again  $v_C(f, x_i)$  using the corresponding frequency  $f$ . In the frequency domain, cross-spectrum

with  $v_C(f, x_1)$  input and  $v_C(f, x_2)$  output, when  $\gamma$  is sufficiently small ( $\gamma \ll 1$ ), is expressed by the following equation (for convenience, the number of targets is one).

$$\begin{aligned} C(f, x_1, x_2) &= \frac{|G(f)|^2 v_C^*(f, x_1) v_C(f, x_2)}{|G(f)|^2 v_C^*(f, x_1) v_C(f, x_1)} \\ &= \frac{v_C^*(f, x_1) v_C(f, x_2)}{v_C^*(f, x_1) v_C(f, x_1)} \\ &\approx e^{j(\alpha_2 - \alpha_1)} \{1 + \gamma(e^{j(\beta_2 - \alpha_2)} - e^{j(\beta_1 - \alpha_1)})\} \end{aligned} \quad (6)$$

with

$$\alpha_i = 2\pi \int_0^{\frac{T}{f_W}(f-f_1) - \frac{x_i}{c}} f(\tau) d\tau + \theta, \quad \beta_i = 2\pi \int_0^{\frac{T}{f_W}(f-f_1) - \frac{2d_n - x_i}{c}} f(\tau) d\tau + \theta + \phi. \quad (7)$$

The square (i.e., cross-power) of the absolute value of  $C(f, x_1, x_2)$  can be written as follows:

$$|C(f, x_1, x_2)|^2 \approx 1 + 2\gamma \{ \cos(\alpha_2 - \beta_2) - \cos(\alpha_1 - \beta_1) \}, \quad (8)$$

where

$$\alpha_i - \beta_i = 2\pi \frac{2(d - x_i)}{c} f - 2\pi \frac{2(d - x_i)}{c} \frac{f_W}{T} \frac{d}{c} - \phi. \quad (9)$$

Therefore, by subtracting the average of  $|C(f, x_1, x_2)|^2$  from Eq.(8), the  $\Delta$ cross-power  $\Delta p(f, x_1, x_2)$  is obtained as follows:

$$\begin{aligned} \Delta p(f, x_1, x_2) &= 2\gamma \left\{ \cos \left( 2\pi \frac{2(d - x_2)}{c} f - 2\pi \frac{2(d - x_2)}{c} \frac{f_W}{T} \frac{d}{c} - \phi \right) \right. \\ &\quad \left. - \cos \left( 2\pi \frac{2(d - x_1)}{c} f - 2\pi \frac{2(d - x_1)}{c} \frac{f_W}{T} \frac{d}{c} - \phi \right) \right\}. \end{aligned} \quad (10)$$

In Eq. (10), the first and second terms represent the power fluctuations induced by interferences between the transmitted and reflected waves. The components of interference are periodic functions, and their periods are inversely proportional to the distance  $d - x_1$  between microphone 1 and target and the distance  $d - x_2$  between microphone 2 and target. It can be seen that by using the cross-spectrum, it is possible to eliminate the influence of the transmitted signal and the influence of the measuring system.

If we apply the Fourier transform to the  $\Delta p(f, x_1, x_2)$  with respect to frequency  $f$ , we obtain

$$P(x) = \int_{f_1}^{f_N} \Delta p(f, x_1, x_2) e^{-j2\pi \frac{2x}{c} f} df. \quad (11)$$

The absolute value  $|P(x)|$  is called the range spectrum, and the location of its peak is the estimated value  $d$  of the distance from the microphone to the target. Thus, the ideal distance spectrum  $|P(x)|$  has two peaks  $x = d - x_1$  and  $d - x_2$ . If two microphones are set closely ( $|x_1 - x_2| \ll \frac{c}{2f_W}$ ), then two peaks are reduced to single peak at  $x = d - \frac{x_1 + x_2}{2}$ .

### 3. Computer simulations

#### 3.1 Simulation conditions

To confirm the validity of the proposed method, we performed computer simulations. The simulation conditions are shown in Table 1. The distance between the microphone 1 and target is set to 0.5 m,

and the positions of microphones 1 and 2 are located at  $x_1 = 0.0$  m and  $x_2 = 0.006$  m, respectively. The cosine linear chirp is adopted as a transmitted wave, as shown in Fig.2.

Table 1: Simulation conditions.

Sound source	Linear chirp signal
Measurement time	0.464 s
Sampling frequency	44.1 kHz
Data points in time domain	2048
Data points in frequency domain	2048
Frequency bandwidth	5490.9 Hz (2153.3 ~ 7644.2 Hz)
Minimum measurable distance	0.03 m
Sound speed	340 m/s
Reflection coefficient	0.05

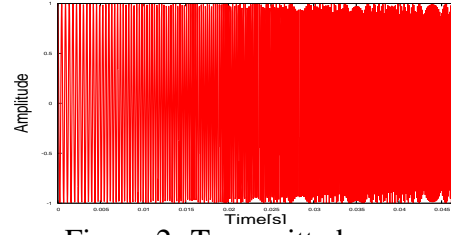
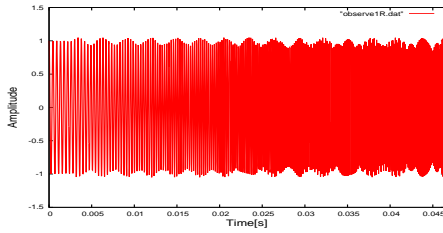


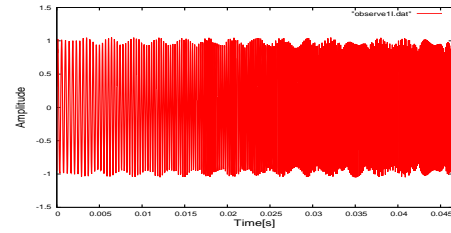
Figure 2: Transmitted wave.

### 3.2 Simulation results

The composite waves at microphones 1 and 2 are shown in Figs.3(a) and 4(a), respectively. These composite waves are transformed by Hilbert transform filter as shown in Figs.3(b) and 4(b). With use of these results,  $\Delta$ cross-power is obtained in Fig.5(a). By applying Fourier transform to this  $\Delta$ cross-power, range spectrum  $|P(x)|$  can be given in Fig.5(b). In this figure, since the estimated distance is 0.49 m, the estimated distance catches the true value 0.5 m within the error of 0.03 m which is the minimum measurable distance. This might demonstrate the validity of the proposed method.

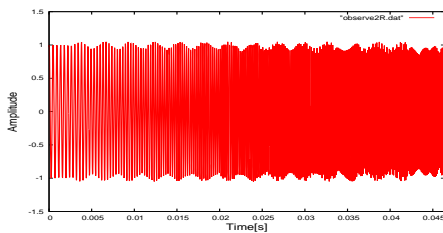


(a) For cosine chirp wave (Real part).

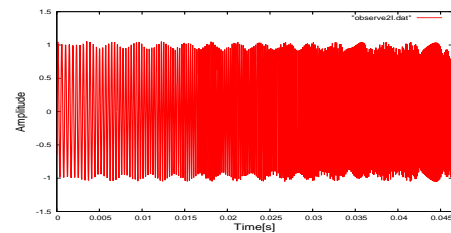


(b) For transformed wave (Imaginary part).

Figure 3: Composite waves at microphone 1.

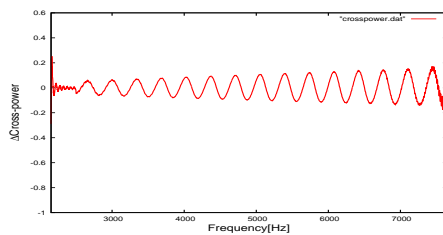


(a) For cosine chirp wave (Real part).

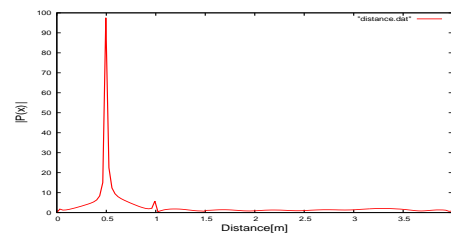


(b) For transformed wave (Imaginary part).

Figure 4: Composite waves at microphone 2.



(a)  $\Delta$  cross-power.



(b) Range spectrum.

Figure 5: Results in simulation.

## 4. Experiment in a real sound field

### 4.1 Experimental conditions

The parameters and conditions of the transmitted wave were the same as in the simulation. The equipment used in the experiment is shown in Table 2. A plywood board was adopted as a target. In this experiment, from the viewpoint of knowing the influences of the measuring system and transmitted sound, the impulse response is obtained by TSP (Time Stretched Pulse) method [12] (note that the number of synchronous additions is 1) and is convoluted with the transmitted sound.

Table 2: Experimental apparatus.

Target	Plywood square (H:30cm×W:30cm×D:0.5cm)
Audio interface	ROLAND, UA-55
Loudspeaker	YAMAHA, MSP5 STUDIO
Microphone	AUDIO-TECHNICA, AT9904
Microphone amplifier	PAVEC, MA-2016C

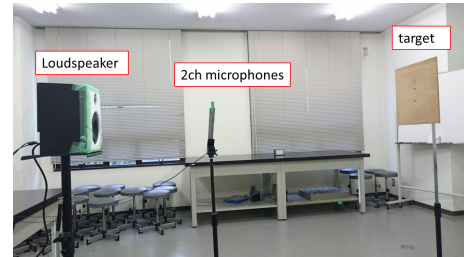
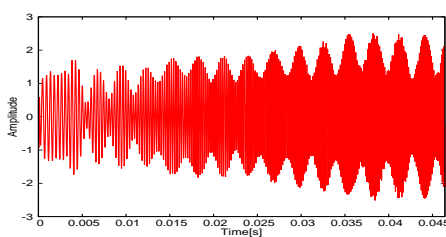


Figure 6: Experimental environment.

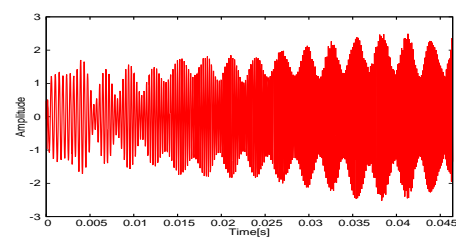
### 4.2 Experimental results

For the linear cosine chirp, the actually observed waves at microphones 1 and 2 are shown in Figs.7(a) and 8(a), respectively. These observed waves are transformed by Hilbert transform filter as shown in Figs.7(b) and 8(b). First, we applied the conventional ADM method to the observed wave of the microphone 1. The  $\Delta$ power spectrum obtained by subtracting the average value from the power spectrum of the observed wave is shown in Fig.9(a). Using this result, the range spectrum is depicted in Fig.9(b). From Fig.9(b), in this range spectrum, a peak appears around 0.5 m of the true value, but it is found that a larger peak appears around 0 m. This seems to be due to the trend in the  $\Delta$ power in Fig.9(b), that is, the influence of the measuring system.

By using the cross-spectral method,  $\Delta$ cross-power is obtained in Fig.10(a). Applying the Fourier transform to this  $\Delta$ cross-power and taking its absolute value yield to the range spectrum  $|P(x)|$  as shown in Fig.10(b). In this result, there is a good agreement between the estimated and true values, because the estimated distance is 0.49 m (while the true distance is 0.5 m and the minimum measurable distance is 0.03 m). From these results, the effectiveness of the proposed method might be demonstrated even in an actual sound field.



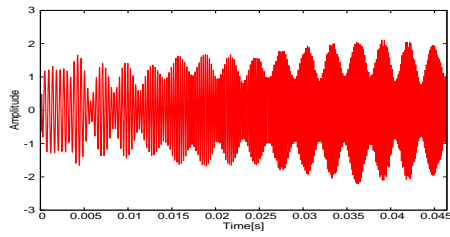
(a) For cosine chirp (Real part).



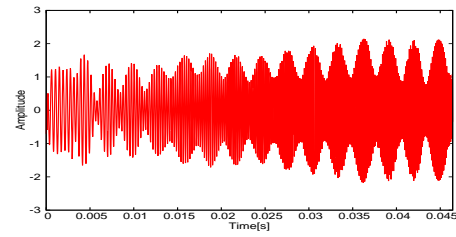
(b) For transformed wave (Imaginary part).

Figure 7: Observed waves at microphone 1.



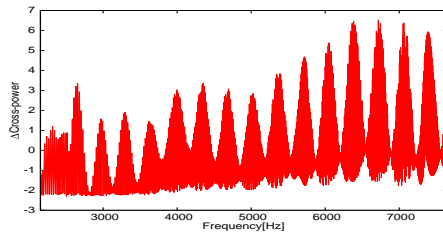
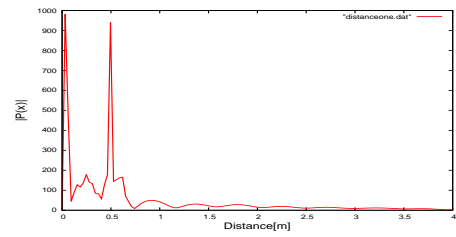


(a) For cosine chirp (Real part).



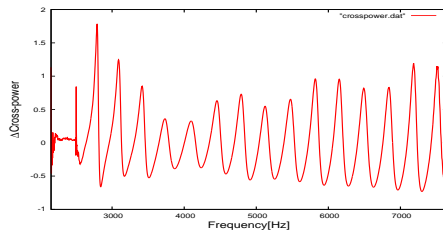
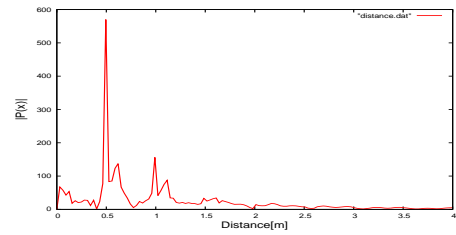
(b) For transformed chirp (Imaginary part).

Figure 8: Observed waves at microphone 2.


 (a)  $\Delta$  power spectrum.


(b) Range spectrum.

Figure 9: Experimental results by the conventional method.


 (a)  $\Delta$  cross-power.


(b) Range spectrum.

Figure 10: Experimental results by the proposed method.

## 5. Conclusion

We have proposed an ADM method using cross-spectrum between analytic signals at 2ch microphones based on 2ch observations and their Hilbert transforms. Furthermore, we tried to confirm the effectiveness of the proposed method in principle by performing simulation and measurement in the actual sound field.

In the actual sound field, we could estimate the distance with an error within the minimum measurable distance of 0.03 m for  $d = 0.5$  m. In the conventional method, when the influence of the measuring system is not removed, it was possible to obtain a peak at the position around 0.5 m, but the peak around 0 m prevented the distance measurement. On the other hand, when the influence of the measuring system by the cross-spectral method was removed, the peak around 0 m was reduced and the peak at the 0.5 m position was made remarkable.

Since the measurement accuracy and robustness against noise of the proposed method also depends on the shape of the target and the magnitude of the reflection coefficient, details of measurement precision and effect of noise are to be studied in the future.

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