24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

FORMULATING HYBRID MODELS FOR PREDICTION OF RAILWAY INDUCED VIBRATION LEVELS IN BUILDINGS

Kirsty A. Kuo, Geert Lombaert, and Geert Degrande
Department of Civil Engineering, KU Leuven, Belgium
email: kirsty.kuo@kuleuven.be

Accurate predictions of railway induced vibration levels at grade and in buildings are much-needed in urban areas when planning the construction of new infrastructure. The use of numerical models is hampered by the introduction of simplifying modelling assumptions and uncertain parameter inputs. Hybrid models have the potential to allay these issues by combining field measurements and state-of-the-art numerical methods. These models are based on the separation of source excitation, propagation path and building response terms. This paper presents several examples of hybrid models that can be used when a new railway is being built in an urban environment, or when a new building is being constructed near an existing railway. The effects of term separation are explored and guidelines for effective hybrid model implementation are given.

Keywords: railway-induced vibration; dynamic soil-structure interaction, hybrid modelling, vibration predictions

1. Introduction

In 2017, there will be many landmark events occurring in the global rail industry. These include: the opening of the first phase of a new standard gauge railway connecting Kenya, Uganda, Rwanda, and South Sudan; the introduction of the first new Crossrail trains in London; breaking the ground for a new metro in Melbourne, Australia; and the issue of a tender for construction of a Kuala Lumpur-Singapore high speed line. Concurrently, new residential and commercial buildings will be constructed next to and above existing railways. All of these projects will have significant impact on the lives of the people living and working near the lines, with one of the major environmental issues being the noise and vibration produced by train passages. Vibration level limits may vary from country to country, but regardless of location, any efforts to meet these limits require accurate predictions of railway induced vibration levels.

A number of forward numerical models for predicting vibration levels in buildings have been developed. Coulier et al. [1] uses a 2.5D coupled FE-BE methodology to model a conventional ballasted railway track at grade that is fully coupled to a four storey portal frame founded on embedded strip foundations. A decoupled approach where the vehicle-track response is computed first to obtain the dynamic loads acting on the soil surface or a tunnel invert, and then these loads are applied in order to obtain the free field and building response is more commonly used. Examples of this include Kouroussis et al. [2] who model a private residence next to a tramway using a coupled lumped mass model and a finite/infinite element model; Stupazzini and Paolucci [3] use a beam-on-elastic-foundation to calculate the loading that is applied to a tunnel-soil-structure model that uses spectral elements. A further degree of decoupling is introduced by Vogiatzis [4], who calculates vibration levels on a tunnel-soil interface using a FE model, propagates the vibrations through the soil using analytical equations, and then uses empirical formulae to estimate the propagation of vibrations through the building.

Despite the considerable progress that has been, and is being, made, these models continue to require detailed parameter inputs and large modelling efforts, whilst delivering large prediction uncertainties. For this reason a hybrid modelling procedure is being developed that combines numerical analyses with field measurements, in the hope that this will reduce prediction uncertainty and provide a more flexible, and easier-to-implement, method of vibration prediction.

This paper describes how the hybrid modelling approach can be applied to predict railway induced vibration levels in buildings. A hybrid model framework is described in the following section, and ways of applying this framework to new-build scenarios are discussed. Section 3 develops the hybrid model formulations for two cases: one where no railway yet exists at the site of interest, and one where no building yet exists at the site of interest.

2. Hybrid model framework

The hybrid model framework proposed here is based on the general form recommended by the ISO 14837-1 standard [5], which expresses the magnitude of the quasi-stationary response $A(f)$ as the sum of three terms:

$$A(f) = S(f) + P(f) + R(f) \tag{1}$$

where $S(f)$ represents the source strength, $P(f)$ characterises the propagation path, and $R(f)$ is the receiver term. All three terms are expressed in decibels, as a function of frequency f, and can be considered to be uncoupled only in some situations for simplified models. To obtain the vibration velocity level at a given frequency, each of these terms should be calculated at the same frequency, which is strictly speaking not valid for moving sources due to the Doppler effect.

This framework forms an ideal basis for hybrid models, as each of the terms can be determined using field measurements, or calculated using numerical procedures. For example, consider the situation where a railway does not yet exist at the site of interest, or has only been partially constructed, and vibration predictions are needed in a nearby building. A generic hybrid model relevant to this situation is:

$$A^{\mathrm{HYB}}(f) = S^{\mathrm{NUM}}(f) + P^{\mathrm{EXP}}(f) + R^{\mathrm{EXP}}(f) \tag{2}$$

where the superscripts HYB, NUM and EXP represent hybrid, numerical, and experimental (measured) means of calculating the vibration terms. The numerical model of the source provides a means of easily assessing the effect of alterations to the track, such as the installation of track-based mitigation measures like resilient fasteners, rail pads and floating slab track, and alterations to the train, such as reduced wheel/rail roughness and increased train speed. The use of a measured propagation term offers resilience in the case of a site with complex soil stratification, avoiding both the expense of extensive soil characterisation tests and the modelling effort required to implement multiple soil layers. Likewise, the measured building response bypasses the need for a detailed structural model that accounts for dynamic soil-structure interaction. However, in situ measurements are still required to obtain transfer functions that characterise the propagation path and the receiver, and to provide dynamic soil characteristics that can be used in the numerical model to account for track-soil interaction.

A second generic hybrid model involves determination of the source and propagation terms using field measurements, and numerical prediction of the building response:

$$A^{\mathrm{HYB}}(f) = S^{\mathrm{EXP}}(f) + P^{\mathrm{EXP}}(f) + R^{\mathrm{NUM}}(f) \tag{3}$$

This model is suited to the scenario of a partially constructed, or yet-to-be constructed building that is sited near a railway. Furthermore, this model can also be used when in situ measurements of the building response cannot be obtained, such as when sensitive environmental conditions exist and/or manufacturing processes must not be disturbed. The use of a numerical prediction of the receiver term provides a means of assessing the effect of installing mitigation measures such as damping treatments, localised stiffening or mass addition, and, in critical conditions, base isolation. This type of hybrid model can also significantly reduce the numerical model complexity by avoiding the need for characterisation of the track parameters and propagation path and inclusion of the train and track elements. Some soil characterisation tests will nevertheless be required to estimate the dynamic soil-structure interaction present in the receiver term, but these tests are expected to be more localised (and therefore less expensive) than those required for a complete numerical model.

Other permutations of hybrid models that combine field measurements with numerical models can also be envisaged and applied to relevant scenarios. For example, a numerical model for the propagation term together with measurements of the source and receiver could be used to assess possible mitigation measures that act to disrupt the transmission path, such as open trenches, wave barriers and wave impeding blocks.

3. Hybrid model implementation

3.1 Governing equation

Hybrid models can be implemented using any prescribed method of determining the source, propagation and receiver terms. The empirical procedure proposed by the FRA [6] is one such method, and for the case where the railway and building are both present, is expressed as:

$$\mathrm{L_v}(\mathbf{x}_\mathrm{b}) = \mathrm{L_F}(\mathbf{X}, \mathbf{x}_1) + \mathrm{TM_L}(\mathbf{X}, \mathbf{x}_1) + \mathrm{C_b}(\mathbf{x}_1, \mathbf{x}_\mathrm{b}) \tag{4}$$

where $\mathbf{X}$ is a vector that collects all the source points, located on the rail heads, that are used for the field measurements. The term $\mathrm{L_v}(\mathbf{x}_\mathrm{b})$ is the vibration velocity level at the receiver point $\mathbf{x}_\mathrm{b}$ in the building, and is measured in decibels at one-third octave band intervals. This is illustrated in Figure 1.

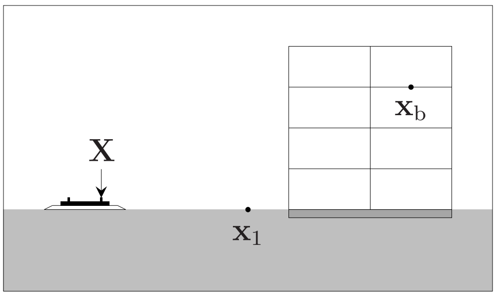

Figure 1: Position of the source and receiver points for the FRA procedure when both railway and building are present.

The excitation force, represented by the equivalent force density level $\mathrm{L_F}(\mathbf{X}, \mathbf{x}_1)$, is calculated as the difference between the measured vibration velocity level at some point on the ground surface $\mathbf{x}_1$ and the line source transfer mobility level: $\mathrm{L_F}(\mathbf{X}, \mathbf{x}_1) = \mathrm{L_v}(\mathbf{x}_1) - \mathrm{TM_L}(\mathbf{X}, \mathbf{x}_1)$. This expression represents the equivalent fixed line source that results in the same vibration velocity level as the train passage. The force density level depends on both the actual force generated at the wheel/rail interface and the dynamic characteristics of the transit structure (that is, the tunnel or ballast and the soil).

The vibration propagation from the track, through the soil to the receiver point on the soil surface is contained within the line source transfer mobility term $\mathrm{TM_L}(\mathbf{X}, \mathbf{x}_1)$. This involves the superposition of point source transfer mobility levels $\mathrm{TM_P}(\mathbf{x}_k, \mathbf{x}_1)$ for a series of n equidistant source points with spacing h [7]:

$$\mathrm{TM_L}(\mathbf{X}, \mathbf{x}_1) = 10 \log_{10} \left[h \sum_{k=1}^{n} 10^{\frac{\mathrm{TM_P}(\mathbf{x}_k, \mathbf{x}_1)}{10}} \right] \tag{5}$$

The receiver term $\mathrm{C_b}(\mathbf{x}_1, \mathbf{x}_\mathrm{b})$ is defined by the FRA as a combination of dimensionless adjustment factors used to account for ground-building foundation interaction and amplification or attenuation of vibration amplitudes as vibration propagates through buildings. These adjustment factors are added to the ground-surface vibration at location $\mathbf{x}_1$ near the building, to estimate the response inside the building, at location $\mathbf{x}_\mathrm{b}$. The FRA defines three adjustment factors: (a) those that represent the change in the incident ground-surface vibration due to the presence of the building foundation, (b) the attenuation of vibration as it travels from foundation to the upward floors, assumed at a rate of 1 to 2 dB per floor, and (c) amplification of approximately 6 dB in the frequency range of the fundamental floor resonances (15-20 Hz for wood-frame, 20-30 Hz for reinforced concrete slabs). For (a), zero correction is applied when estimating basement floor vibration or vibration of at-grade slabs.

A recent study by Kuo et al. [8] develops new expressions for this coupling loss term that account more fully for the dynamic soil-structure interaction that is present. For example, the coupling loss can be defined as the difference in vibration velocities at some point in the building $\mathrm{L_v}(\mathbf{x}_\mathrm{b})$ and at some point on the ground surface between the track and the building $\mathrm{L_v}(\mathbf{x}_1)$:

$$\mathrm{C_b}(\mathbf{x}_1, \mathbf{x}_\mathrm{b}) = \mathrm{L_v}(\mathbf{x}_\mathrm{b}) - \mathrm{L_v}(\mathbf{x}_1) \tag{6}$$

This equation accounts for how the vibration levels attenuate as the vibration propagates through the building foundation and floors. A second definition of the coupling loss accounts for not only this attenuation through the building, but also the effect of the building as a scatterer of an incident vibration field:

$$\mathrm{C_b}(\mathbf{x}', \mathbf{x}_\mathrm{b}) = \mathrm{L_v}(\mathbf{x}_\mathrm{b}) - \mathrm{L_v}(\mathbf{x}') \tag{7}$$

Here $\mathbf{x}'$ denotes a receiver point in the free field (i.e. in the absence of a building), whereas in equation (6) $\mathbf{x}_1$ denotes a receiver point located on the ground surface near a building. Kuo et al. [9] show that the closer the surface measurement point moves towards the building, the greater the divergence of these two coupling loss definitions, due to the dynamic soil-structure interaction that is occurring.

In the following two subsections, we explore how the hybrid model can be implemented using the FRA empirical procedure together with these definitions of the coupling loss. Two new-build scenarios are considered: firstly, where a railway does not yet exist at the site of interest (Case 1); and secondly, where a building does not yet exist at the site of interest (Case 2).

3.2 Case 1: no railway

In this case, a numerical prediction of the source term is combined with propagation and receiver terms that are determined using field measurements. As the railway does not yet exist at the site of interest, it is not possible to determine the propagation term using source excitations located on the rail heads, or indeed anywhere on the track itself. This means equation (4) needs to be adapted to make use of alternative source locations $\mathbf{X}_1$, located on the ground surface next to the track site. Kuo et al. [7] give such an expression for free field vibrations, but this includes only the source and propagation terms. So the expression is augmented here to include the receiver term by means of the second definition of the coupling loss given in equation (7):

$$\mathrm{L_v}(\mathbf{x}_\mathrm{b}) = \mathrm{L_F}(\mathbf{X}_1, \mathbf{x}') + \mathrm{TM_L}(\mathbf{X}_1, \mathbf{x}') + \mathrm{C_b}(\mathbf{x}', \mathbf{x}_\mathrm{b}) \tag{8}$$

The coupling loss term definition given in equation (7) requires the vibration velocity levels within the building due to a train passage, but these levels cannot be determined as the railway does not yet exist at the site of interest. Instead, an approximation of the coupling loss that removes the requirement of a train passage by using source excitation points located away from the track, given in Kuo et al. [8], is used:

$$\mathrm{C_b}(\mathbf{x}', \mathbf{x}_\mathrm{b}) = \mathrm{TM_L}(\mathbf{X}_1, \mathbf{x}_\mathrm{b}) - \mathrm{TM_L}(\mathbf{X}_1, \mathbf{x}') \tag{9}$$

This expression has been derived based on the assumption that the source term is uncoupled from the building and independent of distance, which is appropriate to use given the general framework of uncoupled source, propagation and receiver terms on which this hybrid model formulation is based. It provides a close approximation to the 'true' (fully-coupled) coupling loss term, regardless of the location of the receiver points in the building and on the ground's surface, and the soil type [9]. Substituting this coupling loss into equation (8) results in the hybrid model equation:

$$\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b}) = \mathrm{L_F^{NUM}}(\mathbf{X}_1, \mathbf{x}') + \mathrm{TM_L^{EXP}}(\mathbf{X}_1, \mathbf{x}_\mathrm{b}) \tag{10}$$

As can be seen from the superscripts, the first term in equation (10) represents the source excitation, and is determined using a numerical model of the train, track and soil. The second term represents a combining of the propagation and receiver terms into a single line source transfer mobility that is to be measured in situ. Figure 2 illustrates this hybrid model. The source excitation points $\mathbf{X}_1$ are at the same locations relative to the track site for both the numerical model and the measurement site.

(a) (b)

Figure 2: Case 1: (a) the numerical model; and (b) the measurement site.

The numerical model requires train, track, and soil parameters in order to determine the force density as per the FRA definition as the difference between the free field vibration velocity and the line source transfer mobility:

$$\mathrm{L_F^{NUM}}(\mathbf{X}_1, \mathbf{x}') = \mathrm{L_v^{NUM}}(\mathbf{x}') - \mathrm{TM_L^{NUM}}(\mathbf{X}_1, \mathbf{x}') \tag{11}$$

The force density as defined here is dependent upon the free field measurement location $\mathbf{x}'$. Kuo et al. [7] use both measured data and the results of numerical modelling to determine the force density at various distances $\mathbf{x}'$ from the track. These results are reproduced in Figure 3. It can be seen that the force density magnitude can vary by up to 20 dB at frequencies below 25 Hz and above 50 Hz due to the location of the receiver point. The source term is therefore not completely decoupled from the propagation path and, as a result, a hybrid prediction of the vibration velocity level in the building $\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b})$ that is made using equation (10) will have an inherent dependence on the choice of free field location $\mathbf{x}'$. Whilst this is far from ideal, removing this irregularity would require some means of determining the source excitation independently of the propagation path. It is not possible to achieve this decoupling of the source and propagation terms when using the FRA procedure.

At the measurement site, the line source transfer mobility $\mathrm{TM_L^{EXP}}(\mathbf{X}_1, \mathbf{x}_\mathrm{b})$ is determined using source excitations near the location of the future track. If the distance between $\mathbf{X}_1$ and the building is large, it may be difficult to measure the response in the building due to the source excitations. The effect of moving $\mathbf{X}_1$ away from the track site and towards the building is not yet known, and the dependence of the hybrid model prediction on the location of $\mathbf{X}_1$ is a topic for further investigation.

It should also be noted that it is possible to repeat the formulation of equations (8)-(11) using a surface receiver $\mathbf{x}_1$ located at some point between the track and the building in the place of the free field receiver $\mathbf{x}'$, resulting in:

$$\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b}) = \mathrm{L_F^{NUM}}(\mathbf{X}_1, \mathbf{x}_1) + \mathrm{TM_L^{EXP}}(\mathbf{X}_1, \mathbf{x}_\mathrm{b}) \tag{12}$$

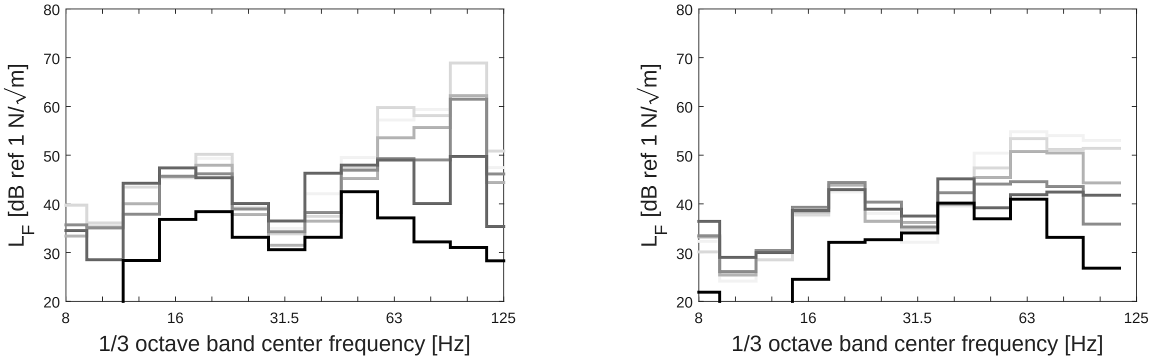

Figure 3: (a) Measured and (b) predicted force density level for sources located adjacent to the track, based on the response at $6\,\mathrm{m}$, $12\,\mathrm{m}$, $24\,\mathrm{m}$, $32\,\mathrm{m}$, $48\,\mathrm{m}$, and $64\,\mathrm{m}$ during the passage of an IC train (198 km/h). The distance from the track is indicated by the line shade, where a darker shade indicates a smaller distance. Reproduced from Kuo et al. [7].

This equation is the same as equation (10), except that the source term has been determined using a surface receiver near a building. As the presence of the building is not expected to have an influence on the source term, there is little benefit to be had in using this expression.

3.3 Case 2: no building

In this case, the source and propagation terms are determined using field measurements, and the receiver term is calculated using a numerical model. As the railway exists at the measurement site, alternative source locations are not required and the governing equation for this model is:

$$\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b}) = \mathrm{L_F^{EXP}}(\mathbf{X}, \mathbf{x}') + \mathrm{TM_L^{EXP}}(\mathbf{X}, \mathbf{x}') + \mathrm{C_b^{NUM}}(\mathbf{x}', \mathbf{x}_\mathrm{b}) \tag{13}$$

Substituting the definition of the force density $\mathrm{L_F^{EXP}}(\mathbf{X}, \mathbf{x}') = \mathrm{L_v^{EXP}}(\mathbf{x}') - \mathrm{TM_L^{EXP}}(\mathbf{X}, \mathbf{x}')$ into equation (13) results in:

$$\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b}) = \mathrm{L_v^{EXP}}(\mathbf{x}') + \mathrm{C_b^{NUM}}(\mathbf{x}', \mathbf{x}_\mathrm{b}) \tag{14}$$

This expression constitutes a field measurement that accounts for both the source excitation and the propagation of vibration into the free field, and a coupling loss term, calculated using a numerical model, that accounts for both how the free field vibration is affected by the presence of the building and how the vibration attenuates through the building.

There are two possible means of quantifying the building's coupling loss term. Firstly, in Case 2a, the difference between the vibration velocity levels in the building and in the free field is used:

$$\mathrm{C_b^{NUM}}(\mathbf{x}', \mathbf{x}_\mathrm{b}) = \mathrm{L_v^{NUM}}(\mathbf{x}_\mathrm{b}) - \mathrm{L_v^{NUM}}(\mathbf{x}') \tag{15}$$

This will require two numerical models: one that contains the train, track, and soil, to obtain the free field vibration level $\mathrm{L_v}(\mathbf{x}')$; and the other that contains the train, track, soil and building, to obtain the building vibration level $\mathrm{L_v}(\mathbf{x}_\mathrm{b})$. Producing these two numerical models is a complex and time-consuming process, particularly as many parameter inputs will be needed. However, the advantage of using this definition of the coupling loss becomes clearer when it is incorporated into equation (14):

$$\mathrm{L_v^{HYB}}(\mathbf{x}_\mathrm{b}) = [\mathrm{L_v^{EXP}}(\mathbf{x}') - \mathrm{L_v^{NUM}}(\mathbf{x}')] + \mathrm{L_v^{NUM}}(\mathbf{x}_\mathrm{b}) \tag{16}$$

This equation represents a numerical prediction of the vibration velocity level in the building $\mathrm{L_v^{NUM}}(\mathbf{x}_\mathrm{b})$ together with a correction term $[\mathrm{L_v^{EXP}}(\mathbf{x}') - \mathrm{L_v^{NUM}}(\mathbf{x}')]$ that accounts for the measured vibration levels in the free field. The existence of this correction term should mitigate against the prediction errors that are introduced through modelling simplifications and lack of detailed parameter knowledge of

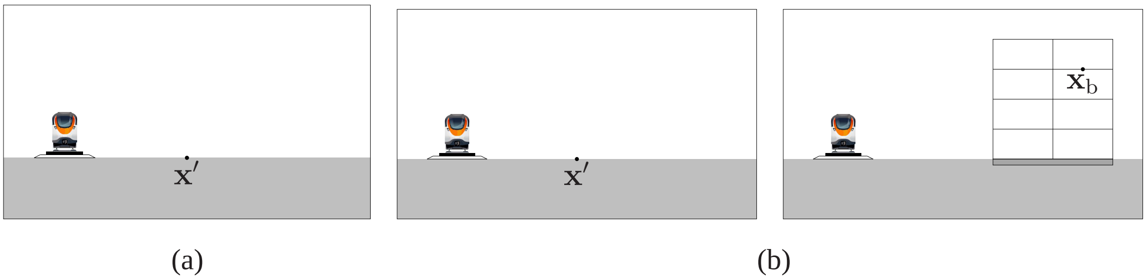

Figure 4: Case 2a: (a) the measurement site; and (b) the two numerical models.

the track and soil, making this hybrid model a powerful and useful means of improving prediction accuracy by incorporating field measurements into state-of-the-art models. Figure 4 illustrates this hybrid model. It should be noted that the location of $\mathbf{x}'$ in relation to the future building site has not been specified, however, it is expected that the closer $\mathbf{x}'$ is to the building site the more effective the use of the correction term will be.

A different means of characterising the building's coupling loss factor is used in Case 2b. Following Kuo et al. [9], an approximate expression that removes the need of a train passage by using source points located away from the track is used:

$$\mathrm{C}_{\mathrm{b}}^{\mathrm{NUM}}(\mathbf{x}', \mathbf{x}_{\mathrm{b}}) = \mathrm{TM}_{\mathrm{L}}^{\mathrm{NUM}}(\mathbf{X}_1, \mathbf{x}_{\mathrm{b}}) - \mathrm{TM}_{\mathrm{L}}^{\mathrm{NUM}}(\mathbf{X}_1, \mathbf{x}') \tag{17}$$

Substituting this into equation (14) gives:

$$\mathrm{L}_{\mathrm{v}}^{\mathrm{HYB}}(\mathbf{x}_{\mathrm{b}}) = \mathrm{L}_{\mathrm{v}}^{\mathrm{EXP}}(\mathbf{x}') + \mathrm{TM}_{\mathrm{L}}^{\mathrm{NUM}}(\mathbf{X}_1, \mathbf{x}_{\mathrm{b}}) - \mathrm{TM}_{\mathrm{L}}^{\mathrm{NUM}}(\mathbf{X}_1, \mathbf{x}') \tag{18}$$

and Figure 5 illustrates this hybrid model. One of the primary advantages of this model is that numerical models do not involve simulation of the train and track, which will simplify the modelling effort required both in terms of the model complexity and the parameter inputs. However, it is expected that there will be some dependence of the modelling results on the location of the source excitation $\mathbf{X}_1$. This dependence has not yet been quantified, and as with Case 1, requires further investigation. Moving the location of the free field receiver $\mathbf{x}'$ closer to the building site will allow the measured vibration levels to encompass a greater propagation distance, which may act to reduce the prediction uncertainty associated with the modelling of dynamic soil behaviour.

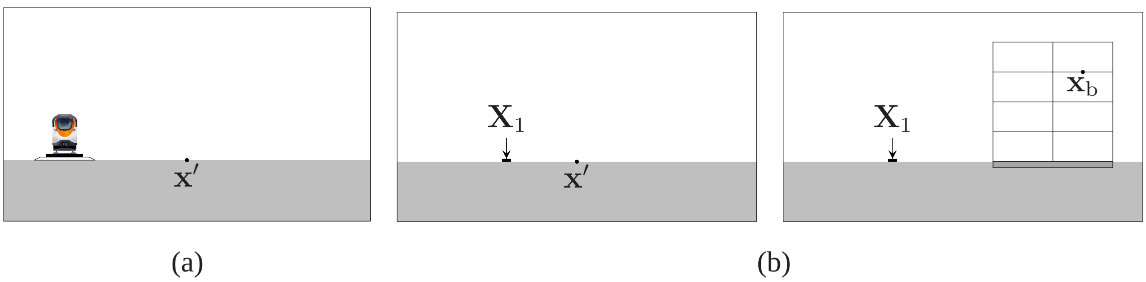

Figure 5: Case 2b: (a) the measurement site; and (b) the two numerical models.

4. Conclusions

This paper has considered two new-build scenarios, and has presented relevant hybrid equations that can be used to predict railway induced vibration levels in buildings. The first case combines a measured transfer mobility from ground surface to the building with a numerical prediction of the force density level. This provides a means of predicting building vibrations when the track is not yet constructed, or only partially constructed. The second case combines a measured vibration velocity level in the free field with a numerical prediction of the building's coupling loss. This provides a means of predicting vibration levels in a building that has not yet been constructed. In the second case, two different methods can be used for quantifying the coupling loss. Having formulated these expressions, further investigations will be undertaken to determine the effect of the location of the source and receiver points on the hybrid predictions of vibration velocity levels.

Acknowledgements

The first author is a postdoctoral fellow of the Research Foundation Flanders (FWO). The support of FWO is gratefully acknowledged.

REFERENCES

1. Coulier, P., Lombaert, G. and Degrande, G. The influence of source–receiver interaction on the numerical prediction of railway induced vibrations, *Journal of Sound and Vibration*, **333** (12), 2520–2538, (2014).

2. Kouroussis, G., van Parys, L., Conti, C. and Verlinden, O. Prediction of ground vibrations induced by urban railway traffic: an analysis of the coupling assumptions between vehicle, track, soil and buildings, *International Journal of Acoustics and Vibration*, **18** (4), 163–172, (2013).

3. Stupazzini, M. and Paolucci, R. Ground motion induced by train passage in urban area, Sas, P. and Bergen, B. (Eds.), *Proceedings of ISMA2010 International Conference on Noise and Vibration Engineering*, Leuven, Belgium, September, pp. 3547–3558, (2010).

4. Vogiatzis, K. Protection of the cultural heritage from underground metro vibration and ground-borne noise in Athens centre: the case of the Kerameikos Archaeological Museum and Gazi Cultural Centre, *International Journal of Acoustics and Vibration*, **17** (2), 59–72, (2012).

5. International Organization for Standardization, (2005), *ISO 14837-1:2005 Mechanical vibration - Ground-borne noise and vibration arising from rail systems - Part 1: General guidance*.

6. Hanson, C., Towers, D. and Meister, L. U.S. Department of Transportation, Federal Transit Administration, Office of Planning and Environment, Transit noise and vibration impact assessment, (2006).

7. Kuo, K., Verbraken, H., G., D. and Lombaert, G. Hybrid predictions of railway induced ground vibration using a combination of experimental measurements and numerical modelling, *Journal of Sound and Vibration*, **373**, 263–284, (2016).

8. Kuo, K., Lombaert, G. and Degrande, G. Characterisation of building response to railway induced vibration, Cagliari, Sardinia, Italy, April, Proceedings of the Third International Conference on Railway Technology: Research, Development and Maintenance, (2016).

9. Kuo, K., Lombaert, G. and Degrande, G. Quantifying dynamic soil-structure interaction for railway-induced vibrations, *Proceedings of the 10th European Conference on Structural Dynamics: Eurodyn 2017*, Rome, Italy, September, submitted, (2017).