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CONCERTED COLLAPSE OF CAVITIES IN ULTRASONIC CAVITATION

K.A. MØRCH

LABORATORY OF APPLIED PHYSICS I

TECHNICAL UNIVERSITY OF DENMARK

DK-2800 LYNGBY, DENMARK

Introduction

Cavitation erosion is caused by the collapse of cavitation bubbles. The pressure wave produced by collapse of a single cavity has been computed for spherical cavity collapse by Hickling and Plesset (1964) and the jet impact pressure in non-spherical collapse by Plesset and Chapman (1971). In both cases the peak pressures obtainable at a solid surface seem on the verge of being able to cause anything but elastic deformation to metals of even moderate hardness. However, it is known that plastic deformation of metals of considerable hardness begins at the onset of cavitation exposure already (Hansson, Klæstrup Kristensen and Mørch, 1978) and therefore mechanisms intensifying the cavity collapse must exist. In connection with observed bending of the trailing edge of propeller blades, Wijngaarden (1964) made calculations of pressure increase at a plate by collective collapse of a uniform layer of cavities considering the equations of liquid motion and the Rayleigh equation of cavity collapse.

In the present paper the effects of the collapse of a cavity cloud is considered on the basis of the pressure waves emitted from each cavity at collapse. Though the principles used are valid generally the computations are concerned with the hemispherical cavity cloud formed at an oscillating ultrasonic horn.

The model

We consider a longitudinally vibrating ultrasonic horn submerged in a liquid. During each cycle of oscillation a hemispherical cloud of cavitation bubbles develops and collapses, its centre being at the plane tip of the horn. The cavities develop uniformly while the horn is accellerated away from the liquid. When this accelleration decreases, collapse starts simultaneously over the whole outer shell of cavities under the influence of the ambient pressure, (Hansson, Mørch and Preece, 1977), Fig. 1.

This pressure propagates into the cloud triggering the collapse of consecutive shells of cavities. Due to the low sound velocity in a bubbly liquid the outer shell collapses completely before the next shell etc. and accordingly the collapse pressure waves from the individual cavities in a collapsing shell form a convergent pressure wave which propagates inward intensifying the collapse of successive shells and eventually it increases the pressure on the surface of the horn. This model is supported by the photographic collapse sequence by Ellis (1956) shown as Fig. 15 in his paper.

It is supposed that when collapse begins the radius of the cavity cloud is R and that spherical cavities of initial radius r_0 are uniformly distributed in all directions, arranged in concentric hemispherical shells with surface density N cavities per unit area, the total number of shells thus being $R\sqrt{N}$, Fig. 1. The collapse pressure Δp is the difference between the ambient pressure and the vapour pressure.

Exact calculation of the collapse of the cloud demands detailed knowledge of the collapse pressure waves produced by the individual cavities, their propagation and their absorption by successive cavity shells. Therefore we restrict the present calculations to two extreme idealized cases:

(a) that the inward radiated energy from simultaneous collapse of a shell of cavities is transferred into collapse energy of the next shell, and

(b) that the converging pressure wave emitted from a collapsing shell of cavities is propagated toward the centre of the cloud without being attenuated by the cavities present in the liquid.

Case (a)

The potential energy associated with the outer shell of cavities (index 1) is

$$E_1 = 2\pi R^2 N(\Delta p \tfrac{4}{3}\pi r_o{}^3) = \tfrac{8}{3}\pi^2 R^2 r_o{}^3 \Delta p N. \qquad (1a)$$

It is assumed that this energy is transferred by a factor γ to the collapse of the next shell (index 2) etc., whence

$$E_2 = 2\pi \left(R - \frac{1}{\sqrt{N}}\right)^2 N(\Delta p \tfrac{4}{3}\pi r_o{}^3) + \gamma \tfrac{8}{3}\pi^2 R^2 r_o{}^3 \Delta p N \qquad (1b)$$

$$E_{R\sqrt{N}} = \tfrac{8}{3}\pi^2 r_o{}^3 \Delta p \left[1 + \gamma \cdot 2^2 + \ldots + \gamma^{R\sqrt{N}-1} \cdot (R\sqrt{N})^2\right] \qquad (1c)$$

If we look at the energy amplification obtained at collapse of the individual cavities in subsequent shells we find an amplification factor F_i for each of the cavities in shell number i

$$F_i = \frac{E_i}{2\pi\left(R - \frac{i-1}{\sqrt{N}}\right)^2 N \cdot \Delta p \frac{4}{3}\pi r_o{}^3} \qquad (2)$$

The transfer factor γ depends on the type of cavity collapse. In case of spherical collapse more than half the energy in the spherical collapse wave is radiated outward and thus $\gamma \lesssim \frac{1}{2}$ is to be expected. However, the cavities collapse in a pressure gradient field ($dp/dR > o$), and therefore non-spherical collapse with jet formation in inward direction is expected, which is also supported by Ellis' photographs. This means that a majority of the "cavity energy" is radiated inward, $\frac{1}{2} < \gamma < 1$.

For a cavity cloud: $R = 1.5$ mm, $r_o = 0.1$ mm, $N = 7.1$ cav./mm² and $\Delta p = 0.1$ MN/m², which approximates the cloud photographed by Ellis, we find:

shell i		1	2	3	4
F_i	$\gamma = \frac{1}{2}$	1	1.9	3.1	7.3
	$\gamma = 1$	1	2.8	7.3	30

The results indicate that the collapse energy of the innermost cavities is enhanced by a factor of about 10 by transfer of energy from the cavities at larger distances from the solid surface. The simultaneous collapse of the inner shell leads to a pressure increase over the central area of the solid surface determined by addition of the pressure pulses from each cavity collapse, which is considered in case (b).

Case (b)

In this case the pressure wave set up by simultaneous collapse of a shell of cavities is considered to be propagated unaffected by remaining cavities inside the cloud and therefore we can limit the computations to effects on the solid surface of the pressure wave mentioned.

We consider small cavities of initial radius r_0 forming a hemispherical shell of radius R, with its centre on the solid surface of the horn. By collapse each cavity is supposed at time t = 0 to emit a spherical pressure wave into the interior of the volume bounded by the shell. The pressure increment at the distance r_0 from each collapse centre due to the cavity itself is termed $dp_{r_0}(t)$. At a distance r from the collapse centre this pressure wave becomes

$$(dp_r)_{t = t_1} = \frac{r_0}{r} (dp_{r_0})_{t = t_1 - \frac{r}{c}} \qquad \text{for} \quad r_0 \ll r, \tag{3}$$

where c is the velocity of sound in the liquid.

As before the number of cavities per unit area is considered to be N. At a point K on the wall situated the distance κ from the centre of the shell the pressure is built up from all waves having reached K. The wavefronts coming from a semicircular segment of radius $R \cdot \sin\theta$ perpendicular to the plane shown in Fig. 2 will be simultaneous and their number is $N \cdot \pi R^2 \sin\theta d\theta$. It is necessary that $t_1 > r_K/c$ (Fig. 2). We find that

$$r_K = \sqrt{R^2 + \kappa^2 - 2R\kappa\cos\theta} \tag{4}$$

which implies

$$1 \geq \cos\theta \geq \frac{R^2 + \kappa^2 - (t_1 c)^2}{2R\kappa} \qquad \text{for} \quad t_1 c \leq R + \kappa \tag{5a}$$

and

$$0 \leq \theta \leq \pi \qquad \text{for} \quad t_1 c \geq R + \kappa \tag{5b}$$

Further we have to know the time variation of the pressure wave at $r = r_o$ and we assume a hyperbolic (p,t) function

$$dp_{r_o} = p_o\left[\frac{C}{t + B} - A\right] , \qquad (6)$$

where p_o is the peak pressure and A, B and C are constants. Such a pressure pulse is shown in Fig. 3, in which the duration of the pulse is supposed to be 1 μsec and the pulse width at half the peak value is 10 nsec as found by Hinsch and Brinkmeyer (1976). At the arbitrary distance r the pressure is determined from eqs.(3) and (6).

By integration over $0 \leq \theta \leq \theta_{max}$, where θ_{max} is determined by eqs. 5a,b, the total pressure p_K in point K due to simultaneous cavity collapse over the shell is found to be

$$(p_K)_{t=t_1} = \int_{\cos\theta = 1}^{\cos\theta_{max}} N\pi \frac{R r_o p_o}{\sqrt{1 + (\frac{\kappa}{R})^2 - 2\frac{\kappa}{R}\cos\theta}} \left[\frac{C}{t_1 - \frac{R}{c}\sqrt{1 + (\frac{\kappa}{R})^2 - 2(\frac{\kappa}{R})\cos\theta} + B} - A\right] \sin\theta d\theta, \qquad (7)$$

which yields

$$(p_K)_{t=t_1} = \pi N \frac{R^2 r_o}{\kappa} p_o \left[\frac{cC}{R}\ln\frac{B + t_1 - \frac{R}{c}(1 - \frac{\kappa}{R})}{B} + A(1 - \frac{\kappa}{R} - \frac{t_1 c}{R})\right]$$

for $t_1 c \leq R + \kappa$ (8)

and

$$(p_K)_{t=t_1} = \pi N \frac{R^2 r_0}{\kappa} p_0 \left[\frac{cC}{R} \ln \frac{B + t_1 - \frac{R}{c}(1 - \frac{\kappa}{R})}{B + t_1 - \frac{R}{c}(1 + \frac{\kappa}{R})} - 2A\frac{\kappa}{R} \right]$$

for $t_1 c \geq R + \kappa$. (9)

Using these formulae on the outer shell of cavities in Ellis' cavity cloud it is found that the pressure on the surface of the horn develops as shown in Fig. 4.

A peak pressure of up to 6-7 p_0 is found in the centre where all the wavefronts meet simultaneously and within a radius of 150 µm p_0 is temporarily exceeded. During the build-up phase the pressure is everywhere above 0.1 p_0. This is also seen from Fig. 5, where the pressure variation with time is shown at selected radii κ = 0, 0.5 and 1.0 mm.

From the calculations of Hickling and Plesset (1964) the value of the peak pressure p_0 produced at Δp = 1 atm by spherical collapse of a single cavity at $r = r_0$ is of order 10^3 atm, and in nonspherical collapse due to a

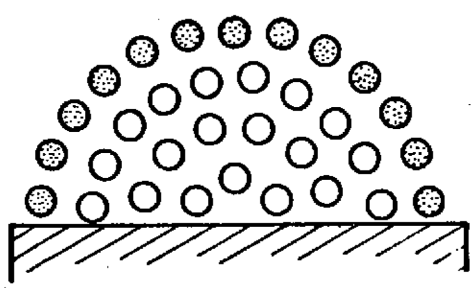

Fig. 1. Hemispherical cavity cloud. Collapse starts at outer cavity shell.

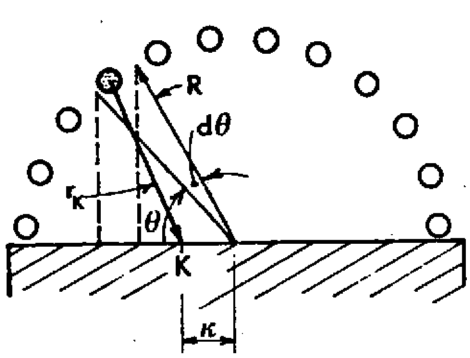

Fig. 2. Nomenclature for hemispherical shell of simultaneously collapsing cavities.

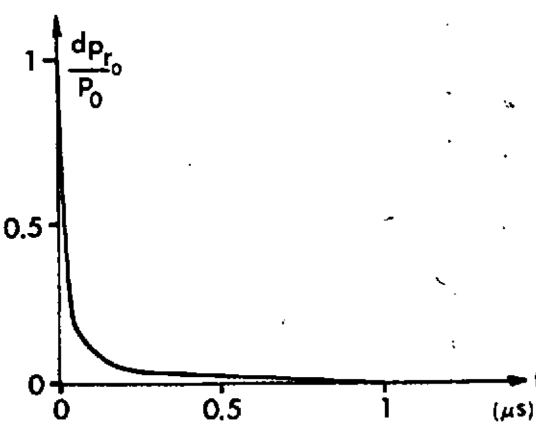

Fig. 3. Pressure pulse at $r=r_0$ from collapse of a cavity. $A=B\cdot 10^6$ (s) = 0.010204, $C=0.010308\cdot 10^{-6}$ (s).

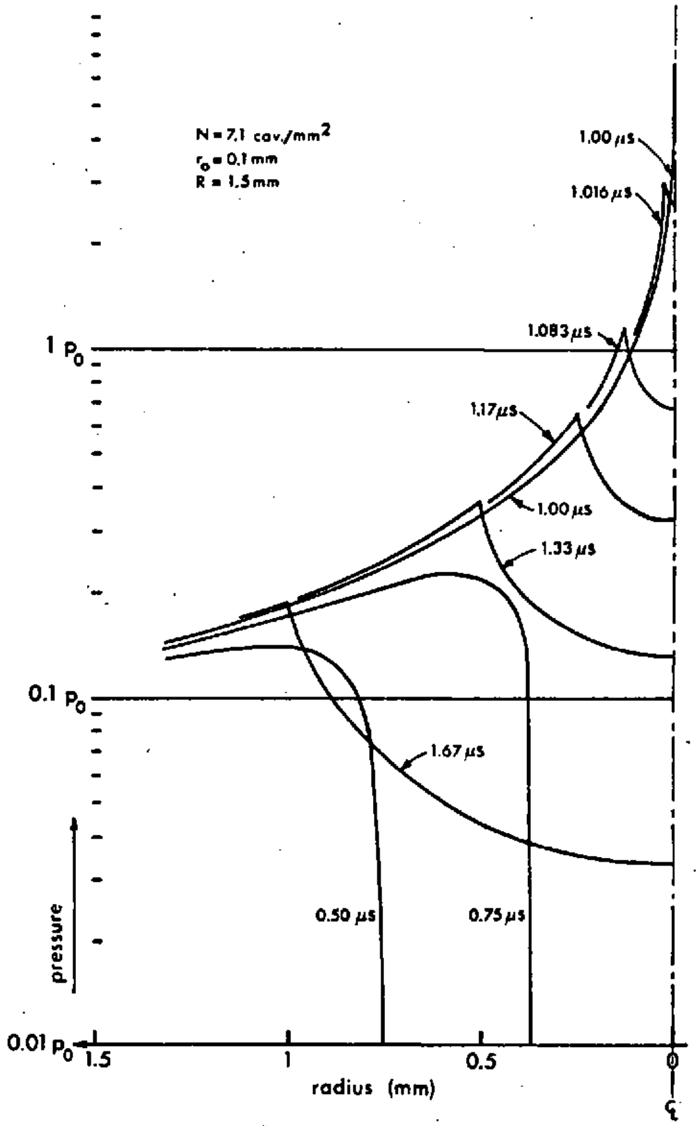

Fig. 4. Pressure distribution on the solid surface at different times after collapse.

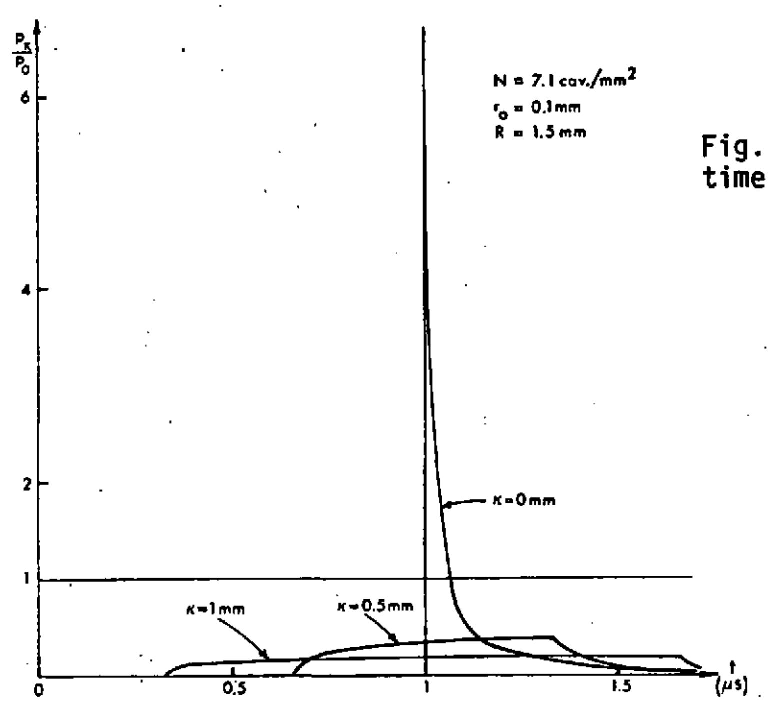

Fig. 5. Pressure variation by time at fixed values of κ.

pressure gradient, where a jet impinges the opposite cavity wall, peak pressures of the same order can be expected.

Conclucion

The hemispherical cloud of cavities formed at an oscillating ultrasonic horn starts collapsing simultaneously over its outer boundary, the collapse spreading inward to the center of the cloud. By total transfer of inward radiated energy from one shell to the next the energy level of the innermost shell of cavities is found to be considerably increased, typically by an order of magnitude, and the collapse pressure waves from the simultaneous collapse of these cavities will form an intensive converging pressure wave propagating towards the centre of the cloud. From the pressures produced by simultaneous collapse of a single shell of cavities without energy absorption by cavities inside the shell long range effects in intensity as well as in duration and space markedly exceeding those due to a single wall-near cavity are found. These effects of concerted cavity collapse are able to explain that cavitation damage can occur on metals of considerable hardness right from the beginning of cavitation exposure.

Taking into consideration that absorption from one shell to the next is only partial the principles of calculation presented in this paper form the basis of a more detailed analysis. Such an analysis can be formulated to apply also to collapse of arbitrary configurations of cavity clouds as found in flow cavitation.

REFERENCES

Ellis, A.T. 1956 Cavitation in Hydrodynamics. NPL. Her Majesty's Stationery Office.

Hansson, I., Kristensen, J.K. and Mørch, K.A. 1978 J. Phys. D., Applied Phys. 11.

Hansson, I., Mørch, K.A. and Preece, C.M. 1977 Proc. Ultrasonics International. Guildford: IPC Sci. Techn. Press.

Hickling, R. and Plesset, M.S. 1964 Phys. Fluids. 7, 7.

Hinsch, K. and Brinkmeyer, E. 1976 SPIE. High Speed Photography (Toronto). 97, 166-171.

Plesset, M.S. and Chapman, R.B. 1971 J. Fluid Mech. 47, 283-290.

van Wijngaarden, L. 1964 Proc. 11. Int. Congr. Appl. Mech. (Munich), 854-861.

DISCUSSION

Dr. Crum suggested that the more-or-less uniform erosion obtained on ultrasonic probes operated into liquids at high intensities seemed to be in conflict with the Author's idea that a general concerted collapse would concentrate the effect into small areas. The Author replied that many people had observed more intense erosion pitting near the centre of a circular probe-face, with the peripheral area hardly affected.