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GENETIC ALGORITHMS FOR CHOOSING SOURCE LOCATIONS IN ACTIVE CONTROL SYSTEM

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1. INTRODUCTION

Active noise control (ANC) is a technique whereby the average level of a soundfield is reduced by destructive interference from a number of controlled "secondary" sources of sound. The optimal choice of secondary source locations in an active noise control system is a difficult problem. The noise reduction that can be achieved using an active control system will depend on these secondary source locations, but typically the secondary sources could be positioned in many possible locations. When implementing a practical active noise control system, it is impossible to measure the reductions resulting from all possible locations of secondary sources, so that secondary source locations are selected from the most practicable positions, although there is still generally a larger number of possible positions than secondary sources which can be controlled. An engineering judgement must be made as to which locations are likely to produce the best attenuation.

At low frequencies in an enclosure this could be based on getting a reasonably uniform distribution of secondary sources throughout the space, so that the maximum number of acoustic modes can be independently controlled. At higher frequencies it appears to be better to position the secondary sources as close as possible to the assumed primary excitation source, which is generally a distribution of velocity over the boundaries of the enclosure. These guidelines were used to position the 16 secondary loudspeakers in a series of flight trials of active noise control in a propeller aircraft described by Elliott et al [3,4]. It was still necessary, however, to investigate the performance of about 26 different combinations of loudspeaker positions before a satisfactory performance could be achieved, and this is clearly an expensive process if all experiments have to be carried out in flight. There is also no guarantee that the final loudspeaker positions used were in any sense optimal.

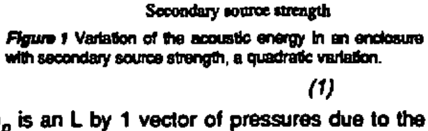


Figure 1 Variation of the acoustic energy in an enclosure with secondary source strength, a quadratic variation.

It should be emphasised that once the position of the secondary sources has been decided, the calculation of their optimal source strength is a quadratic optimisation problem (Figure 1) which can be readily solved analytically. We follow the approach of Nelson & Elliott [1] in formulating the active noise control problem in enclosures. The harmonic pressure measured at each microphone position is the sum of pressures due to primary and secondary sources, so it can be written as

$$\mathbf{p} = \mathbf{p}_p + \mathbf{Z}\mathbf{q}_s \qquad (1)$$

where **p** is an L by 1 vector of complex total pressures, $\mathbf{p}_p$ is an L by 1 vector of pressures due to the primary source alone, **Z** is an L by M matrix of complex acoustic transfer impedances and $\mathbf{q}_s$ an is M by 1 vector of complex secondary source strength.(All quantities in the following equations are assumed complex, vectors are denoted by bold lower case symbols and matrices are by bold upper case symbols). If the number of microphone(L) is greater than the number of secondary source(M), the problem is overdetermined and least squares method can be used. A cost function given by the sum of the modulus squared pressures is

$$J = \mathbf{p}^H \mathbf{p} \tag{2}$$

Thus, from equation(1) and equation(2)

$$J = [\mathbf{q}_s^H \mathbf{Z}^H \mathbf{Z} \mathbf{q}_s + \mathbf{q}_s^H \mathbf{Z}^H \mathbf{p}_p + \mathbf{p}_p^H \mathbf{Z} \mathbf{q}_s + \mathbf{p}_p^H \mathbf{p}_p] \tag{3}$$

Equation(3) is a standard Hermitian quadratic form in which $\mathbf{Z}^H\mathbf{Z}$ is positive definite. Thus, J has a global minimum when the vectors of complex secondary source strength is [1].

$$\mathbf{q}_{opt} = -[\mathbf{Z}^H \mathbf{Z}]^{-1} \mathbf{Z}^H \mathbf{p}_p \tag{4}$$

which results in the minimum value of the cost function given by

$$J_{\min} = \mathbf{p}_p^H [\mathbf{I} - \mathbf{Z}(\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H] \mathbf{p}_p \tag{5}$$

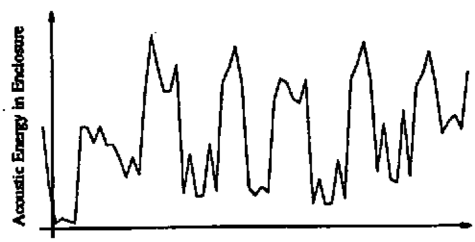
The cost function with no control is $J_{p_p} \doteq \mathbf{p}_p^H \mathbf{p}_p$. Thus a theoretical limit on the maximum achievable attenuation is given by

$$\text{Attenuation(dB)} = 10\log_{10}\left(\frac{J_{p_p}}{J_{\min}}\right) = 10\log_{10}\left(\frac{\mathbf{p}_p^H \mathbf{p}_p}{\mathbf{p}_p^H [\mathbf{I} - \mathbf{Z}(\mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H] \mathbf{p}_p}\right) \tag{6}$$

Also, the sum of the squared optimum source strengths, called the control "effort", is

$$\mathbf{q}_{opt}^H \mathbf{q}_{opt} = \mathbf{p}_p^H \mathbf{Z}[\mathbf{Z}^H \mathbf{Z}]^{-2} \mathbf{Z}^H \mathbf{p}_p \tag{7}$$

The variation of attenuation with the locations of the optimally adjusted secondary sources is, however, very definitely not a quadratic optimisation problem. As the position of any one source is varied, for example, the attenuation will typically rise and fall, with numerous maxima and minima as shown in Figure 2. The object of this paper is to study the use of various algorithms to solve this optimisation problem, and in particular to investigate the use of Genetic Algorithms (GAs) in this application.



Position of optimally adjusted secondary source

Figure 2 Variation of the acoustic energy in an enclosure with secondary source position , a non-quadratic variation.

2. VARIATION OF PERFORMANCE WITH NUMBER OF SECONDARY SOURCES

A multi-channel active control system has been constructed, which has 32 error microphones and 16 secondary source loudspeakers, for in-flight experiments on the active control of propeller-induced passenger cabin noise [3,4]. A wooden laboratory mock-up of an aircraft interior has also previously been constructed for experiments with this control system. The internal dimensions of enclosure were $2.2m \times 2.2m \times 6m$ (Figure 3). The walls were lined with 1-inch open cell foam in front of a 1-inch cavity and the floors were carpeted in order to increase the acoustic

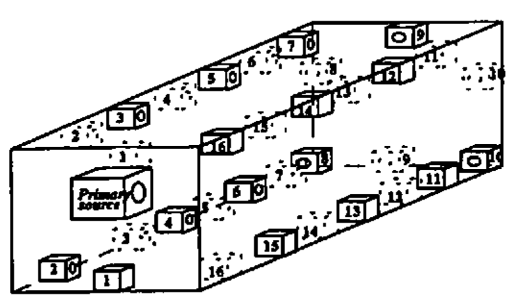


Figure 3 Experimental loudspeaker positions where position set 1 is drawn in solid line and position set 2 in dashed line

damping. Figure 3 shows two sets of positions of the 16 loudspeakers used as the secondary sources. The 32 microphones used as error sensors were uniformly placed at standing head height. The primary acoustic field was generated by a loudspeaker (of 300*mm* diameter) in the enclosure, driven from an oscillator which also provided the reference signal for the control system.

By suitable partitioning of the measured data (acoustic impedance transfer matrix and primary field pressure vector), the attenuation can be calculated for all possible combinations of secondary source positions using equation (6). For example, for the loudspeakers in the positions corresponding to set 1 in Figure 3, the attenuations have been calculated for all combinations of loudspeaker positions such that the total number of secondary sources is varied from 1 to 16. In each case there exist ${}_{16}C_1$, ${}_{16}C_2$,..., ${}_{16}C_{16}$ combinations of secondary source position. The results are summarised in Table 1 for an excitation frequency of 88 Hz.

Attenuation	Total number of loudspeakers															
[dB]	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	1	10	35	67	67	36	10	1
29	0	0	0	0	0	0	0	1	31	180	343	294	141	39	4	0
28	0	0	0	0	0	0	1	36	209	402	386	265	113	18	1	0
27	0	0	0	0	0	0	9	99	318	513	471	209	34	3	0	0
26	0	0	0	0	0	0	24	197	499	555	306	109	30	4	0	0
25	0	0	0	0	0	5	89	306	466	376	261	151	61	13	1	0
24	0	0	0	0	1	26	135	291	312	372	314	155	33	2	0	0
23	0	0	0	0	9	63	148	190	344	523	444	173	30	2	0	0
22	0	0	0	1	10	29	71	337	898	1042	550	138	19	1	0	0
21	0	0	0	1	3	18	193	816	1390	1041	377	90	14	1	0	0
20	0	0	0	0	1	49	338	949	1126	564	184	40	3	0	0	0
19	0	0	0	0	6	68	440	1126	1082	536	153	22	1	0	0	0
18	0	0	0	1	13	146	711	1103	722	317	93	11	0	0	0	0
17	0	0	0	1	29	223	716	915	622	251	49	3	0	1	0	0
16	0	0	0	9	58	271	710	1052	944	558	241	76	13	0	0	0
15	0	0	1	3	31	325	924	1141	696	230	51	2	0	0	0	0
14	0	0	0	1	67	416	928	848	456	166	26	2	0	0	0	0
13	0	0	0	10	134	670	1323	1053	351	72	38	11	1	0	0	0
12	0	0	1	28	282	969	1234	790	447	228	44	2	0	0	0	0
11	0	0	3	78	443	1074	1336	953	426	66	2	0	0	0	0	0
10	0	0	11	134	589	1115	983	402	63	4	0	0	0	0	0	0
9	0	2	44	221	677	1024	549	133	17	2	0	0	0	0	0	0
8	0	5	60	289	558	531	221	37	11	0	0	0	0	0	0	0
7	1	12	45	135	341	300	90	36	6	0	0	0	0	0	0	0
6	0	1	23	132	262	159	72	23	2	0	0	0	0	0	0	0
5	0	5	51	164	210	129	107	34	1	0	0	0	0	0	0	0
4	1	10	46	128	180	214	72	2	0	0	0	0	0	0	0	0
3	1	11	67	162	246	146	15	0	0	0	0	0	0	0	0	0
2	1	20	57	109	124	28	1	0	0	0	0	0	0	0	0	0
1	1	20	115	200	93	10	0	0	0	0	0	0	0	0	0	0
0	11	34	36	13	1	0	0	0	0	0	0	0	0	0	0	0
Number of loudspeaker combinations	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1

Table 1 Numbers of loudspeaker combinations giving specified attenuation levels when the total number of secondary loudspeakers were restricted to 1, 2,..., 16. Data set 1, 88 Hz excitation.

This table shows the occurrence of attenuation (represented by rounded values) according to the number of secondary source positions used. The maximum attenuations which can be achieved with different numbers of secondary loudspeakers are plotted in Figure 4, which also shows the corresponding results for excitation frequencies of 176 Hz and 264 Hz. At 88 Hz, 8 well positioned loudspeakers can give an attenuation which is within about 2 dB of that achieved with 16 loudspeakers. At the higher frequencies, the absolute levels of attenuation are smaller, and the achievable attenuation continues to increase with the total number of loudspeakers up to the maximum of 16.

It was also noted that at the lower frequency (88 Hz), there are relatively few combinations of 8 loudspeaker positions which give high attenuations, indicating that the loudspeakers need to be at very specific locations. At higher frequencies(with more acoustic modes excited) the exact positioning is less critical and there are a larger numbers of loudspeaker locations which give good reductions.

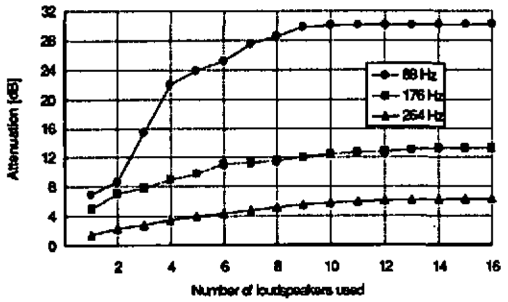
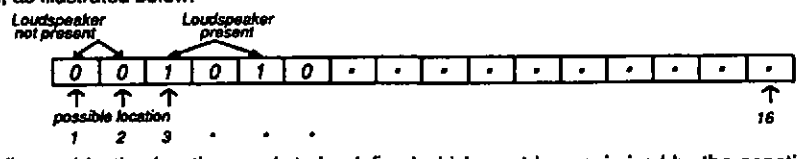


Figure 4 Maximum attenuations obtained from exhaustive searching at 88, 176 and 264 Hz according to the number of loudspeakers used

3. USE OF GENETIC ALGORITHMS IN FINDING BEST 8 LOUDSPEAKER POSITIONS FROM 16

The purpose of this work is to apply genetic algorithms to the problem of finding optimum secondary source loudspeaker locations in a feedforward active noise control system. The use of genetic algorithms in finding optimal actuator locations in the feedback control of structural vibration have been reported in [9], in which the best 3 locations from a possible 8 were calculated for which there are 56 combinations. In active noise control we are typically interested in optimising the position of a larger number of secondary sources from a much larger number of possible locations. In the final example reported in this paper the problem of finding the best 8 locations from a possible 32 is studied, for which there are over 10^7 possible combinations.

We must first code the loudspeaker positions in a way which can be used by a genetic algorithm. A binary coding has been used, with each possible secondary source position being indicated by a bit position in a binary string, with each bit having a value of *0* or *1* indicating the absence or presence of a source, as illustrated below.



Secondly an objective function needs to be defined which must be maximised by the genetic algorithm. The reduction in the sum of the squared outputs of the microphone array, in dB, as described in equation (6), is called the attenuation below, and was used as the objective function, or "*fitness*", in the genetic algorithm.

3.1 INTRODUCTION TO GENETIC ALGORITHMS

Genetic algorithms (GAs) are robust stochastic global optimisation procedures for finding the global maximum (or minimum) of a multimodal function. GAs require the natural parameter set of the optimisation problem to be coded as a finite-length string containing alphanumeric characters called genes(bits). Genetic algorithms starts with a population of randomly selected strings. The fitness value for each member of this initial population, for each initial string, is then calculated. The strings in this first generation are then selected at random, but with a probability proportional to their fitness in order to perform a reproduction operation to generate the next generation of strings. After selection, various genetic operators such as Crossover and Mutation are used to extract common properties shared by two good strings which are then "*mated*" and used to provide selected offsprings for the next generation. The process is repeated until convergence is achieved to a population dominated by the global maximum of the fitness function or satisfying user defined conditions. Genetic algorithms are robust and computationally simple to implement and are not limited by restrictive assumptions(continuity, existence of derivative, unimodality, etc.) about the searching space.

A simple genetic algorithm used in many practical problems is composed of three operators : Reproduction, Crossover and Mutation [5]. Reproduction is a process in which individual strings are used again at random in the next generation according to their objective function value (fitness value). Hence, strings with higher fitness have a higher probability of composing one or more offspring in the next generation. One of the easiest way of implementing this operator is to create a biased roulette wheel where each string in the current population has a roulette wheel slot sized proportional to its fitness. Every spin of the weighted wheel then yields the next candidate. In this way, strings with higher fitness have a higher number of offspring in the succeeding generation but the process of selection is still random. In a crossover operation, newly reproduced strings are "mated" at random. Each pairs of strings undergoes crossover as follows : an integer number *k*, which determines the crossover site, is selected at random between 1 and the string length less one, with uniform probabilities. Two new strings are created by swapping all genes behind the crossover site(called as a "tail" of a string). For example, assume *string A='0101'*, *string B='0110'* and crossover site *k=3*. The crossover operation yields two new strings (*A'* and *B'*) as follows where the crossover site is marked with a dotted line.

<Before>		<After>
A = ' 0 1 0 ⁞ 1 '	$\xrightarrow{\text{crossover}}$	A' = ' 0 1 0 ⁞ 0 '
B = ' 0 1 1 ⁞ 0 '		B' = ' 0 1 1 ⁞ 1 '

The mutation operator also plays an important role in the operation of genetic algorithms. In the simple genetic algorithm, mutation is the random alteration of the value of genes in the strings, usually with a small probability. In the binary coding of a string, this simply means changing a '1' to a '0' and vice versa. The mutation operator helps protect a genetic algorithm against converging to local optimal points in the searching space. It acts as an insurance policy against premature converging behaviour. Empirical studies of genetic algorithm have shown that good mutation probability is on the order of one mutation per thousand position transfers [6].

3.2 A SPECIFIC CONSTRAINT IN APPLYING GENETIC ALGORITHMS TO ANC PROBLEM

The secondary source loudspeakers used were coded using a binary format, i.e. a string *'0000000011111111'* means that the first 8 speakers(from loudspeaker No.1 to No.8) are not used and last 8 loudspeakers(from No.9 to No.16) are used. Because of the restriction that the total number of loudspeakers used should be a constant, general genetic algorithms could not be used directly without modifications. For instance, consider a simple example of selecting 3 locations from 6 and assume string *A='000111'* and string *B='111000'* are mated in a crossover operation. For the given two strings, any crossover site will cause the two strings after crossover to not have three '1's in any strings. If the crossover site is 3, for example, string *A* becomes *'000000'* and string *B* becomes *'111111'*. A number of methods were investigated of applying the constraint that the total number of sources remain constant during crossover [7]. The most successful was to identify the total number of pairs of genes (bits) which were different in the two parent strings, and to swap a random but even number of these pairs between the two strings to form the next generation, while maintaining a constant total number of secondary sources ("Modified random reordering"). Furthermore, in the mutation operation, a mutation of one gene in a string should be accompanied by another mutation of a different gene which has different value to keep the constant number of '1's in the string. That is, a mutation of one gene in a string from '1' to '0' should be followed by another mutation of one gene in the string from '0' to '1'.

On the basis of the simple genetic algorithm, various modifications were investigated such as the "Penalty Method" [5] , "Constrained random crossover" [7], "Modified random reordering" [7], "Elitist model" [5], "Variable mutation probability" [7], "Linear scaling" [5], etc. 9 modified genetic algorithms were developed using the above techniques which incorporate this constraint and the results were compared to each other(see reference [7] for the details of each algorithms and their comparative performance). Among all these possibilities, algorithm SG33 showed the best performance in which "linear scaling", "stochastic remainder selection without replacement", "Modified random reordering", "Variable mutation probability" and "Elitist model" techniques were used.

An example of the result of using algorithm SG33 to choose the best 8 loudspeaker positions from a possible 16 in set 1 is shown in Figure 5, which also shows the results of using three other variations on this algorithm. The maximum attenuation resulting from any of the 30 strings in each generation is plotted against generation number, averaged over the results of 20 applications of the genetic algorithm. From the possible 12870 combinations, the algorithm finds a loudspeaker arrangement which has an attenuation within about 0.5 dB of the best possible (from Figure 4) after an average of 8 generations, i.e. after evaluating the results of 240 strings.

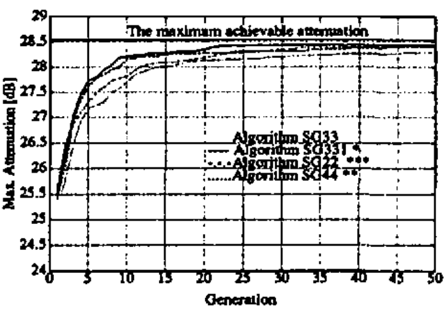


Figure 5 Simulation results using genetic algorithms for the problem of finding optimal 8 secondary source locations from 16 (position set 1 at 88Hz).
* SG331 : SG33 + fixed mutation probability scheme.
** SG44 : SG331 + constrained random crossover.
*** SG22 : SG44 + simple random crossover & mutation + no elitist model

4. USE OF GENETIC ALGORITHMS IN FINDING BEST 8 LOUDSPEAKER POSITIONS FROM 32

If L microphones and M secondary source loudspeakers are used in the control system, the measured impedance matrix (Z) has dimensions L × M, in equation (1). The **Z** matrix can be augmented simply by attaching another **Z** matrix for other set of secondary source positions to the original **Z** matrix, or can be reduced simply by deleting some columns from the original. For example, two different transfer impedance matrices Z1 (LxM) and Z2 (LxN), measured for two different sets of secondary source positions, can be combined to form an L by M+N **Z** matrix under the assumption of linear superposition of acoustic response. A reduced L by M-N **Z** matrix can also be obtained by eliminating N columns from the original. Predicted maximum attenuations can thus be calculated for any combination of loudspeakers and microphones from the new **Z** matrix and similarly partitioned primary field vector $\mathbf{p}_p$.

Using the measured transfer impedance data for the two sets of secondary source positions shown in Figure 3, the attenuation of any combination of loudspeakers from both sets can be calculated. In this way the maximum attenuation which can be obtained by choosing 8 loudspeaker positions from the possible 32 in both sets has been calculated. This is a much larger problem than that discussed above, with 10,518,300 possible combinations. Figure 6 shows the results of using the genetic algorithm SG33 in this case, in which the objective function (fitness) was taken as the sum of the mean square pressures at the 32 microphones at 88 Hz. An exhaustive search of all possible combinations was also run for this case to establish the best possible performance, a procedure which would be prohibitively expensive under normal circumstances since it requires the equivalent of a months running time on a 486PC !

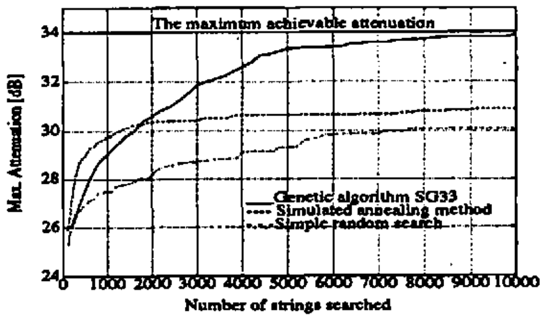


Figure 6 Algorithm performance comparison : Genetic algorithm SG33, Simulated annealing method and simple random searching for the problem of finding optimal 8 secondary source positions from 32 at 88Hz(averaged result of 20 runs per each method).

Figure 6 also shows the results of a simple random searching in which the initial population generating procedure used in the GAs is repeated every generation so that all the strings are generated in a purely random way without any genetic operations. Also, a simulated annealing method [8] was investigated for comparison. The simulated annealing method is an optimisation technique suitable for very large scale problems. For this problem, the initial set of loudspeaker positions is selected randomly and the objective function is calculated. New solutions are generated by replacing two location with dissimilar values at random and monitoring the change in the objective function value. If the new configuration produces a higher attenuation value, it is accepted unconditionally. If not, it is accepted with probability $e^{(-E/T)}$, where E is the change in the objective function and T is a "temperature" selected such that the initial acceptance rate is approximately 50 percent. If the new solution is not accepted, the previous configuration is restored. If the convergence criteria for a particular value of T is satisfied or the maximum number of iterations has been reached, T is reduced by a user specified amount, decreasing the probability of acceptance, and the best solution is used as a starting point. Final convergence occurs when the solution remains unchanged for a predetermined number of times. Since too rapid "cooling" increases the probability of converging on a local optimum and too slow cooling causes high cost in the searching, a balance must be obtained by adjusting the parameters. In the simulations for this problem, an initial value of T=50 as above was used and then T was decreased by assigning each subsequent value to be 90 percent of the current value. Also, the number of iterations required at each temperature was set to 200 and the maximum number of iteration success in each temperature was set to 20. The result of the simulated annealing method was compared with those of genetic algorithms, in Figure 6. Algorithm SG33 showed better performance than simulated annealing method in this case, although it cannot be definitely concluded that genetic algorithm will always out perform

No.	Binary coded strings representing secondary source positions (1:loudspeaker present 0:loudspeaker not present)	Attenuation (dB)	Number of cases found from 20 runs by SG33
1	11000000000000101010100000010100	34.004	15
2	11000000000010100110100000000100	33.651	17
3	11001100000000101010000000000100	33.609	2
4	11000000000010101010100000000100	32.971	18
5	11000000000000100110100000010100	32.949	18
6	01010001000000101010000000000101	32.932	4
7	01010000000000101010001000000101	32.841	1
8	01010000000000101010000010000101	32.646	2
9	11000010000001100010100000000100	32.622	7
10	01000000000000101010100010000101	32.613	1

Table 4 Best 10 strings found by exhaustive searching for the problem of finding optimal 8 locations from 32 at 88Hz and number of cases found by 20 runs of the genetic algorithm SG33.

simulated annealing methods since algorithm SG33 was developed through many refining procedures to produce the best results but the simulated annealing method was not so exhaustively developed.

Table 4 shows the 10 strings found by the exhaustive search which produced the best attenuations at 88 Hz. Also shown is the number of times each of these strings was identified over 20 runs of the genetic algorithm SG33 which were allowed to run for 50 generations with 200 strings per generation.

Algorithm SG33 was also used (for a total of 100 generations with 200 strings per generation) to find *optimal* 8 locations, in which the objective function was taken as the sum of the mean square pressures at the 32 microphones at the three *harmonic* frequencies 88 Hz, 176 Hz and 264 Hz. The results indicated that best performance may be obtained with the combination of loudspeaker numbers 2, 3, 5 and 15 in position set 1 and number 1, 3, 14 and 16 loudspeakers in the position set 2 (Figure 3). It is interesting to note that these loudspeaker positions are not just those closest to the primary source. An experiment was then performed using loudspeakers placed at those positions in which an adaptive algorithm [2] was used to adapt the outputs to the loudspeakers at the three frequencies to minimise the output from the 32 microphones. The results, together with the predicted reductions, are shown in Table 5. Despite the small differences between calculated and measured attenuations, the genetic algorithm had clearly found a set of secondary source positions which gave very good reductions in practice. There may be many reasons for the differences between expected and measured values in Table 5. Among them, the most significant reason is thought to be inconsistencies in the data for the primary pressure field. In fact, there exist two different primary field vectors ; measured with the loudspeakers in the position set 1 and position set 2. The two vectors were expected to be identical, in practice, however the acoustic field formed by the primary source was slightly changed by the secondary source loudspeakers located in the two different sets of positions. Averaged values of the primary field were used in the objective function calculations in the genetic algorithms used, which were thus not entirely consistent with the practical measurement conditions. The differences between calculated and measured attenuations were, however, less than 1 dB at all frequencies.

Frequency [Hz]	Predicted Attenuation[dB] (from GA search)	Measured Attenuation[dB] (from experiments)
88	30.98	30.0
176	10.67	10.5
264	7.25	8.0

Table 5 Attenuation in a model cabin using 8 secondary sources found by genetic algorithm SG33.

5. DISCUSSIONS AND CONCLUSIONS

In Figure 7, the control efforts (equation (7)) are plotted according to the descending order of attenuation values (equation (6)) for the best 100 cases for the problem of finding the best 8 secondary source positions from 16 using data taken from position set 1 at 88 Hz. This shows that the optimal control effort is very similar for many loudspeaker combinations, but substantially higher for others. Among these 100 cases, experiments were performed for the best 7 secondary source position sets and showed good agreements(within 0.5 dB difference) between the predicted and

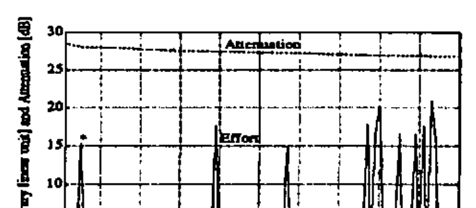


Figure 7 Attenuations and corresponding Efforts plot for the best 1000 cases in attenuation found by exhaustive searching for the problem of finding optimal 8 locations from 16(position set1 at 88Hz).

measured attenuations in all but one case. In that case (marked with • in the Figure 7), the effort required was relatively high, the measured attenuation was 3 dB less than that predicted and a much longer time was needed for the control system to converge. In order for the genetic algorithm not to select such ill conditioned solutions [2], it may be advantageous to include an effort term in the objective function used to assess the fitness of each string.

As a way of confirming the robustness of the positions for the secondary sources, found by minimising the sum of the mean square pressures at the three harmonic frequencies using genetic algorithm SG33, the primary source fundamental frequency used in the experiments was changed from 88 Hz to 89 Hz and to 87 Hz. As shown in Table 6, the measured reductions produced by the ANC system under these conditions were still high, even though the transfer impedance matrix used by the control system was learned at 88 Hz. We have thus found that the source locations found by the genetic algorithm are reasonably robust to small changes in the conditions of the experiment, but this robustness has not been exhaustively explored, and further work on this aspect of the problem is currently underway.

Primary source fundamental frequency [Hz]	89	87
Attenuation at 1st harmonic [dB]	30.2	29.8
Attenuation at 2nd harmonic [dB]	8.9	10.9
Attenuation at 3rd harmonic [dB]	6.2	7.6

Table 6 Experimental results of attenuation for the first three harmonics (where primary source was driven at 89 Hz and 87 Hz each) in a model cabin using 8 loudspeakers.

In this paper we have concentrated on finding the optimal secondary source locations for a feedforward active control system designed to control a pure tone enclosed soundfield (an interior problem). We have found that genetic algorithms provide an efficient and robust search procedure for such problems, which appears to perform better than random searching or simulated annealing algorithms. Similar techniques could be used to find the optimal secondary source locations in the control of sound radiation from a body (an exterior problem), or in the active control of vibration using mechanical actuators. The genetic algorithms developed above may be also applied to find optimal error microphone positions so that the total number of microphones could also be optimised. The method could also be extended to the active control of broadband noise using either feedforward or feedback methods.

6. REFERENCES

[1] P A Nelson & S J Elliott, 'Active control of sound', *Academic press*, (1992)

[2] S J Elliott, C C Boucher & P A Nelson,. 'The behavior of a multiple channel active control system', *IEEE Transaction on Signal Processing* vol. 40, no. 5 (1992)

[3] S J Elliott, P A Nelson, I M Stothers & C C Boucher, 'Preliminary results of in-flight experiments on the active control of propeller-induced cabin noise', *Journal of Sound and Vibration*, vol. 128, 355~357 (1989)

[4] S J Elliott, P A Nelson, I M Stothers & C C Boucher, 'In-flight experiments on the active control of propeller-induced cabin noise', *Journal of Sound and Vibration*, vol. 140, 219~238 (1990)

[5] D E Goldberg, '*Genetic algorithms in search, optimization, and machine learning*', Addison-Wesley (1989)

[6] K A De Jong, 'An analysis of the behavior of a class of genetic adaptive systems', *Doctoral dissertation*, University of Michigan (1975)

[7] K H Baek, 'Genetic algorithms for choosing source locations in active control system', *MSc thesis*, University of Southampton(ISVR) (1993)

[8] W H Press et al, 'Combinatorial Minimisation : Method of Simulated Annealing', *Numerical Recipes*, Cambridge University Press, 326~334 (1987)

[9] S S Rao & T S Pan, 'Optimal placement of actuators in actively controlled structures using genetic algorithms', *AIAA journal*, vol.29, no.6, 942~943 (1991)