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OPTIMUM THICKNESS DISTRIBUTION OF DAMPING LAYERS FOR SQUARE PLATES

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INTRODUCTION

Effective design of viscoelastic damping layer treatments for plates and panels often requires that the added weight (or cost) of the damping material be minimized, while maximizing the added damping. In this paper, we consider the optimum distribution of thickness of unconstrained damping layer treatments applied to one side of edge-fixed and simply-supported square plates vibrating in the lower flexural modes. While the optimum distribution of damping material along a beam has been studied by several investigators, this is believed to be the first application of optimization methods to the design of damping layer treatments for plates. The general approach used is described, and the optimum distribution of damping material is presented for several select cases.

GENERAL APPROACH

An expression for the loss factor of plates with partial unconstrained-layer viscoelastic damping treatments applied to one side can be obtained from the results presented in [1]. In applying these results to the current problem, the plate surface is represented by a 4x4 grid of area elements of equal size. The thickness of the damping layer is assumed to be constant over each element of area, but variable from element to element. It is further assumed that the thickness distribution is symmetrical with respect to axes with origin at the center of the plate and aligned with its edges.

*This work was completed while the authors were associated with the Department of Ocean Engineering, Florida Atlantic University. Their support is gratefully acknowledged.

Many unconstrained-layer damping treatments satisfy the thin, flexible layer conditions $n \ll 1$ and $nh \ll 1$, where n is the ratio of the real part of the complex modulus of the viscoelastic layer to the elastic modulus of the plate and h is the ratio of the thickness of the layer to that of the plate. For this case, the system loss factor, η, for a plate with m surface elements is given by the expression

$$\eta/\eta_c = (n \sum_{j=1}^{m} \alpha_j L_j)/(1 + n \sum_{j=1}^{m} \alpha_j L_j) \tag{1}$$

where η_c is the loss factor for the damping material. In this expression,

$$\alpha_j = (4h_j^3 + 6h_j^2 + 3h_j)(1-\nu_p^2)/(1-\nu_c^2) \tag{2}$$

is a thickness parameter and

$$L_j = [\int_{A_j} (\nabla^2 \omega)^2 dA - \int_{A_j} (\omega_{xx}\omega_{yy} - \omega_{xy}^2) dA] / \int_{A_p} (\nabla^2 \omega)^2 dA \tag{3}$$

is a parameter involving integrals of the mode shape over the element area A_j and plate area A_p.

It is apparent from Eq.(1) that the loss factor can be maximized by maximizing the function $F(h_j) = \Sigma\alpha_j L_j$. Maximization of the loss factor while minimizing the added weight (volume) of the damping material leads to a constrained optimization problem, which was handled numerically using an algorithm similar to those presented in [2,3]. The constraints used are that the thickness ratios be positive and finite and that the total volume of damping material for any one given maximum allowable thickness ratio be equal to that for a damping treatment of uniform thickness equal to that of the plate (h=1). Details of the algorithm used are given in [4].

RESULTS

Optimum distributions of thickness of unconstrained-layer damping treatments were obtained for the first three modes of flexural vibration of edge-fixed and simply-supported square plates with maximum admissible coating to plate thickness ratios ranging from 1.0 to 2.0.

The mode shapes were assumed to be the same as for the bare plate and were taken in the form of a single product of the corresponding mode shapes for beams. The viscoelastic material was assumed to be incompressible ($\nu_c = 0.5$), with $\nu_p = 0.3$ and the modulus ratio $n = 0.01$, which is representative of a commercially available damping material on an aluminum plate.

The results, listed in Tables 1 and 2, are expressed in terms of the percentage increase in damping that can be achieved over that obtained with the same amount of damping material applied to a uniform thickness over the entire plate:

$$\% \text{ Difference} = 100\ (\eta_{optimum} - \eta_{uniform})/\eta_{uniform} \qquad (4)$$

Table 1. Optimum Coating Thickness Ratios for Simply-Supported Plates

Max Allow. Thk. Ratio (h)	Optimum Thickness Ratios:				Modes: First		Second		Third	
	h_1	h_2	h_3	h_4	η/η_c	% Dif.	η/η_c	% Dif.	η/η_c	% Dif.
1.0	1.0	1.0	1.0	1.0	.1362	.00	.1362	.00	.1362	.00
1.1	1.1	1.1	.7	1.1	.1494	9.66	.1500	10.08	.1409	3.41
1.2	1.2	1.2	.4	1.2	.1682	23.44	.1692	24.15	.1537	12.81
1.3	1.3	1.3	.1	1.3	.1912	40.36	.1925	41.27	.1728	26.80
1.4	1.4	1.2	.0	1.4	.2090	53.39	.2108	54.72	.1813	33.10
1.5	1.5	1.0	.0	1.5	.2260	65.88	.2285	67.72	.1873	37.51
1.6	1.6	.8	.0	1.6	.2461	80.66	.2492	82.92	.1980	45.31
1.7	1.7	.6	.0	1.7	.2688	97.27	.2723	99.87	.2126	56.04
1.8	1.8	.4	.0	1.8	.2933	115.27	.2972	118.14	.2305	69.18
1.9	1.9	.2	.0	1.9	.3191	134.24	.3233	137.32	.2510	84.21
2.0	2.0	.0	.0	2.0	.3458	153.81	.3502	157.05	.2733	100.62

Table 2. Optimum Coating Thickness Ratios for Edge-Fixed Plates

Max.Allow. Thk. Ratio (h)	Optimum Thickness Ratios:				Modes: First		Second		Third	
	h_1	h_2	h_3	h_4	η/η_c	% Dif.	η/η_c	% Dif.	η/η_c	% Dif.
1.0	1.000000	1.000000	1.000000	1.000000	.1363	.00	.1363	.00	.1363	.00
1.1	1.100000	1.100000	1.100000	.665972	.1557	14.26	.1564	14.77	.1512	10.94
1.2	1.200000	1.200000	1.200000	.331944	.1790	31.34	.1801	32.18	.1716	25.89
1.3	1.300000	1.300000	1.298080	.000000	.2050	50.42	.2065	51.53	.1958	43.69
1.4	1.400000	1.400000	1.089740	.000000	.2142	57.15	.2276	67.02	.2044	50.01
1.5	1.500000	1.500000	.881410	.000000	.2282	67.47	.2513	84.41	.2178	59.85
1.6	1.600000	1.600000	.673077	.000000	.2464	80.83	.2770	103.28	.2353	72.69
1.7	1.700000	1.700000	.464744	.000000	.2680	96.63	.3042	123.23	.2561	87.95
1.8	1.800000	1.800000	.256410	.000000	.2920	114.26	.3323	143.88	.2794	105.06
1.9	1.900000	1.900000	.048077	.000000	.3177	133.14	.3609	164.90	.3045	123.45
2.0	2.000000	1.839740	.000000	.000000	.3310	142.85	.3720	173.02	.3172	132.77

Figure 1 shows the general trend of the optimum damping layer thickness distributions. It also indicates the location over the plate quadrant of the area elements 1 - 4 for which the optimum thickness ratios are listed in Tables 1 and 2.

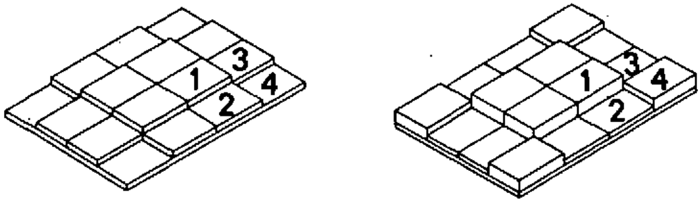

Figure 1. Optimum Coating Thickness Distribution for a Simply-Supported Plate (left) and Edge-Fixed Plate (right).

CONCLUDING REMARKS

The results presented reveal that the system loss factor can be increased by as much as 100% or more by optimizing the thickness distribution of the damping treatment. Also revealed are the regions of the plate where added damping treatments are most effective. These latter results should prove very useful in the design of partial damping layer treatments for plates.

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