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DAMPING OF SQUARE PLATES WITH CONSTRAINED-LAYER DAMPING TREATMENTS

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INTRODUCTION

The damping of plates with constrained-layer viscoelastic damping treatments has been considered by numerous investigators. However, because of mathematical difficulties, analytical solutions have been obtained only for plates with simply-supported edge conditions. Finite element programs capable of analyzing such structures are available, but typically they are expensive to execute if one considers a wide range of parameter values. In this paper, we present the results of a project to obtain in a computationally-efficient manner approximate solutions for the natural frequencies and loss factors of simply-supported and edge-fixed square plates vibrating in the lower flexural modes. A modified Rayleigh-Ritz procedure was used. The general approach is described, and the results are compared with theoretical and experimental results from the literature.

GENERAL APPROACH

It is shown in [1] that the natural frequency and loss factor of a viscoelastically damped system can be obtained using the Rayleigh-Ritz procedure and the energy expressions for the corresponding elastic system with the appropriate elastic moduli replaced by the corresponding complex moduli of the viscoelastic components. The resulting system eigenvalues are complex-valued quantities of the form $E = R + iI$, where

$$\eta = I/R \tag{1}$$

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in the system loss factor and

$$f = 2\pi R^{\frac{1}{2}} \quad (2)$$

in the system natural frequency.

Expressions for the kinetic and potential energies for a three-layer system consisting of an elastic plate, a viscoelastic damping layer and an elastic constraining layer were derived [1]. Small displacement theory was used, and the following assumptions were made:

1. Within each layer, the in-place displacements vary linearly through the thickness.
2. Plane stress conditions exist in all three layers.
3. Changes in layer thicknesses are negligible.
4. All layers are homogeneous and isotropic.
5. Transverse shear stresses in the plate and constraining layer are negligible.
6. Perfect continuity of displacements exists at the interfaces between layers.

These same assumptions have been used in numerous studies of layered plates.

As a result of these assumptions, the strain energy consists of the bending and stretching energies of the three layers and the transverse shear energy of the viscoelastic core. These quantities can be expressed in terms of five independent displacement variables: the transverse displacement and two components of in-plane displacement for the plate and for the constraining layer. The assumptions made preclude the possibility of prescribing different boundary conditions on the transverse displacement for the different layers, although different conditions can be imposed on the in-plane displacements.

The in-plane displacements were assumed in the form of a series of products of the eigenfunctions for the axial vibration of rods, while the lateral displacement was taken in the form of a series of products of the eigenfunctions for the lateral vibration of beams. Four terms were taken in each of the series, resulting in a total of twenty undetermined coefficients in the assumed displacements and twenty degrees of freedom in the plate model. Space limitations preclude listing of the assumed displacement functions here. Suffice it to say that for the simply-supported case, both the plate and the constraining layer were assumed hinged along the edges. For the edge-fixed case, the constraining layer was assumed to be free to undergo in-plane displacements at the edges (zero in-plane tractions at the boundary). These boundary conditions were chosen for compatibility with those used in the investigations [2 - 4] from which comparison data were obtained.

A computer program was written to evaluate the energy expressions and to execute the Rayleigh-Ritz procedure. This program made use of a standard existing algorithm for complex-valued eigenvalue problems. The eigenvalues obtained from the program correspond to various bending, stretching and shear modes of the plate, and these can be easily identified by examination of the corresponding eigenvectors. Only the bending modes were of interest in this investigation.

RESULTS

Natural frequencies and loss factors for the first three flexural modes of simply-supported and edge-fixed square plates were computed using the approach described and the results compared with those from the literature. Table 1 provides a comparison of results with those of Reddy et. al. [2] for an edge-fixed aluminum plate, 2mm thick and 500 mm on a side, with a 2-mm thick aluminum constraining layer and a polyvinyl chloride viscoelastic damping layer of various thicknesses, h[2].

Table 1. Natural frequencies and loss factors for edge-fixed square plates.

Mode	$h^{(2)}$ (mm)	f(hz)			η		
		Reddy[2] (expt.)	Reddy[2] (theory)	Current (theory)	Reddy[2] (expt.)	Reddy[2] (theory)	Current (theory)
1	3	92	112	120	0.14	0.22	0.26
	4	93	112	118	0.18	0.24	0.27
	5	98	111	118	0.17	0.25	0.27
2	3	189	194	201	0.14	0.20	0.22
	4	158	190	198	0.15	0.21	0.23
	5	124	180	196	0.13	0.22	0.23
3	3	245	256	270	0.12	0.19	0.19
	4	250	248	265	-	0.21	0.20
	5	250	242	261	0.13	0.22	0.21

The theoretical results in [2] were obtained using finite element methods. The current and the finite element predictions are in reasonably close agreement, but both yield frequencies and loss factors that are higher than the experimental values. However, the experimental results presented in [2] are suspect, since the natural frequencies reported for the simply-supported case are higher than those for the edge-fixed case. Unfortunately, no other experimental data was available for comparison. Other comparisons made showed excellent agreement between the theoretical results from [3] and [4] and those obtained from the current theory.

CONCLUDING REMARKS

It has been demonstrated that the Rayleigh-Ritz procedure can be used to obtain approximate solutions for the natural frequencies and loss factors of plates with constrained-layer damping treatments. This procedure has certain computational advantages over finite element methods, and higher-order effects such as core stretching and in-plane and rotatory inertia can be easily included.

The shear displacement in the core depends upon the in-place displacements in the plate and constraining layer. Accordingly, the in-place displacements must be approximated with reasonable accuracy. However, several examples considered indicated that good results can be obtained using only a one-term solution for the transverse displacement. This reduces the computational time significantly.

REFERENCE

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