BRITISH ACOUSTICAL SOCIETY

71/17

"Finite Element Techniques in Structural Vibration"

March 24th, 25th, 1971.

DYNAMIC AND STATIC ANALYSIS OF DOUBLY CURVED HONEYCOMB SANDWICH PLATES BY THE FINITE ELEMENT METHOD

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Summary

The free vibration and static characteristics of doubly curved honeycomb sandwich plates are studied by the finite element method using an element having five and seven degrees of freedom per node. Both constant and parabolic variation of strain in the core is used. Various parametric studies are conducted to determine their effects on free vibrations of sandwich plates.

1. INTRODUCTION

In many of the high intensity acoustic environments encountered in aerospace structures, honeycomb sandwich plates are the lightest structures to withstand the acoustic pressures. The usual form of construction has thin face plates (aluminium, titanium or stainless steel) separated by an aluminium or glass fibre honeycomb core. In this configuration the direct forces due to bending are carried in the face plates and the shear forces are carried in the core.

The majority of analytical work to date has been on flat honeycomb sandwich plates |1,2,3| with very little on singly |4,5| or doubly curved plates |6,7|. The purpose of this paper is therefore to provide a method for determining the dynamic and static properties of curved honeycomb plates such as might be used in aircraft structures.

2. POTENTIAL ENERGY EXPRESSION, EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The free vibration and static characteristics of curved sandwich plates can be determined by using the theorem of stationary potential energy. The potential energy functional of sandwich plates is written as:

$$\pi = U + V + T \qquad (1)$$

where U, V, T are the strain energy, potential energy and kinetic energy respectively of the sandwich plate. These expressions are derived using strain-displacement relationship given by |8|:

$$U = \tfrac{1}{2}\int_0^b\int_0^a \{D|(\overset{\prime}{u} + \frac{w}{R_1})^2 + (\overset{\circ}{v} + \frac{w}{R_2})^2 + 2\nu(\overset{\prime}{u} + \frac{w}{R_1})(\overset{\circ}{v} + \frac{w}{R_2}) + (\frac{1-\nu}{2})(\overset{\circ}{u} + \overset{\prime}{v})^2| +$$

$$K|(\overset{\prime}{\psi})^2 + (\phi^{\circ})^2 + 2\nu\overset{\prime}{\psi}\phi^{\circ} + \frac{(1-\nu)}{2}(\overset{\circ}{\psi} + \overset{\prime}{\phi})^2| + Gx(\overset{\prime}{w} + \psi - \frac{u}{R_1})^2 +$$

$$Gy(\overset{\circ}{w} + \phi - \frac{v}{R_2})^2 \}\, dxdy$$

$$V = \frac{1}{2}\int_0^b\int_0^a q.w\ dx\ dy;\ T = -\frac{1}{2}\omega^2 \int_0^b\int_0^a |M(u^2+v^2+w^2)+I(\psi^2+\phi^2)|\ dx\ dy \quad (2)$$

where

$$D = \frac{2Eh_2}{(1-\nu^2)};\ K = \frac{2Eh_2h_1^2}{(1-\nu^2)};\ M = 2h_1\rho_1+2h_2\rho_2;\ I = \frac{2}{3}h_1^3\rho_1+2h_1^2h_2\rho_2;$$

$$Gx = \begin{Bmatrix} 2h_1\ Gxz \\ \frac{5}{3}h_1\ Gxz \end{Bmatrix};\ Gy = \begin{Bmatrix} 2h_1\ Gyx \\ \frac{5}{3}h_1\ Gyz \end{Bmatrix};\ (\ )' = \frac{\partial}{\partial x};\ (\ )^{o} = \frac{\partial}{\partial y} \quad (3)$$

The plate consists of two equal face plates of thickness h_2 and a middle layer core of thickness $2h_1$ which acts as a low density stabiliser for the outer facing and resists bending deformation.

Carrying out the first variation as shown by |9| on (1) and (2) leads to the equations of motion and boundary conditions. The corresponding boundary conditions for the system are obtained by satisfying the line integrals. Any possible boundary conditions along a co-ordinate line x = constant can be expressed as:

$$(u' + \frac{w}{R_1}) + \nu(\overset{o}{v} + \frac{w}{R_2}) = 0 \text{ or } u = 0;\ \overset{o}{u} + v' = 0 \text{ or } v = 0;\ w' + \psi - \frac{u}{R_1} = 0 \text{ or } w = 0;$$

$$\psi' + \nu\phi^{o} = 0 \text{ or } \psi = 0;\ \overset{o}{\psi} + \phi' = 0 \text{ or } \phi = 0.$$

Boundary conditions along a co-ordinate line y = constant can be expressed as:

$$\overset{o}{u} + v' = 0 \text{ or } u = 0;\ (\overset{o}{v} + \frac{w}{R_2}) + \nu(u' + \frac{w}{R_1}) = 0 \text{ or } v = 0;\ \overset{o}{w} + \phi - \frac{v}{R_2} = 0 \text{ or } w = 0;$$

$$\overset{o}{\psi} + \phi' = 0 \text{ or } \psi = 0;\ \phi^{o} + \nu\psi' = 0 \text{ or } \phi = 0 \quad (4)$$

3. SELECTION OF THE ASSUMED DISPLACEMENT FIELD

In order to develop a two-dimensional curved sandwich element, the displacement function must be chosen such that continuity along element boundaries must be maintained and for practical reasons the presence of a rigid body mode must be assured (in a rigid body mode there should be no straining anywhere in the region). The related minimum potential energy functional requires only the first derivatives in the unknown displacement fields to be evaluated. In the finite element method, this means that continuity of the displacement field is only required. Based on these requirements, a five degrees of freedom element has been developed and the displacement vector is given by $\{u\} = \{u\ \psi\ v\ \phi\ w\}$. However due to the shear properties of the honeycomb core it was found necessary to use at least seven degrees of freedom per node to achieve a reasonable representation. The displacement vector for seven degrees of freedom is $\{u\} = \{u\ \psi\ v\ \phi\ w\ \overset{o}{w}\ w'\}$ and the results obtained with this element are compared with those yielded by the five degrees of freedom per node element in the following section.

4. RESULTS AND CONCLUSION

The finite element solution results quoted here were obtained for a quarter plate because the plate retains complete symmetry. The plate was subdivided into rectangular elements and four separate problems were solved. Table (1) shows a comparison between the finite element solution and the Lagrange's multiplier method |2| for a flat sandwich plate. It is clear that for relatively coarse mesh sizes, the displacement function incorporating seven degrees of freedom per node yields results which converge rapidly to the results given in |2|. On the other hand the five degrees of freedom per node element requires much finer subdivision of the structure in order that reasonable accurate results are obtained. It may also be noted that the finite element solution is lower than that of |2|; this is due to the fact that in the former case, the contribution of flexural, inplane and shearing motion were taken into consideration whereas the primary concern in |2| was only the flexural motion.

Table (2) shows a comparison between the finite element solution and that of |5| for a singly curved sandwich plate. It may be seen that the finite element solution is lower than that of |5| for both functions used. However it appears that the two theoretical results presented are not bracketed with the experimental data.

Table (3) shows the central deflection w of a flat clamped sandwich plate subjected to a uniform load of intensity q lb/in^2. The results are compared with those published in |10| and |11|. It may be recalled that |10| used the method of successive approximation while |11| used the finite element method with an element having twenty degrees of freedom per node. The results from seven degrees of freedom element representation shows good agreement with those given in |11|.

ACKNOWLEDGEMENT

The author wishes to thank Professor B.L. Clarkson, Dr.M.Petyt and Dr.R.G. White for this helpful comments during the preparation of this paper.

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Table(1) - Natural Frequencies Hz

Finite Element five degrees/node (Symmetric modes)		Finite Element Seven degrees/node		Ueng Results (2)			
25 elements parabolic strain	36 elements parabolic strain	16 elements constant strain	16 elements parabolic strain	Upper bound	Lower Bound	Experimental	mode w(m,n)
69.69	65.17	56.04	55.59	53.040	53.005	48	(1,1)
-	-	86.36	85.56	91.980	91.878	84	(2,1)
-	-	137.24	135.01	138.845	138.538	127	(1,2)
227.53	199.62	142.25	140.67	149.002	148.501	137	(3,1)
-	-	163.77	160.81	161.338	160.814	148	(2,2)
-	-	213.67	206.26	206.00	205.538	192	(3,2)
-	-	227.32	224.08	-	-	-	(4,1)
287.75	269.124	258.36	251.64	256.124	255.300	238	(1,3)
-	-	291.75	284.77	264.904	264.013	249	(2,3)
379.55	351.40	329.40	319.69	300.803	299.876	285	(3,3)

Footnote to Table(1):-

$a = 67.75$ in, $b = 43.5$ in, $2h_1 = 0.250$ in, $h_2 = 0.016$ in, $\rho_1 = 4.02 \times 10^{-6}$ lb.sec^2/in^3 $\rho_2 = 2.48 \times 10^{-4}$ lb.sec^2/in^3 Gxz = 19500 lb/in^2, Gyz = 7500 lb/in^2, $E = 10^7$ lb/in^2, $\nu = 0.34$

Table(2)

Natural frequencies Hz

Finite Element seven deg. Per node	Plumblee Results (5)			
16 elements constant strain	Sine Function	Beam Function	Experimental	Mode w(m,n)
564.48	754	599	421	(1,1)
694.38	916	825	640	(1,2)
969.13	1145	1099	839	(1,3)
1028.72	-	-	-	(2,1)
1140.54	-	-	-	(2,2)
1289.84	-	-	-	(1,4)
1333.12	-	-	-	(2,3)
1582.55	-	-	-	(2,4)
1638.22	1712	1732	1387	(1,5)

$a = 16.5$ in, $b = 23.00$ in, $2h_1 = 0.372$ in, $h_2 = 0.016$ in, $\rho_1 = 5.25 \times 10^{-6}$ lb.sec^2/in^3, $\rho_2 = 4.15 \times 10^{-4}$ lb.sec^2/in^3, Gxz = 18000lb/in^2 Gyz = 9050 lb/in^2, $E = 1.62 \times 10^7$ lb/in^2, $\nu = 0.322$, $R_2 = 84$ in.

Table(3)

The central deflection of a square sandwich plate

Load Parameter Q / $w/2h_1$	Author's Results		Reference |11|	Reference |10|
	16 elements	25 elements	16 elements	
10	0.746095	0.801182	0.813	0.815
20	1.49219	1.60236	1.71	1.75

$a = b = 50$ in, $2h_1 = 1.0$ in, $h_2 = 0.015$ in, Gxz = Gyz = 50000lb/in^2, $E = 10^7$ lb/in^2, $\nu = 0.30$