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FREE VIBRATIONS OF CURVED SANDWICH BEAMS

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Summary

The finite element displacement method is used to investigate the free vibration characteristic of curved sandwich beams. Three displacement models were used incorporating an element having three, four and five degrees of freedom per node and various parametric studies made to investigate their effect on the natural frequencies.

1. INTRODUCTION

The curved sandwich beam shown in Fig.(1) consists of two comparatively thin faces of equal thickness separated by a thick layer (core) of weak but very light weight material. One of the main advantages of this type of construction is the large distance between the faces which means that the core must be rigid enough in a direction perpendicular to the faces to prevent crushing. Moreover sandwich plates and beams are known to possess high resistance to fatigue failure under acoustic excitation [1-3]. Therefore a knowledge of the natural frequencies and mode shapes of curved sandwich beams is important if an understanding of their resistance to acoustic fatigue is required. In this paper the finite element displacement method to study the dynamic behaviour of sandwich beams is used. A displacement function having three, four and five degrees of freedom per node is used. One of the elements which utilises four degrees of freedom per node is used to investigate the various parameters associated with sandwich construction.

2. POTENTIAL ENERGY, EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The free vibration characteristic of a curved sandwich beam may be analysed by using the principle of minimum total potential energy. According to this principle the total potential has a stationary value for any variation of the dependent variables v and w. The total potential may be written as:

$$\pi = U + T \qquad (1)$$

where U and T are the total strain and kinetic energy expressions. These may be written as:

$$U = \frac{1}{2}\int_0^s \{\alpha^2[(\frac{\partial v}{y})^2+(\frac{w}{R})^2] + \gamma^2(\frac{\partial^2 w}{\partial y^2}) + \beta^2(2v + h.\frac{\partial w}{\partial y})^2\}\, dy$$

$$T = -\frac{1}{2}\,\omega^2 \int_0^s (Q_1 v^2 + Q_2 w^2)\, dy \qquad (2)$$

where
$$\alpha^2 = 2Et \; ; \; \beta^2 = \left(\frac{1}{t_c} + \frac{t_c}{4R^2}\right) . G_c \; ; \; \gamma^2 = \frac{Et^3}{6} \; ; \; h = t + t_c \; ;$$

$$Q_1 = 2t\rho_f + \frac{1}{3}\rho_c \; t_c \; ; \quad Q_2 = 2t\rho_f + \rho_c t_c \qquad (3)$$

Carrying out the first variation on equations (1) and (2) as shown by [5], equations of motion and boundary conditions may be deduced.

Equations of motion can be written as:

$$\frac{\partial^2 v}{\partial y^2} + \frac{1}{\alpha^2}(\omega^2 Q_1 - 4\beta^2)v - \frac{2h\beta^2}{\alpha^2} . \frac{\partial w}{\partial y} = 0 \qquad (4)$$

$$\frac{\partial^4 w}{\partial y^4} - \frac{h^2\beta^2}{\gamma^2} . \frac{\partial^2 w}{\partial y^2} + \frac{1}{\gamma^2}\left(\frac{\alpha^2}{R^2} - \omega^2 Q_2\right)w - \frac{2h\beta^2}{\gamma^2} . \frac{\partial v}{\partial y} = 0 \qquad (5)$$

The corresponding boundary conditions for clamped-clamped sandwich beam may be written as:

At $y = 0$ and $y = s$ (6)

$$v = w = \frac{\partial w}{\partial y} = 0$$

The corresponding boundary conditions for a cantilever sandwich beam may be written as:

At $y = 0$

$$v = w = \frac{\partial w}{\partial y} = 0 \qquad (7)$$

At $y = s$ i.e. at the free end

$$\frac{\partial v}{\partial y} = \frac{\partial^2 w}{\partial y^2} = 0 \text{ or } \{2\gamma^2 \frac{\partial^3 w}{\partial y^3} + 2\beta^2 h(2v + h . \frac{\partial w}{\partial y})\} = 0 \qquad (8)$$

3. ASSUMED DISPLACEMENT FIELD

The accuracy of the finite element solution for solving structural problems depends mainly on the proper selection of the assumed displacement field. If the displacement functions are not properly selected, the results may not converge to the correct solution. The cirteria given in [6] are that the next to the highest derivatives occurring in the strain energy expression must be continuous and also rigid body motion must be adequately represented at least approximately. Based on these requirements the following three displacement functions are used:

(a) Three degrees of freedom per node

$$v = A_1 + A_2 y$$
$$w = A_3 + A_4 y + A_5 y^2 + A_6 y^3 \qquad (9)$$

(b) Four degrees of freedom per node

$$v = A_1 + A_2 y + A_3 y^2 + A_4 y^3$$
$$w = A_5 + A_6 y + A_7 y^3 + A_8 y^3 \qquad (10)$$

(c) Five degrees of freedom per node

$$v = A_1 + A_2 y + A_3 y^2 + A_4 y^3$$
$$w = A_5 + A_6 y + A_7 y^3 + A_8 y^3 + A_9 y^4 + A_{10} y^5 \qquad (11)$$

The constants $(A_1, \ldots A_6)$, $(A_1, \ldots A_8)$, $(A_1, \ldots A_{10})$ in equations (9), (10) and (11) may be evaluated at the two ends of the beam in terms of $v, w, \frac{\partial w}{\partial y}$; $v, \frac{\partial v}{\partial y}, w, \frac{\partial w}{\partial y}$; $v, \frac{\partial v}{\partial y}, w, \frac{\partial w}{\partial y}, \frac{\partial^2 w}{\partial y^2}$ respectively.

It can also be deduced that continuity of displacement and radial slope are maintained for the three elements derived.

4. RESULTS AND DISCUSSIONS

The convergence of the solution for the three displacement functions derived earlier with the increased number of elements for a sandwich beam clamped at both ends is shown in Table 1. The mode shapes are indicated by the number of half waves occurring for the radial displacement w. It can be seen that the finite element solution converges monotonically from above, and the two elements incorporating four and five degrees of freedom per node converges very rapidly even when the number of elements is small. The three degrees of freedom element appears to be conservative and needs much finer subdivision of the beam in order that reasonably accurate results are obtained. Therefore the use of curvature $\left(\frac{\partial^2 w}{\partial y^2}\right)$ as an additional degree of freedom in element (C) improves the estimate of natural frequencies and mode shapes even when small number of elements are used.

As the exact natural frequencies for clamped-clamped or cantilever curved sandwich beam are not known, there is no standard of reference against which the results may be compared. Therefore it is justifiable to compare frequencies obtained by these elements with that of straight uniform sandwich beam. Recently [4] computed the frequencies and mode shapes for a uniform cantilever sandwich beam and the results are shown in Table 2. The results obtained with the first two elements converge rapidly with the increased number of elements to that given by [4]. It may also be seen that the element incorporating four degrees of freedom per node yields acceptable results even for coarse beam subdivision.

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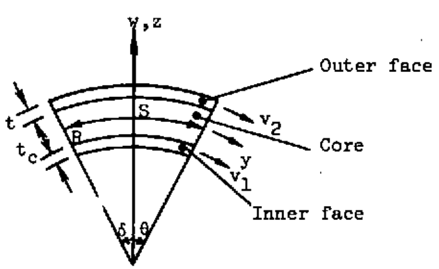

Fig. 1 Curved sandwich beam

TABLE 1

Natural frequencies (Hz) of clamped-clamped curved sandwich beam.
$S = 28$ in., $t_c = 0.50$ in., $t = 0.018$ in., $E = 10^7 lb/in.^2$,
$G_c = 1200 lb/in.^2$, $\rho_c = 3 \times 10^{-6} lb.sec^2/in.^3$, $\rho_f = 2.48 \times 10^{-4} lb.sec^2/in.^3$, $\frac{1}{R} = 0.00595 in.^{-1}$

mode no. / no. of elements	3 degrees/node			4 degrees/node		5 degrees/node	
	7	10	14	4	7	4	7
1	269.914	266.829	265.389	265.191	264.516	265.357	264.864
2	556.479	537.651	528.974	531.678	524.905	527.522	522.887
3	975.852	926.278	903.677	921.623	895.280	890.007	887.142
4	1472.24	1379.92	1338.03	1413.83	1325.83	1315.32	1307.97
5	2013.81	1870.81	1806.49	2027.34	1792.92	1772.83	1758.96
6	2623.43	2381.53	2293.47	2921.45	2282.97	2265.62	2215.91

TABLE 2

Natural frequencies (Hz) of a uniform cantilever sandwich beam.
Dimensions and material properties are the same as Table 1. $\frac{1}{R} = 0.0$

mode no. / no. of elements	3 degrees/node		4 degrees/node	
	10	14	7	10
1	34.044	34.010	33.983	33.973
2	202.935	201.622	200.729	200.536
3	529.664	522.727	518.474	517.259
4	951.211	932.511	921.943	918.138
5	1431.183	1395.626	1377.005	1368.510
6	1941.827	1886.452	1860.201	1844.203
7	2465.954	2389.757	2359.067	2331.721
8	2994.001	2896.899	2880.003	2824.088

mode no. / no. of iteration	MEAD RESULTS [4]			
	r = 20	r = 15	r = 10	r = 5
1	34.242	34.293	34.132	34.298
2	201.85	202.38	202.05	204.76
3	520.85	521.68	527.37	523.46
4	925.40	925.74	951.90	823.40
5	1381.30	1382.10	1452.00	1974.00
6	1867.20	1872.00	2006.00	-
7	2374.90	2391.70	2593.10	-
8	2905.80	2950.20	3472.10	-