A SIMPLE NOVEL METHOD FOR DIRECT EVALUATION OF THE ACOUSTIC SOURCE IMPEDANCE OF THE EXHAUST SYSTEM OF A SINGLE CYLINDER ENGINE

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In this paper, a novel method is proposed for direct evaluation of the engine source impedance, making use of the values of the in-cylinder pressure and temperature at the time of the exhaust valve/port opening (blow-down condition). The source impedance just downstream of the valve throat is the sum of the cylinder compliance and valve impedance (resistance and inertance). The cylinder compliance and valve impedance both vary with respect to crank angle. Therefore, source impedance is estimated by taking the reciprocal of the crank-angle-area-weighted average of the admittance of the source while exhaust valve is open, where at every crank angle step the instantaneous cylinder volume is taken for estimation of the compliance of cylinder and the instantaneous valve flow area and mass flow rate for the valve impedance. The source impedance of a single cylinder engine is estimated at different engine speeds and it is shown that the little variations in it due to different engine speeds do not matter for the insertion loss estimation. Further a simple, albeit approximate, parametric expression of source impedance is derived in terms of cylinder capacity for ready reference of the muffler designers.

Keywords: Source Impedance; Exhaust Muffler; Single Cylinder Engine; Insertion Loss

1. Introduction

Insertion loss is the true measure of a muffler's acoustical performance. Its estimation requires a prior knowledge of the acoustic source impedance. Thus, the insertion loss of a muffler depends not only on the muffler but also on the source to which it is applied for controlling the noise in the transmission path [1].

The value of source impedance has always been a concern for analysis of mufflers. Kathuriya and Munjal [2-3] proposed the two load method for experimental evaluation of the aeroacoustic source characteristics of a pulsating gas flow. They showed that it can be applied either with the impedance tube method or the method of external measurements.

Ross and Crocker [4] applied a standing wave tube technique for determination of acoustic source impedance of a multi-cylinder engine for different engine loads and speeds. Later, Prasad and Crocker [5, 6] used the transfer function method (with a random excitation auxiliary source) to determine the source impedance of a multi-cylinder engine and showed that measured impedance results in reasonable prediction of insertion loss of a muffler as well as the radiated muffled sound pressure level. It may be noted that direct method [4-6] used for estimating the source impedance invariably needs an auxiliary source, and the primary source has to be made inactive. However, indirect methods [7-10] do not need an auxiliary source, but yield negative source resistance for most of the frequencies. The possible causes of negative source resistance were reviewed by Ih and Peat [11].

Making use of a modified two-load method, Hota and Munjal [12] conducted comprehensive parametric studies with curve fitting, and thus derived approximate empirical relations for the aero-acoustic source strength level in terms of engine speed, air-fuel ratio, the number of cylinders and engine capacity for exhaust system of compression ignition engines. However, they made use of a grossly simplified and arbitrary expression for source impedance [12, 1]. Kant and Munjal [13] did a similar study on single cylinder gasoline and diesel engines. These relations, however, have met with limited success.

It is reported in the literature that indirect (two-load or multi-load) method often leads to the prediction of negative source resistance which is obviously absurd. In fact, it casts a shadow of doubt on the accuracy of the reactive part of the source impedance and the source strength as well. On the other hand, the direct method always gives positive source resistance. However, the direct method reported in the literature, carried out experimentally or analytically, has certain limitations which make the method not suitable for estimation of source impedance of IC engines [1].

In this paper, the equivalent internal impedance (source impedance) of a time-variant internal combustion engine is estimated making use of a hypothetical auxiliary source. The variation of source impedance for different engine speeds is investigated. Further, for a range of different capacities of a single cylinder engine, a parametric expression of impedance is derived, which is probably the first of its kind in the literature. These expressions can be used for the estimation of insertion loss of mufflers applied to single cylinder internal combustion engines.

We start with derivation of a general expression for insertion loss of a muffler making use of the electro-acoustic analogy.

2. Electro-Acoustic Analogy and Insertion Loss

A schematic of the IC engine exhaust system is given in Fig. 1(a) and the equivalent electro-acoustic circuits of the exhaust system are given in Figs. 1(b) and 1(c) for pressure representation (Thevenin equivalent) of the source and velocity representation (Norton equivalent) of the source, respectively. In the pressure and velocity representations of the source, source characteristics are: (p_s, Z_s) and (v_s, Z_s), respectively, with $v_s = p_s/Z_s$ for the two representations to be equivalent.

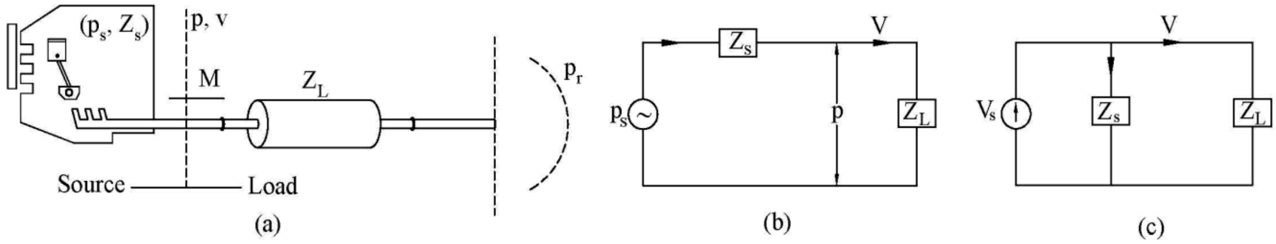

Figure 1: Schematic of the typical IC engine exhaust system: (a) Acoustic source-load system, (b) Electro-acoustic circuit with pressure representation of the source, (c) Electro-acoustic circuit with velocity representation of the source

Insertion loss is defined as the difference between sound power level radiated without and with the muffler, or the difference between the sound pressure level at a far-field point (at the same distance from the exhaust duct orifice) without and with the muffler. It is usually defined either with respect to a reference pipe (a simple pipe is inserted between the source and atmosphere) or without any reference pipe (source-load junction opening directly into the atmosphere). Electro-acoustic circuit representation with a muffler and a simple pipe are shown in Figures 2 (a) and 2 (b), respectively.

As per the definition of insertion loss:

$$IL = 10\log_{10}\left(W_u / W_m\right), \quad dB \tag{1}$$

where subscripts u and m denote 'without muffler' and 'with muffler', respectively.

Acoustic power through a pipe cross-section in the presence of incompressible mean flow is given by [1]

$$W = \int_s I ds = \frac{1}{\rho_0}\left[\langle pv \rangle + \frac{M}{Y_0}\langle p^2 \rangle + MY_0 \langle v^2 \rangle + M^2 \langle pv \rangle \right] \tag{2}$$

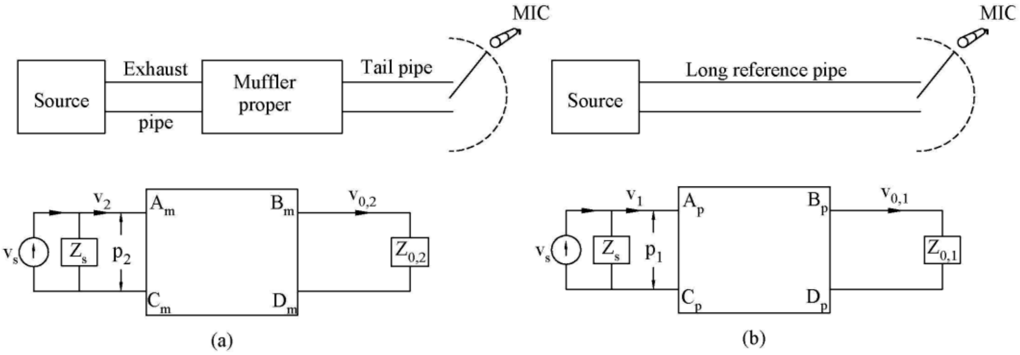

Figure 2: Electro-acoustic circuit representation of exhaust system (a) With muffler, (b) With reference pipe (the four-pole parameter subscripts m and p denote 'muffler' and 'reference pipe', respectively).

Using Eqs. (1) and (2) along with the transfer matrix relation between the upstream and downstream state variables, in accordance with Fig. 2, the expression of insertion loss due to the muffler in the presence of mean flow (Mach number, M) is given by

$$IL = 20\log_{10}\left[\frac{A_m Z_{0,2} + B_m + \left(C_m Z_{0,2} + D_m\right) Z_s}{A_p Z_{0,1} + B_p + \left(C_p Z_{0,1} + D_p\right) Z_s} \left(\frac{\rho_{0,2}}{\rho_{0,1}} \frac{R_{0,1}\left(1 + M_{ep}^2\right) + M_{ep}\left(\left|Z_{0,1}\right|^2 / Y_{ep} + Y_{ep} \right)}{R_{0,2}\left(1 + M_{tp}^2\right) + M_{tp}\left(\left|Z_{0,2}\right|^2 / Y_{tp} + Y_{tp} \right)} \right)^{1/2} \right] \tag{3}$$

where subscripts ep and tp denote exhaust pipe and tail pipe, respectively, and 1 and 2 are used to denote 'without muffler' and 'with muffler', respectively.

Eq. (3) gives a generalized expression for insertion loss with incompressible mean flow. It is clear that the prediction of insertion loss requires the source impedance, Z_s. In the next section estimation of the source impedance by a direct method is presented.

3. A Direct Method for Estimation of Source Impedance

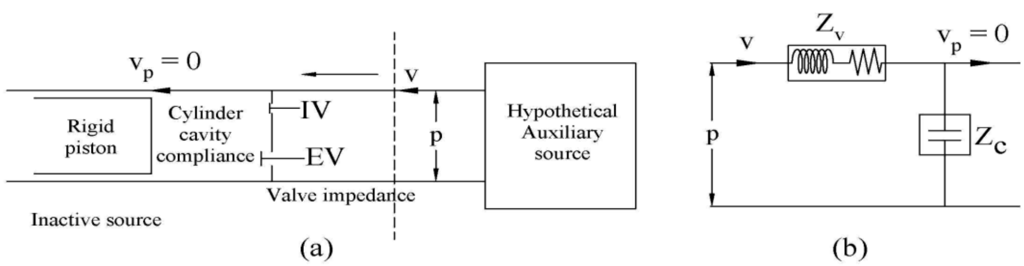

Figure 3: Application of the Direct Load Method for estimation of source impedance: (a) Schematic Diagram with auxiliary source transmitting waves into the inactive real source, and (b) the electro-acoustic circuit representation thereof.

It is evident from Figs. 1(b) and 1(c) that if there were an auxiliary source put somewhere in the load section and the real source were made inactive, and then the ratio of acoustic pressure and acoustic mass velocity, *p/v*, at the source-load junction would be the source impedance, Z_s [14]. It is shown schematically in Figure 3(a), and the corresponding electroacoustic analogous circuit is shown in Fig. 3(b).

In the present study, equivalent source is the same as the actual IC engine source. It sends the same mass flow through the valve as would be produced by the cylinder while exhausting. The cylinder cavity impedance is taken as a lumped compliance while the valve impedance has inertance as well as resistance. The resistance term in the valve impedance is taken from the Elnady *et al.*'s expression for resistance due to bias mean flow in the perforate impedance expression [15]. From Fig. 3(b) it is clear that source impedance is the sum of the valve impedance (inertance and resistance) and cylinder cavity impedance (compliance). Cylinder compliance depends on the instantaneous volume of the cylinder and temperature of the in-cylinder gas. Valve impedance depends on the mass flow rate through exhaust valve, temperature and density of exhaust gas at the valve throat section, and the instantaneous open area of the valve throat section. All these variables, which are needed for the source impedance, are functions of the crank-angle. Thus, the expression for source impedance at a particular position of crank angle θ is given by

$$Z_s(\theta) = -j\frac{c_{c0}^2(\theta)}{\omega V_c(\theta)} + j\omega\frac{t_{v,eff}(\theta)}{S_v(\theta)} + 1.15M_v(\theta)\frac{c_{v0}(\theta)}{S_v(\theta)}, \quad \text{where} \quad M_v(\theta) = \frac{\dot{m}_{ex}(\theta)}{\rho_v(\theta)S_v(\theta)c_{v0}(\theta)} \tag{4}$$

The equivalent source admittance of a time-varying source is evaluated here as the average source admittance over different crank-angle steps while the exhaust valve is open. Symbolically, it is represented by

$$\frac{1}{Z_{s,eq}} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{Z_s(\theta_i)} \tag{5}$$

where n is the number of the crank-angle steps while exhaust valve is open. In the present analysis, a complete cycle of thermodynamic processes is divided into 2^{11} or 2048 crank-angle steps.

The variables in Eq. (4) are obtained from thermodynamics of the cylinder during the exhaust process as explained by Gupta and Munjal [16]. The computation of source impedance requires the valve flow area as a function of crank angle or the valve lift, can be obtained from Heywood [17].

The algorithm to compute the equivalent source impedance consists of the following steps:

i. Input the essential variables: RPM of the engine, bore diameter, length of stroke, compression ratio, connecting rod length, exhaust and intake valve diameters, lift-profiles and valve flow loss coefficients, crank angle corresponding to the exhaust valve opening, intake valve opening and exhaust valve closing, and the in-cylinder gas pressure and temperature at the exhaust valve opening.
ii. Compute the array of crank-angle steps in between EVO and EVC as well as IVO and EVC. Compute the exhaust valve throat area, volume of the in-cylinder gas and its rate of change corresponding to each of the crank angle steps, as given in Heywood [20].
iii. The in-cylinder pressure at the current time step is computed by Eq. (6).

$$p_n = \frac{p_{n-1}}{1-\gamma\frac{\Delta\theta}{\omega}\left(\frac{\dot{m}_n}{m_n}-\frac{\dot{V}_n}{V_n}\right)} \tag{6}$$

First, the current step in-cylinder pressure is computed from the previous step net mass flow rate and the in-cylinder mass value. This pressure is used now to compute the mass flow rate. With this net mass flow rate, the in-cylinder gas mass is computed. The values of the previous step in-cylinder pressure, the current step mass flow rate and the net in-cylinder gas mass are used for computing the corrected value of the in-cylinder gas pressure. This is used to find out the corrected value of net mass flow rate, and which in turn is used to estimate the corrected in-cylinder gas mass.

iv. After the completion of the step computation the source impedance at that instant is computed as per Eq. (4).
v. Finally, Eq. (5) is used for computing the average source impedance.

It can be noted that the estimation of exhaust mass flow rate requires the values of pressure at the downstream of the exhaust valve and upstream of the intake valve (during the valve overlap period) which vary with crank angle. One may assume a constant mean pressure (say 1.06 bar) at the downstream of exhaust valve to compute the average source impedance. It is clear from Fig. 4 that the assumption is reasonable for the source impedance prediction. In the valve overlap period the pressure at upstream of the intake valve is also assumed constant (say 0.98 bar). These assumptions avoid the complete time-domain analysis of the cylinder along with waves in the intake and exhaust systems by means of commercial software like AVL-BOOST.

The small variation in the prediction of source impedance does not matter for the prediction of insertion loss. This is illustrated in Fig. 5 for a simple expansion chamber, where the exhaust and tail pipe diameter is 31 mm, chamber diameter is124 mm, and lengths of exhaust pipe, tail pipe and chamber are 200 mm, 400 mm, and 400 mm, respectively.

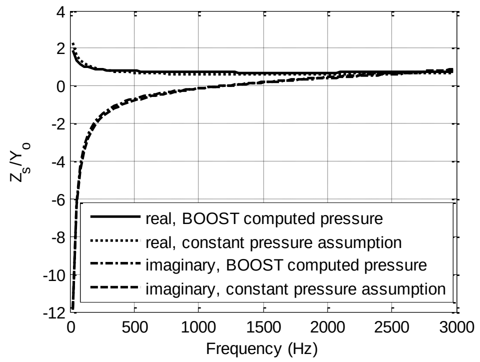

Figure 4: Effect of pressure at the downstream of exhaust valve on the estimated value of the normalized resistance and reactance of the engine running at 3500 RPM

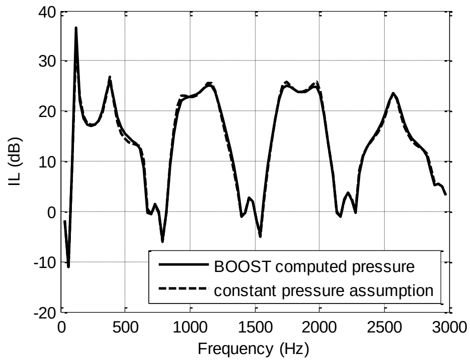

Figure 5: Effect of the static exhaust pressure at the downstream of valve on the estimated insertion loss of the engine running at 3500 RPM

The present study is done on the BOOST Example Manual single cylinder 500 cc gasoline engine. Fig. 6 represents the BOOST model of the engine.

4. Effect of Source Impedance on Insertion Loss

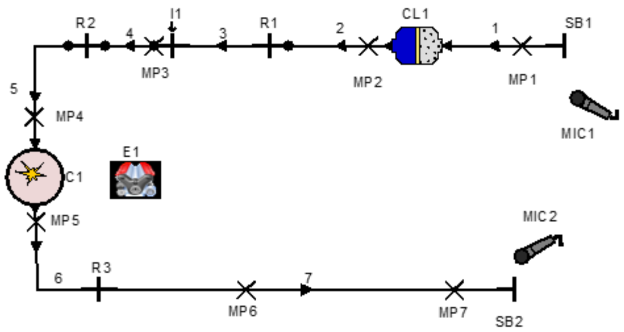

Figure 6: AVL BOOST model of the 500 cc single cylinder gasoline engine

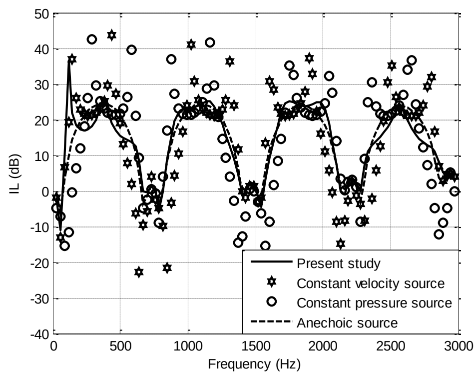

Figure 7: Effect of different source impedance expressions on the insertion loss of the simple expansion chamber muffler when the engine is running at 3500 RPM

The effect of the source impedance on insertion loss is shown in Fig. 7. It may be observed that assumptions of infinite impedance and zero impedance result in large variation in IL whereas the characteristic impedance (anechoic source) assumption yields IL values that are reasonably close to those predicted from the source impedance computed here by means of the direct method, especially at high frequencies. However, the IL values computed by assuming infinite source impedance tally well at very low frequencies only. The reason for this fact is that the cylinder compliance is quite high in the low frequency region as may be observed from Fig. 4.

5. Parametric Study of Source Impedance

The difference in insertion loss of the muffler with different source impedance expressions indicates the importance of the correct estimation of the source impedance. It motivated the authors to do a parametric study of the source impedance. First, speed (RPM) of the engine is varied to see the changes in the source impedance. As shown in Fig. 8, the effect of RPM is not significant on the source impedance and a common fitted value can serve the purpose of estimating the insertion loss with sufficient accuracy, and its adequacy and efficacy for insertion loss is shown in Fig. 9.

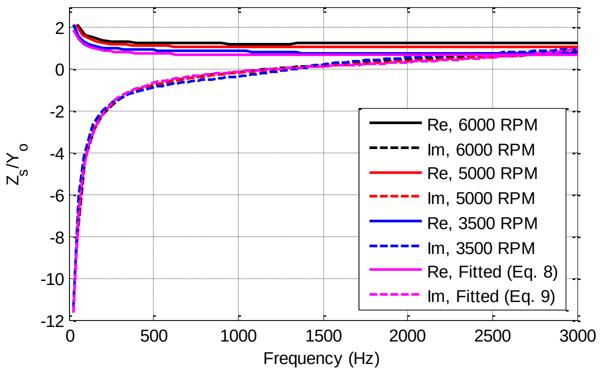

Figure 8: Effect of the engine RPM on the source impedance

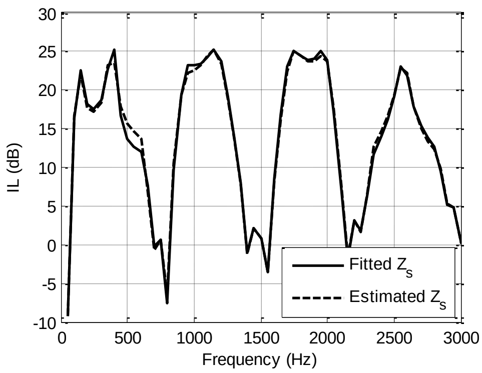

Figure 9: Comparison of the estimated insertion loss with the single fitted source impedance value and the computed value of source impedance, for the engine running at 6000 RPM

Curve fitting for the normalized source impedance is done in the form

$$\frac{Z_s}{Y_0} = \frac{a_R + b_R f}{1 + c_R f + d_R f^2} + j\frac{a_X + b_X f}{1 + c_X f + d_X f^2}, \quad 30\,\text{Hz} \le f \le 3000\,\text{Hz} \tag{7}$$

where coefficients are listed in the last column of Table 1.

It is clear from Fig. 9 that the single fitted value for all the engine speeds for an engine is good enough to predict the insertion loss at different engine speeds. This further motivated the authors to extend the parametric analysis of source impedance of single-cylinder gasoline engine to incorporate the effect of cylinder capacity.

If the cylinder capacity is varied, then the lengths and areas in the model are changed as given below:

i. $L_n = L_o\left(V_c/500\right)^{1/3}$ where V_c is the capacity of the cylinder in cc. L_n and L_o are characteristic lengths; that is, diameters of cylinder, valves and pipes; length of stroke, connecting rod-length, intake and exhaust valve lifts of the new capacity engine, and the base engine of 500 cc, respectively.

ii. $A_n = A_o\left(V_c/500\right)^{2/3}$ where A_n and A_o are, areas of intake and exhaust port areas of the new capacity engine, and the base engine of 500 cc, respectively.

Table 1: Table of coefficients used in the curve fit in Eq. (7) for different values of the cylinder capacity

Coefficients	Cylinder Capacity				
	100 cc	200 cc	300 cc	400 cc	500 cc
a_R	3.806	3.399	3.374	3.340	3.468
b_R	1.23×10^{-2}	1.69×10^{-2}	1.61×10^{-2}	2.088×10^{-2}	2.542×10^{-2}
c_R	3.33×10^{-2}	3.23×10^{-2}	3.31×10^{-2}	3.522×10^{-2}	4.013×10^{-2}
d_R	3.72×10^{-7}	-1.47×10^{-7}	-9.32×10^{-7}	-13.33×10^{-7}	-16.25×10^{-7}
a_X	-81.97	-60.37	-53.44	-42.63	-38.33
b_X	3.86×10^{-2}	3.60×10^{-2}	3.53×10^{-2}	3.35×10^{-2}	3.31×10^{-2}
c_X	11.2×10^{-2}	9.97×10^{-2}	9.21×10^{-2}	7.98×10^{-2}	7.51×10^{-2}
d_X	-2.9×10^{-5}	-2.55×10^{-5}	-2.30×10^{-5}	-1.94×10^{-5}	-1.80×10^{-5}

These coefficients are fitted by the least squares polynomial fit as a function of cylinder capacity. Expressions for normalized resistance and reactance in the frequency range 30 Hz to 3000 Hz are given below in Eqs. (8) and (9).

$$\frac{R_s}{Y_0}=\frac{\left(4.228-0.528v+0.075v^2\right)+\left(0.932+0.301v\right)\times10^{-2}f}{1+\left(3.647-0.398v+0.094v^2\right)\times10^{-2}f+\left(8.21+5.18v\right)\times10^{-7}f^2},\ v=\frac{V(\text{in cc})}{100} \tag{8}$$

$$\frac{X_s}{Y_0}=\frac{\left(-102.2+23.67v-2.19v^2\right)+\left(4.1-0.2764v+0.0236v^2\right)\times10^{-2}f}{1+\left(12.51-1.387v+0.075v^2\right)\times10^{-2}f+\left(-3.404+0.477v+0.031v^2\right)\times10^{-5}f^2},\ v=\frac{V(\text{in cc})}{100} \tag{9}$$

Finally, the expression for the normalized source resistance and reactance given by Eqs. (8) and (9) may be used for estimating the insertion loss of a given exhaust muffler for a single cylinder gasoline engine, without having to follow the algorithm detailed above in Section 3.

The present source impedance formula is used for the estimation of insertion loss of a commercial muffler used in a 100 cc motorcycle engine running at 3120 RPM. The insertion loss of the muffler is shown in Fig 11.

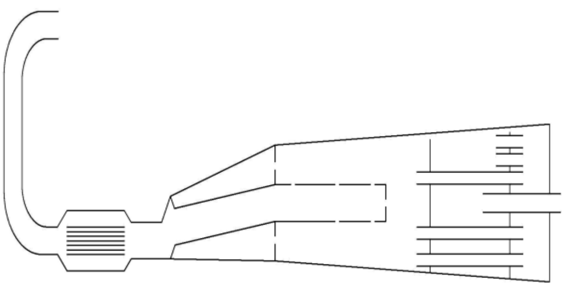

Figure 10: Schematic diagram of commercial muffler used in the 100 cc motorcycle engine

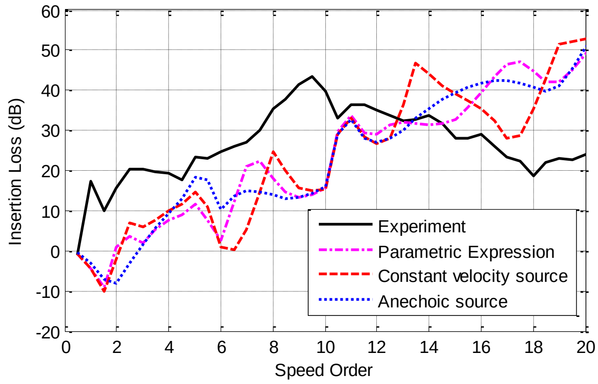

Figure 11: Effect of source impedance on Insertion Loss of a commercial muffler used for 100 cc motorcycle engine running at 3120 RPM with engine load 800 Watt

6. Conclusions

In this paper a novel scheme has been presented for direct evaluation of the source impedance of a single cylinder firing engine. It is free from the anomalous behaviour of the indirect methods that make use of two or more arbitrary loads and result in negative source resistance. It makes use of a hypothetical auxiliary source and does not require the engine bed measurements or complete numerical simulations over several thermodynamic cycles. Both resistance and reactance components of the resulting source impedance turn out to be smooth functions of frequency that allow simple algebraic fits. The source impedance is shown to be more or less independent of the engine speed.

Parametric studies have been conducted for different capacities of a single cylindrical gasoline engine, so as to derive a general expression of the source impedance as a function of the engine displacement as well as frequency. This expression should come handy for muffler designers who can predict the insertion loss of a given muffler with reasonable accuracy.

This simple direct approach is now being extended to multi-cylinder engines.

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