BRITISH ACOUSTICAL SOCIETY

71/29

Meeting on Finite Element techniques in structural vibration

March 1971

Application of Finite Element Method to the Dynamic analysis of tall Structures

by

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1. Introduction.

The application of finite element technique in calculating the response of tall structures subjected to random gust loading is presented. The work relates to the loading parallel to the wind direction and the contribution from vortex shedding effects is not included in the analysis.

Response of structures is evaluated by replacing the distributed pressure field by a suitable representation at a discrete number of points. The structural properties are lumped in a single function called the frequency response function and the summation is taken over the finite number of points rather than over all the modes of interest. Thus there is no need to evaluate the normal modes explicity. We call this method the direct formulation technique (ref.1). A brief description of this method is presented below.

2. Direct Formulation Method

The equation of motion for a structural system can be written in matrix format as follows.

$$M \; z + (1 + ig) \; K \; z = P(t) \qquad \ldots (1)$$

where M = Mass matrix of size n x n

K = Stiffness matrix of size n x n

g = Loss factor

z = 'Displacement vector' of size n x 1

P = 'Force vector' of size n x 1

The force vector relates to the fluctuating drag pressure of air and can be expressed as

$$P = f \; v \qquad \ldots (2)$$

where $f = C_d \, A \, \bar{u} \, \rho$

ρ = Air density

C_d = Coefficient of drag

A = Projected Area

$\bar{u}$ = Mean wind velocity component

v = Fluctuating wind velocity component

In expression (2), the contribution from second order term and the force associated with rate of change of velocity have been omitted for the sake of simplicity. A detailed discussion of their effects is given in ref.(1)

The response cross spectral density can be obtained from expression (1) by taking its fourier transform and multiplying by its complex conjugate.

$$S_y = [H][S_p][H^*] \qquad \ldots (3)$$

where

$$H = \begin{bmatrix} K - \omega^2 M & -g K \\ -g K & K - \omega^2 M \end{bmatrix}$$

and

$$S_p = [f][S_v][f]^T \qquad \ldots (4)$$

S_y, S_p, S_v, are the cross spectral density of displacement, pressure and velocity respectively.

On examination of expression (3), we notice that all the structural properties are lumped in a single parameter H called the frequency response function. The input spectral density function S_p is dependent only on the loading pattern and includes the appropriate correlation between loads and their derivatives.

3. Determination of Suitable Structural Element Size.

In the development of the input spectral density function S_p, no mention has been made about the suitable length of structural element in relation to the pressure correlation. From expression (4), we notice that the pressure cross spectrum is expressed in terms of velocity cross spectrum. In case, we are dealing with homogenous turbulence, and assuming a exponential type of correlation, the velocity spectrum can be expressed as

$$[S_v] = S_1 \; [COR] \qquad \ldots (5)$$

COR is the correlation matrix and its elements are given by $e^{-c\,|z|\frac{1}{\lambda}}$

where S_1 = Direct wind spectrum

z = Separation distance between any set of arbitary points

C = Constant with a value between 6 to 8

λ = Gust Wavelength

It is obvious from expression (5), that the velocity correlation is dependent upon the wavelength of the gust and the separation distance Z. In order to determine the length over which the pressure can be assumed to be fully correlated, consider the curve shown in Fig.(1). The equivalent length can be approximately determined by replacing the area under the shaded curve by an equivalent rectangle and the length scale becomes

$$L \times 1.0 = \int_0^\infty e^{-c\,|Z|\frac{1}{\lambda}}\,dZ \quad \text{or} \quad L = \frac{\lambda}{c} \qquad \ldots (6)$$

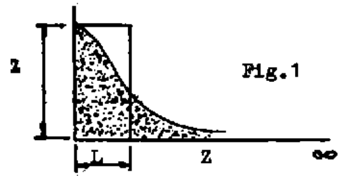

Fig.1

From equation (6), we notice the relationship between the length scale L and the wavelength . We can relate it to various types of correlations as follows

3.1 Full Correlation Case.

The fully correlated case corresponds to c = 0 i.e. non-spatially decaying cross spectral density. This means from equation (6) that as $c \to 0$ then $L \to \infty$.

$$\{S_v\} = S_1 \quad [COR]$$

where all the elements of correlation matrix are unity. It seems therefore that we can take a single element of any suitable size to represent the fully correlated pressure field.

3.2 Partial Correlation Case.

The size of the element depends on the constant c and wavelength λ. For a particular value of c, the size is purely dependent on the wavelength. For smaller wavelengths, we need a fine structural mesh size.

3.3 Zero Correlation Case.

In the case of zero correlation, equation (5) becomes

$$[S_v] = S_1\,[COR] \qquad \text{for } z = 0$$

where the diagonal elements of the correlation matrix are unity and the rest are zero

$$S_v = 0 \qquad \text{for } z \neq 0$$

Thus for z = 0 the exponentially decaying correlation coefficient $e^{-c\,|z|\frac{1}{\lambda}}$ would be satisfied by setting $c \to \infty$ resulting in $L \to 0$. This shows that the element size should be as small as possible to represent the case of zero correlation.

In the design of tall structures subjected to gusts, the maximum contribution to the response is due to the fundamental mode. It means that the long structural wavelength are being excited. Hence with the use of fewer elements, the pressure field can be adequately represented in the case of zero correlation.

4. Comparison of 'Normal Mode' and 'Direct' Formulation Techniques.

A comparison of normal mode and direct formulation technique is illustrated by considering a uniform cantilever beam subjected to a distributed pressure field. The exact value of the spectral density of displacement is obtained by using the normal mode approach (ref.1). In the direct formulation technique, a beam element with three degrees of freedom per node (displacement, slope and curvature) is used and the distributed pressure field is replaced by 'equivalent' nodal forces. The response is calculated by increasing the number of elements to chech the accuracy of the method. It is found that when the pressures are fully correlated, response obtained by direct formulation technique is similar to the exact solution even with the use of 2-elements. When the correlation is zero, the results obtained by direct method converge to exact solution with the increase in number of elements. These results corroborate our discussion of full and zero correlation in the previous section.
An example showing the application of direct formulation technique to an actual structure is given in the main paper to be published by the Journal of Sound and Vibration.

Reference.

1. Handa K.N. Ph.D. thesis, University of Southampton, 1970. An extensive bibliography is given in this reference.