ACOUSTICS: INSIDE OUT AND BACK TO FRONT

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1 INTRODUCTION

The field of Acoustics embraces a surprisingly wide range of disciplines, and, having worked as an academic at the Institute of Sound and Vibration Research (ISVR) for three decades, I have found myself involved in many. Although most of this work has been associated with noise control in some form or another, my passion for all things audio, and especially loudspeakers, has always bubbled to the surface. This passion has been maintained over the years largely by my collaboration with Philip Newell, who sponsored my PhD research, and has remained a great friend and inspiration to this date. Although this paper is not directly about audio, wherever possible, it will include examples of how the various techniques described can be applied to audio problems.

Now in retirement, a look back over my time at ISVR reveals that I have spent a great deal of effort looking at acoustics problems inside out and/or back to front! Two major topics that I have been involved in are Acoustic Reciprocity and Acoustic Inverse Methods: the former concerns the interchange of the positions of sources and receivers of sound, and the latter the inversion of the acoustic path(s) from sources to receivers of sound.

2 ACOUSTIC RECIPROCITY

The principle of acoustic reciprocity is not new and was first proposed in 1860 by Helmholtz in a paper on the acoustic behaviour of open-ended pipes [1]. Point-to-point acoustic reciprocity can be summarised as follows: -

“The acoustic pressure at a point due to a harmonic point monopole source at another point is independent of an interchange of the positions of source and receiver”

Or

$$\mathrm{G}(\mathbf{x}|\mathbf{y}) = \mathrm{G}(\mathbf{y}|\mathbf{x}), \tag{1}$$

where G(**x**|**y**) represents the Green function from position **y** to **x**.

This somewhat obvious statement hides the powerful and (almost) universal application of the principle but, in order to be useful, several conditions need to be met. First, the medium in which the source and receiver are immersed (in many cases, air) must be still; second, any boundaries within the medium must be linearly reacting (no rattles, for example); third, there are no other sources of sound (noise free). These conditions are not unusual and are in fact necessarily applied ‘automatically’ in the solution of most acoustic problems (known as LTI – Linear, Time-Invariant systems).

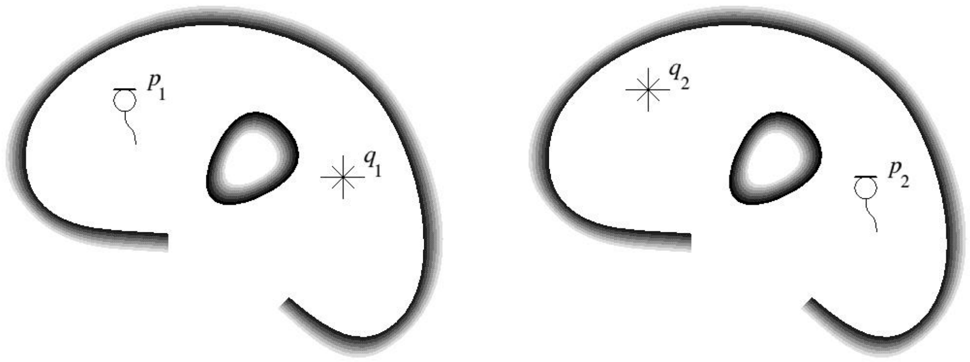


Figure 1 Acoustic Reciprocity

The left side of Figure 1 shows a monopole source of volume velocity q_1 and a receiver at which there is a resultant sound pressure p_1 in the presence of arbitrary boundaries. The right side shows the same system but with the positions of the source and receiver interchanged, giving a sound pressure p_2 at the receiver in response to a volume velocity q_2 at the source.

Referring to Figure 1, it follows by the principle of acoustic reciprocity that

$$\frac{p_1}{q_1} = \frac{p_2}{q_2} \; . \tag{2}$$

The presence of the arbitrary boundaries allows the principle to be extended beyond monopoles to include directional sources and receivers; for example, if a source is positioned on a rigid boundary, it effectively radiates into half-space instead of the full-space of a monopole; all that is required is that the boundaries are unchanged by the position interchange. This is equivalent to ensuring that source #1 has the same directivity as receiver #2 and source #2 has the same directivity as receiver #1.

2.1 Practical Uses for Acoustic Reciprocity

Probably the most useful application for acoustic reciprocity is in situations where a source or receiver of sound is in an inconvenient or inaccessible position. In many noise control problems, it is desirable to isolate the contribution from a particular source to a receiver position - often the ear of a person. This can be done conventionally by placing a calibrated source (loudspeaker) at the source position and measuring the pressure at the receiver position with a microphone. However, it may be impractical to place the loudspeaker at the source position, due to size constraints for example, in which case acoustic reciprocity may be used: A small microphone is placed at the source position and the calibrated loudspeaker is placed at the receiver position; the pressure at the microphone due to operation of the loudspeaker yields the intended pressure contribution by reciprocity.

Acoustic reciprocity can also be used for creative purposes. I was involved some years ago in the recording of a rock band in a factory building. In experimenting with various unusual instrument positions, the lead guitarist found that in order to achieve a particular effect he needed to play his guitar in the toilet! This was impractical for obvious reasons so, armed with my new-found knowledge of reciprocity, I suggested that he place a microphone in the toilet and play outside; to his amazement the resultant effect as recorded was very similar!

My undergraduate final year project was supervised by Professor Frank Fahy, a great inspirator who also later supervised my PhD and much of my post-doctoral research. The project involved applying the principle of acoustic reciprocity to the rifle microphone concept to produce an end-fire directional loudspeaker [2]. One way of realizing a highly directional microphone is to mount a single microphone capsule at one end of a long tube. The tube has a row of holes (or sometimes a slot) along its length into which the sound enters, propagates along the tube, and is picked up by the microphone. If the sound arrives along the axis of the tube, the contribution from each hole arrives at the microphone at the same time as that from the other holes and pick-up sensitivity is high; sound arriving from other angles will result in destructive interference at the microphone between the contributions from the holes and hence lower sensitivity. This creates a virtual end-fire array, with the holes acting as the array elements.

The objective of the project was to invoke the principle of acoustic reciprocity by replacing the microphone capsule with a loudspeaker and testing for the radiated directivity patterns. In true Peter Barnett fashion, a mathematical model of the system was created (in BASIC on a BBC B microcomputer) to optimize the hole size and spacing…

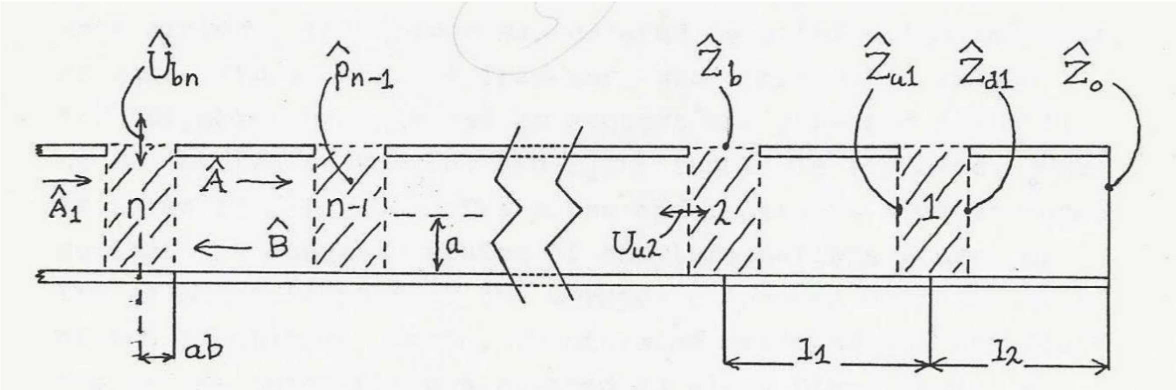


Figure 2 Mathematical Model of End-Fire Radiator

After optimization, a prototype radiator, using a PVC pipe with drilled holes driven by a compression driver, was constructed and tested in the large anechoic chamber at ISVR. Armed with a printout of the theoretical polar plot, I watched as the B&K chart plotter, linked to a turntable, drew the measured plot in real time. I was so astonished by the similarity that I ran to Professor Fahy's office to show him the two plots! "*The mathematical model works!*" I proclaimed to which Frank replied "*of course, that's why we learn how to do it*". My faith in the analytical modelling of acoustics was created at that moment and has stayed with me, unshaken, to this day.

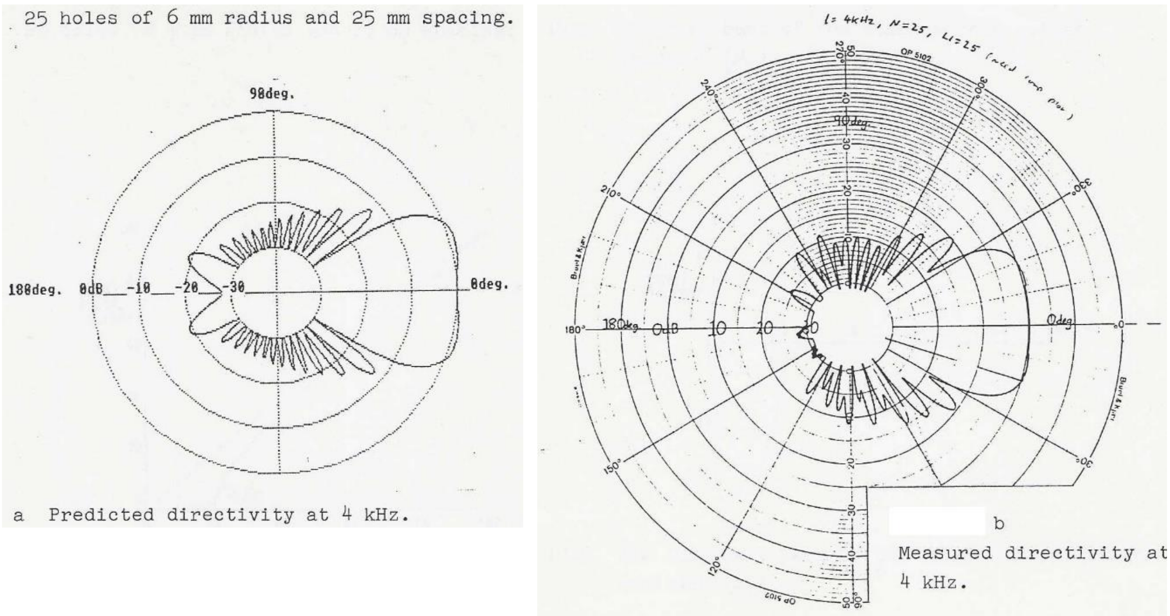


Figure 3 Predicted vs Measured Polar Pattern for End-Fire Radiator

3 VIBROACOUSTIC RECIPROCITY

Most sources of sound involve the vibration of one or more surfaces. Vibroacoustics is the study of the sound radiated (or received) by a vibrating structure and is a topic that kept me occupied (employed) for much of my time as a Research Fellow at ISVR during the 1990s. Vibroacoustic Reciprocity is an extension of acoustic reciprocity where the sources and/or receivers are vibrating surfaces [3, 4, 5].

Consider a vibrating surface divided into a number of small elements. By the principle of superposition, the sound pressure field radiated by the vibrating surface can be estimated by summing the field radiated by all the elemental areas representing the surface:

$$p(\mathbf{x}) \approx \sum_i S_i\, u_i \mathrm{G}(\mathbf{x}|i), \tag{3}$$

where u_i is the normal velocity of surface element i having an area S_i and $\mathrm{G}(\mathbf{x}|i)$ is the Green function representing the sound pressure at position $\mathbf{x}$ due to the volume velocity at element i. Given measurements of the velocities of the surface elements, the contribution of each of these to the total radiated sound field can be estimated if the Green functions G are known. With simple geometries, these Green functions may be calculated analytically, but for arbitrary systems they can be measured by invoking the principle of vibroacoustic reciprocity

Figure 4 shows how vibroacoustic reciprocity works. An element of a vibrating surface is replaced by an equivalent monopole source on a rigid surface; applying acoustic reciprocity, the pressure in the reciprocal case is then the pressure measured on a rigid surface (the ‘blocked’ pressure).

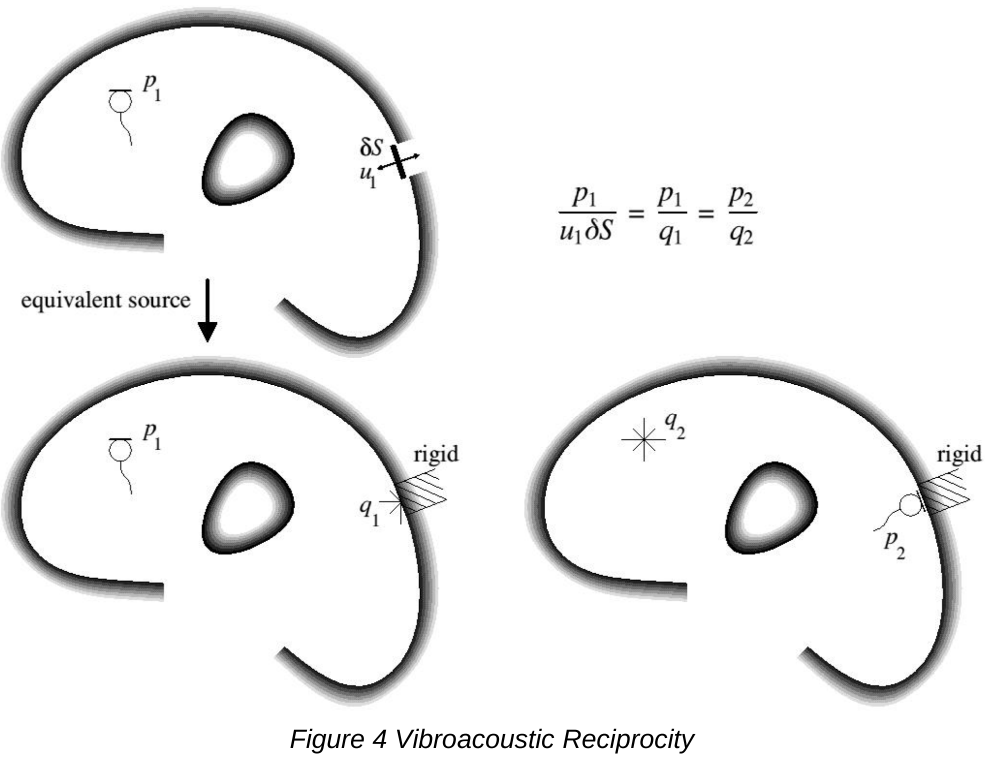


Figure 4 Vibroacoustic Reciprocity

Further extensions to the principle include point forces and normal velocities, and the reader is directed to the references for more information.

3.1 Practical Uses for Vibroacoustic Reciprocity

As part of a Europe-wide project, I was tasked with testing the vibroacoustic reciprocity principle, and in particular, an ISVR-developed Volume Velocity Transducer [5,6,7,8]. The experiment involved estimating the contribution of the vibration of the surfaces of the interior of a truck cab to the sound pressure at the driver's ear. The entire interior of the cab was divided into 1604 elements using sticky tape (Figure 5) and the volume velocity of each element was measured with a loudspeaker operating in the engine position (Figure 6a). A monopole source was then placed at the driver's ear position and the blocked acoustic pressure was measured in the centre of each element using a miniature microphone (Figure 6b). The experiment took a total of six weeks to complete as the portable 2-channel analyser used could only store 30 high-resolution measurements before transferring the data to a computer via the RS232 serial port!

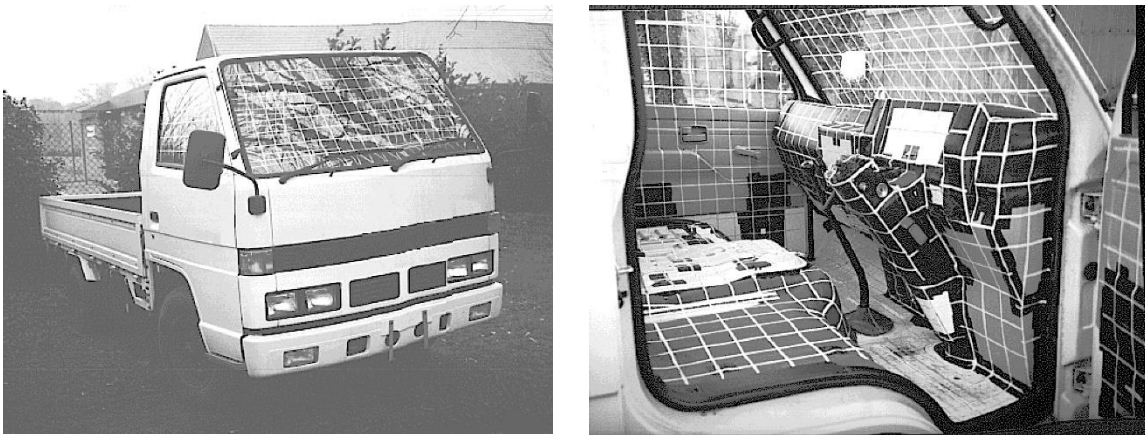
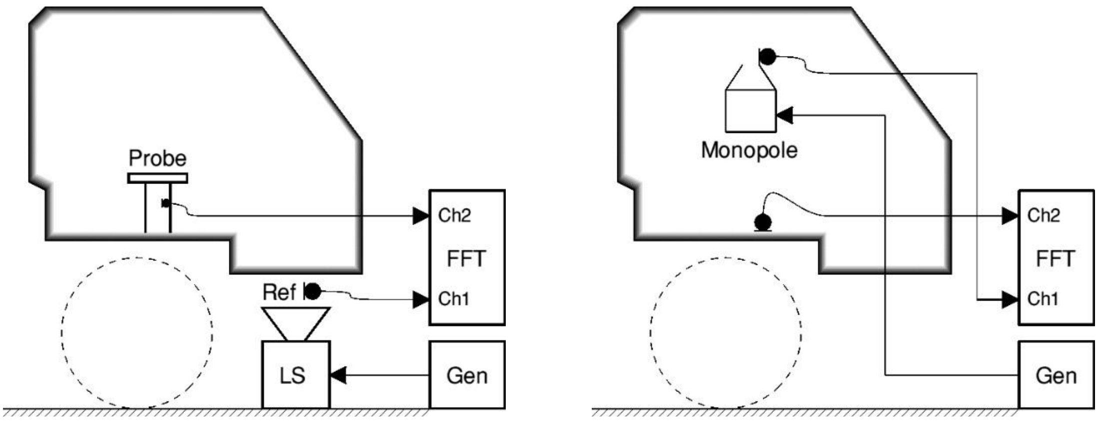


Figure 5 Truck Cab Divided into 1604 Surface Elements



a measuring volume velocity *b measuring blocked pressure*

Figure 6 Application of Vibroacoustic Reciprocity to a Truck Cab Interior

The result of combining 1604 velocity measurements with 1604 blocked pressure measurements is then compared with a single, direct measurement of the pressure at the driver's ear position. Figure 7 shows the comparison.

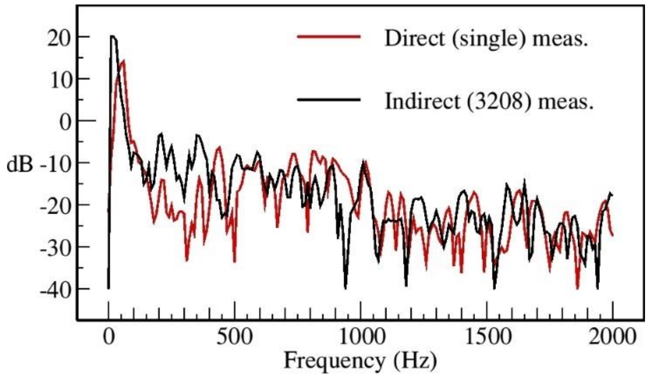


Figure 7 Comparison of 3208 measurement reciprocity-based pressure estimate with directly-measured result – pressure at driver's ear

It may seem a waste of time to take 3208 measurements and arrive at a similar result to a single direct measurement, but it is the analytic properties of the intermediate steps that yield the power of the technique. Figure 8 shows a colour map of the relative contribution of each of the 1602 surface elements to the pressure at the driver's ear; strong contributions from the foot wells, the bottom of the driver's door and between the back of the seats is evident.

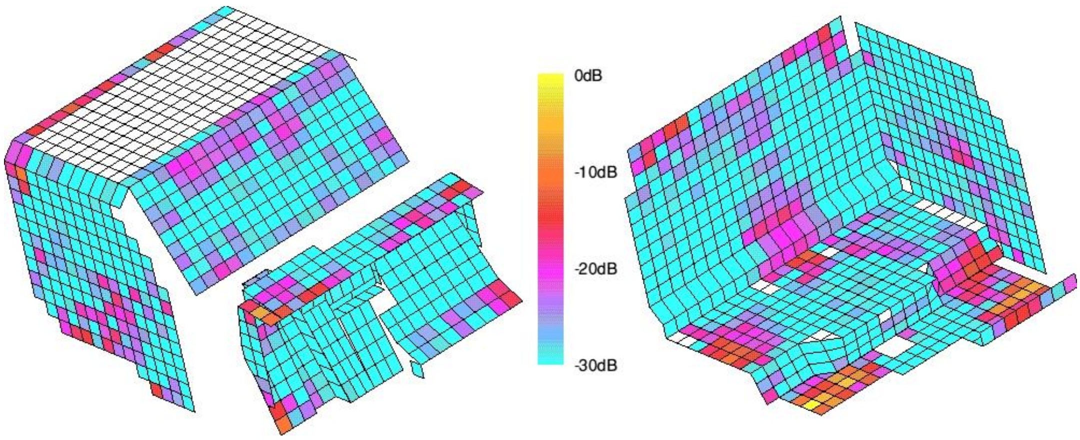
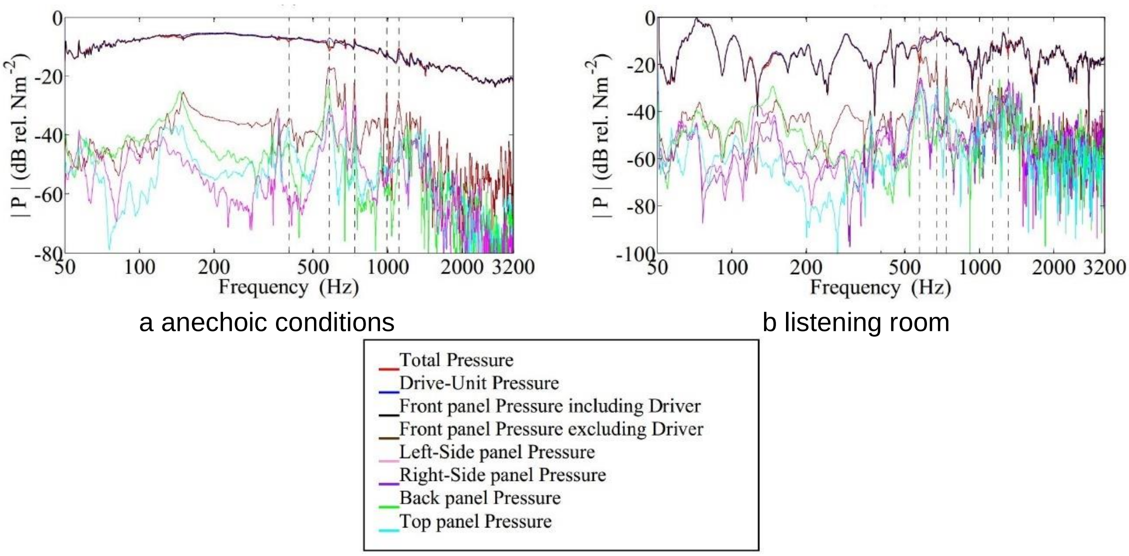


Figure 8 Contribution of Each Surface Element to Sound Pressure at Driver's Ear

Returning to audio. An undergraduate project that I supervised was concerned with estimating the contributions of the walls of a loudspeaker cabinet to the sound heard by a listener in a room. The work resulted in a presentation at Reproduced Sound in 2009 [9]. The entire loudspeaker cabinet, excluding the base but including the driver, was divided into small elements and the vibration of each element was measured using a non-contacting laser vibrometer. The loudspeaker was moved first to an anechoic chamber and then to a listening room and in each room the Green functions from each of the elements to the listening position were measured using reciprocity via the blocked acoustic pressures. Figure 9 shows a comparison of the contributions of each cabinet wall to the sound pressure at the listener's ear under anechoic conditions (a) and in a listening room (b).



a anechoic conditions b listening room

Figure 9 Contribution of Cabinet Walls to Sound Pressure at Listener's Ear

It can be seen in a comparison between Figure 9a and 9b that the peak in the contribution of the cabinet walls at 600Hz makes little contribution to the sound pressure under anechoic conditions but becomes significant in the listening room. It is interesting to note that, whereas the Green functions for the anechoic conditions in Figure 8a could have been predicted using boundary element methods, those for the listening room would have proved far more challenging.

4 ACOUSTIC INVERSE METHODS

In essence, an acoustic inverse method is a means of making estimates of the source(s) of sound from observations of the radiated sound field [10,11]. In other words, "what source(s) produces the sound that I observe?". In its simplest form, a single microphone output is used to make an estimate of the single source that produced the observed pressure. This estimate is only possible if the relative positions of the source and receiver are known, that is we know the Green function between the two points, the forward situation,

$$p(\mathbf{x}) = \mathrm{G}(\mathbf{x}|\mathbf{y})q(\mathbf{y}), \tag{4}$$

then yields the inverse,

$$q(\mathbf{y}) = \mathrm{G}(\mathbf{x}|\mathbf{y})^{-1}p(\mathbf{x})\,, \tag{5}$$

from which the source q can be derived from the pressure p by inverting the Green function **G**. Under free-field conditions, the Green function can be calculated simply from a knowledge of the frequency f, the speed of sound c and the linear distance between the points r:

$$\mathrm{G}_{ff} = j\rho ck\frac{e^{-jkr}}{r}, \tag{6}$$

where $k = 2\pi f/c$ and ρ is the air density. I am known to state that a free space Green function can be measured using just a ruler and a thermometer (c is a function of temperature)!

In many situations, due to the presence of reverberation or background noise, for example, the estimate of q from a single measure of p is unreliable in which case an array of M microphones can be employed. By inverting the Green function of each element of the array, M estimates of q are combined to yield a better estimate of q with reduced contribution from interfering sources. This is the principle of the beamformer array. If the microphones are arranged in a line, the array can be steered in the source direction simply by delaying each microphone output before combining (crude inverse Green functions without the $1/r$ component); this is the delay and sum beamformer that is used extensively to determine the directional position (but not the strength) of a source. In general, the inverse problem of M microphones 'pointing' at a single source q can be written

$$q_{opt} = [\mathbf{g}^{\mathrm{H}}\mathbf{g}]^{-1}\mathbf{g}^{\mathrm{H}}\mathbf{p}, \tag{7}$$

where q_{opt} is the resultant best estimate of q, **g** is a vector of Green functions, **p** is a vector of pressure observations and $^{\mathrm{H}}$ denotes the Hermitian (conjugate) transform of a vector. Note that Equation (7) represents the result of applying a least-squares optimization to the estimate of q.

The acoustic inverse method can be extended to situations where more than one source of sound is operating. In this case, a matrix of Green functions **G**, linking each source with each microphone is required. The forward situation becomes,

$$\mathbf{p} = \mathbf{G}\mathbf{q}, \tag{8}$$

where **q** is now a vector of the required sources. The inverse is then,

$$\mathbf{q}_{opt} = [\mathbf{G}^{\mathrm{H}}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{p}, \tag{9}$$

which becomes a problem of matrix inversion. Crucially, the success (or otherwise) of this matrix inversion depends upon the condition number of the Green function matrix **G**, with a high condition number indicating a high sensitivity to errors in the system and a low condition number indicating a robustness to these errors.

So far, all of the sources have been assumed to be fully correlated with each other. This is not usually the case in real situations where a mix of inter-source correlations are often present. Usefully, the inverse method can re-written assuming arbitrary source correlations thus: -

$$\mathbf{S}_{qq} = G^{+}\mathbf{S}_{pp}\mathbf{G}^{+H}, \tag{10}$$

where $\mathbf{S}_{\mathrm{qq}}$ and $\mathbf{S}_{\mathrm{pp}}$ are matrices of source and pressure cross-spectra respectively. $\mathbf{G}^{+}$ is the pseudo-inverse of **G**, which is equal to $\mathbf{G}^{-1}$ when the number of sources and microphones is the same, but when there are more microphones than sources is given by

$$G^{+} = [\mathbf{G}^{H}\mathbf{G}]^{-1}\mathbf{G}^{H}. \tag{11}$$

Equation (10) represents the most universal form of the inverse method, and many different algorithms have been devised to solve it efficiently and with minimum error.

4.1 Practical Uses for the Acoustic Inverse Method

Aero engines are complicated sources of sound, and the understanding and control of the noise radiated from an aero engine is the subject of on-going study. In 1999, the Rolls-Royce UTC in Gas Turbine Noise was established at ISVR and my role within that organization was heading the development of advanced acoustic measurement techniques. This involved designing, installing and operating large-scale microphone arrays alongside jet engines to attempt to quantify the various noise sources. One example of this was the so-called Polar-Correlation Technique, devised in the 1970s by Marcus Harper-Bourne and Mike Fisher of ISVR / Rolls Royce, using only two microphones; one fixed the other moveable [12]. They found that, assuming the engine and its jet behind it can be represented by a line source, there exists a Fourier relationship between the Cross-spectra between the microphones in a polar arc and the source distribution along the engine axis. This relationship results in a condition number of unity, perfect for reliable inversion. Fisher later devised a formal least-squares solution to the problem which is similar to Equation 10 but with the simplified far-field polar geometry. Some 20 years after the original work, Mike Fisher supervised a further study into the Polar Correlation Technique that found me conducting an experiment on a running jet engine at the outdoor test facility at Hucknall, UK - in the middle of winter! The experiment involved a curved arc of 125 microphones mounted on the ground of the test bed at a radius of 20m from the engine. Figure 10 shows the geometry for the experiment and Figure 11 shows a typical result of the application of the Polar Correlation Technique to the engine noise (with vertical scale removed for confidentiality).

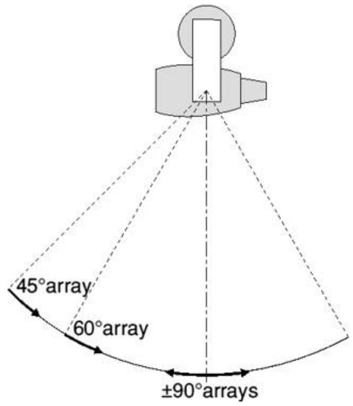


Figure 10 Geometry for Polar Correlation Experiment

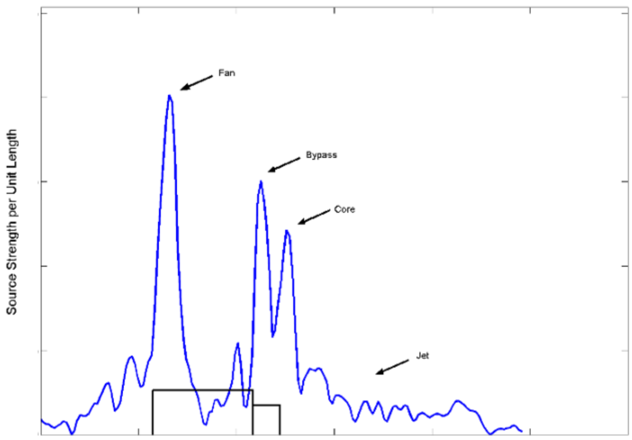


Figure 11 Result of Polar Correlation Experiment

Outdoor engine tests, like the one described above, are expensive and very dependent upon the weather. However, aero engines are routinely tested in indoor test beds so a major part of my involvement with Rolls Royce was on the application of acoustic inverse methods to the measurement of noise from aero engines in indoor test beds. This involves dealing with reverberation and poor signal-to-noise ratios, all without the simple, far-field geometry of the outdoor tests.

Several successful applications involved mounting large spiral-shaped arrays of microphones alongside jet engines in various indoor test beds. The Green functions from the noise sources on the engine to each microphone in the array were measured in-situ, either directly or using reciprocity, so that they include the test bed acoustics. This arrangement resulted in the successful inversion of the Green function matrix to yield realistic estimates of the strengths and mutual correlations of the engine sources. These results were further used to predict the far-field radiated sound pressure which can be compared with that measured directly in an outdoor test to verify the technique. Figure 12 shows the result of such a comparison; the scales have been removed for confidentiality.

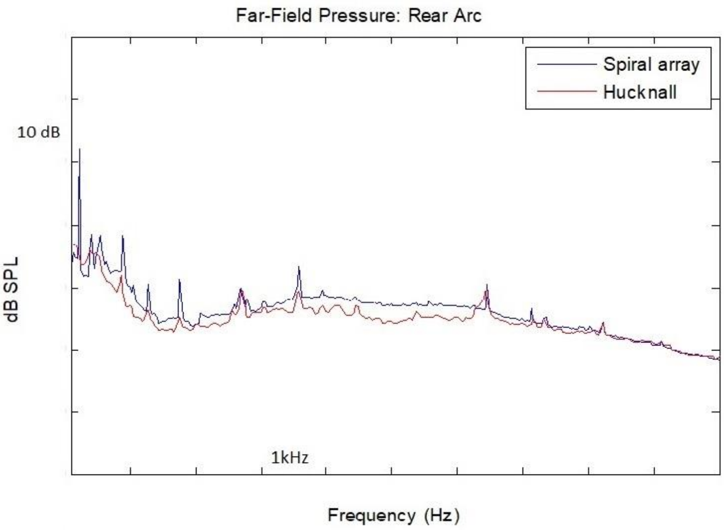


Figure 12 Comparison of Indoor Derived and Outdoor Directly Measured Far-Field Sound Pressure

4.2 Condition of the Green Function Matrix

As stated above, the reliable solution of the acoustic inverse problem is dependent upon the condition of the Green function matrix. This statement holds true for every situation, from the simple delay and sum beamformer pointing at a single source to the most complex of source and microphone arrangements with arbitrary source correlations.

Equation (6) reveals that, under free-space conditions at least, the individual Green functions are strong functions of frequency and geometry. This is also therefore true of the conditioning of the Green function matrix. So, even before a microphone array is built, the range of frequencies over which it is expected to work reliably can be assessed for any given geometry simply by inspecting the condition number of the Green function matrix built up from the system geometry.

4.3 Audio Applications of the Acoustic Inverse Method

Tying together the two topics of this paper, reciprocity and inverse methods, reveals some useful outcomes. A reversal of the microphone and source positions yields the same Green function matrix, and hence the same limitations. Therefore, the techniques used to optimize microphone arrays for measurement purposes can also be used for loudspeaker arrays. The geometry of an array of loudspeakers can be optimized to yield reliable coverage over a number of points, if the condition of the Green function matrix is optimized for the frequency range of interest. Note that this optimization is prior to, and therefore independent of, the signal processing applied to the loudspeakers in the array to achieve the required result. The relationship between the condition of the Green function matrix and the reliability of its inversion can therefore be used to assess the suitability of arrangements of loudspeakers for sound reproduction.

Consider a pair of loudspeakers arranged in front of a listener in the familiar stereo listening setup. There exists a 2x2 Green function matrix linking the two loudspeakers to the two ears of the listener. Figure 13 shows the geometry and condition number of the resultant Green function matrix for a 60-degree loudspeaker span at 2m from the listener.

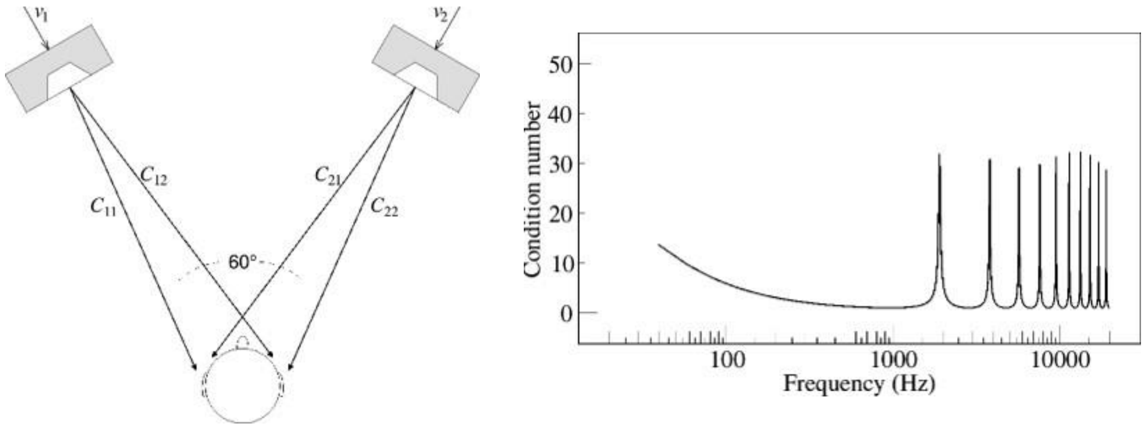


Figure 13 Stereo Loudspeaker Geometry and Condition Number of Green Function Matrix

It is clear from the peaks in the condition number that reproduction over this stereo system breaks down at several spot frequencies; this is a well-known phenomenon with conventional stereo reproduction. During the 1990s, Phil Nelson and his team realized that a change in geometry would bring about an improvement in the condition number, especially at high frequencies. The result is the "Stereo Dipole" arrangement [13, 14]. Figure 14 shows the geometry and condition number for this arrangement.

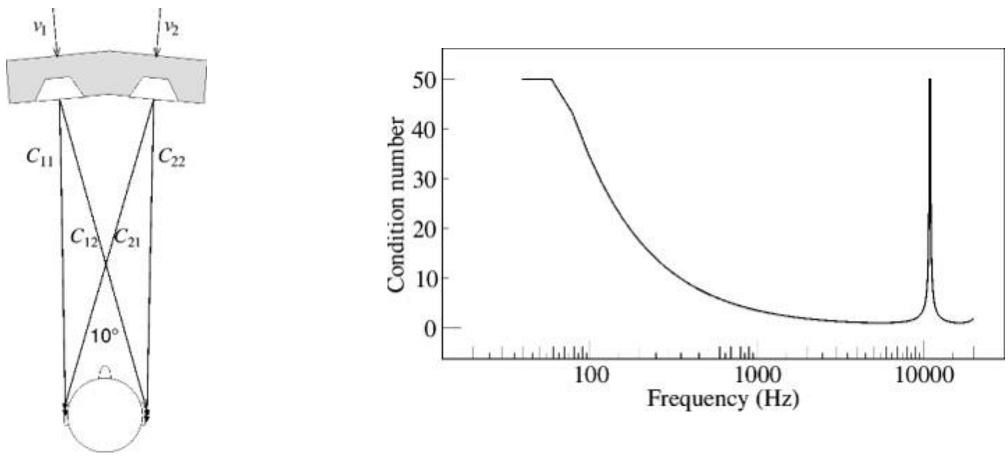


Figure 14 Stereo Dipole Geometry and Condition Number of Green Function Matrix

The reduced span to 10 degrees has improved the condition number at high frequencies but worsened it at low frequencies. The stereo dipole proved to be very good for reproducing binaural recordings over loudspeakers; the good conditioning of the Green function matrix at high frequencies allows for accurate and reliable individual control over the sound at each of the listener's ears. A later development is the "Optimal Source Distribution" [15] which overcomes the low-frequency limitations of the stereo dipole; the details of an example of which is shown in Figure 15.

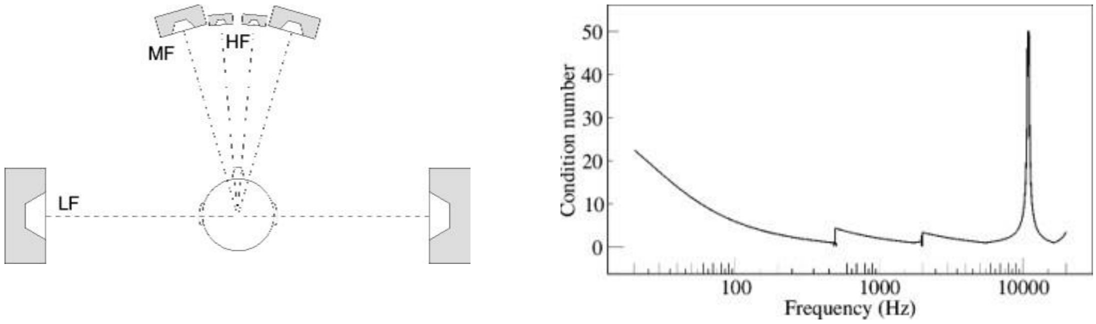


Figure 15 Optimal Source Distribution and Condition Number of Green Function Matrix

5 NONLINEAR HORN MODELLING

During my PhD studies, supervised by Frank Fahy and Chris Morfey and supported by Philip Newell, I developed a finite-element one-parameter model of horns which divided an arbitrary-flared horn into short exponential elements for which an analytic solution exists [16]. Figure 16 shows the division of a horn into elements. The model begins with a knowledge of the radiation impedance of the open 'mouth' of the horn and uses the analytical solution of the horn wave equation to propagate forward and backward waves in each element until the driver 'throat' end of the horn is reached and a throat impedance is established. The signal from an attached driver can then be propagated back along the horn to predict the radiated field.

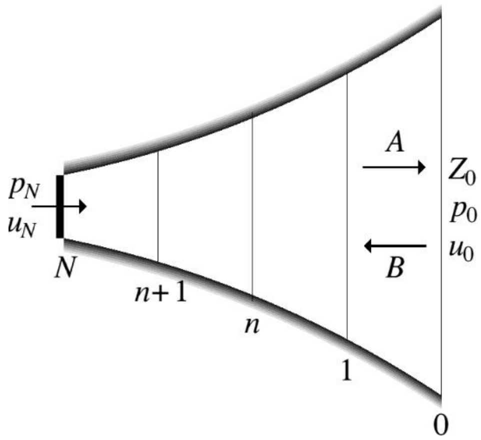
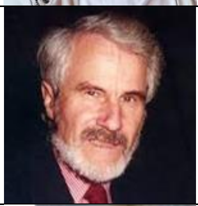


Figure 16 Division of Arbitrary Horn into Exponential Elements

This model, along with most theoretical acoustic models, assumes linearity, which allows for the superposition of waves. However, many horns used for audio operate at high enough sound pressure levels that linear wave behaviour cannot be assumed. That being the case, it cannot be assumed that the forward and backward propagating waves do not affect one another so it is not possible to predict an output from the horn due to a given input. So, continuing with the theme of this paper - doing acoustics 'backwards' - the nonlinear horn model overcomes this limitation by modelling the horn from the 'linear' end (mouth) of the horn back to the nonlinear end, with the linearly propagated waves corrected for nonlinear distortion at the end of each element. The linear calculations are carried out in the frequency domain and the nonlinear distortion in the time domain with the two linked via Fourier transforms. The result is the waveform required at the throat of the horn to achieve a desired waveform at the mouth – truly acoustics done backwards!

6 ACKNOWLEDEGEMENTS

The following people have been key in inspiring and developing my passion for audio and acoustics through the years.

	**Philip Newell**, who sponsored my PhD on horn loudspeakers and is co-author of numerous of my publications on many aspects of audio, including two editions of the book “Loudspeakers”.
	**Frank Fahy**, who supervised my undergraduate project, my PhD and my post-doctoral work on Vibroacoustics. It was Frank who taught me how to do experimentally based research.
	**Chris Morfey**, who supervised my work on nonlinear propagation in horns. Chris exemplified clear thinking.
	**Mike Fisher**, with whom I carried out my early work for Rolls-Royce and who was instrumental in enrolling me as a founder-member of the Rolls-Royce UTC at ISVR.
	**Phil Nelson**, who I worked with on inverse methods and all my later work with Rolls Royce on advanced measurement techniques.
	**Peter Davies**, with whom I also worked on advanced measurement techniques, mainly in hostile environments such as combustion engine exhaust pipes.
	**Ian Piper**, who taught me everything I know about live sound mixing. Ian and I together did the live mixing for hundreds of acts throughout the 1980s.

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