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THE PREDICTION AND MEASUREMENT OF THE THROAT IMPEDANCE OF HORNS

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1. INTRODUCTION

A conventional loudspeaker, consisting of a diaphragm, cone or dome mounted in a baffle, suffers from a poor radiation efficiency at low frequencies where the acoustic wavelength greatly exceeds the length of the diaphragm perimeter. To overcome this problem the diaphragm must be large and heavy (if rigidity problems are to be avoided), resulting in a reduction in output at higher frequencies. This compromise between size and moving mass sets an upper limit on the electro-acoustic efficiency of a conventional loudspeaker drive unit to a mere few percent. Horns attempt to combine the high radiation efficiency of a large diaphragm with the low mass of a small diaphragm in a single unit. This is achieved by coupling a small diaphragm to a large radiating area via a gradually tapering flare. This arrangement can result in electro-acoustic efficiencies of twenty to fifty percent. In the design and analysis of horns, a knowledge of the acoustic impedance presented by a horn flare to a diaphragm, or similar velocity source, at the throat is therefore vital to the assessment of the performance of the horn.

The following paper describes an attempt to predict and measure the throat impedance of a number of practical horns.

2. THE COMPUTER MODEL

2.1 The Horn Equation

The analysis of the sound field in horns found in most standard text books on acoustics (eg. Olson [1]) uses the so-called "Webster Horn Equation" (see Webster [2]). This equation

$$\frac{\partial^2 p}{\partial t^2} - c^2\frac{\partial^2 p}{\partial x^2} - \frac{\rho_0 c}{S(x)}\frac{\partial}{\partial x}\left[\frac{S(x)}{\rho_0 c}\right]\frac{\partial p}{\partial x} = 0 \qquad (1)$$

where c and ρ_0 are the sound speed and density respectively, describes the quasi one-dimensional sound field in a duct which has a cross sectional area S(x) that varies along its length (in this paper referred to as its "area profile"). The main assumption made during its derivation is that the sound field within the duct is a function of only one coordinate and for this to be the case, the area profile must be a slowly varying function of that coordinate. Analytic solutions of equation (1) exist for a small number of area profiles in a small number of coordinate systems (see Eisner [3] for an authoritative discussion), but for the analysis of horns of arbitrary area profile, numerical solution is necessary. The simplest analytical solution of equation (1) is that for an exponential area profile, ie

$$S(x) \propto e^{mx} \quad (2)$$

The coefficient m, which can be interpreted as

$$m = \frac{1}{S(x)} \frac{dS(x)}{dx} , \quad (3)$$

is known as the flare rate and is constant throughout the horn. Substitution into equation (1) and subsequent solution for p yields

$$p = Ae^{(a+ib)x} + Be^{(a-ib)x} , \quad (4)$$

where

$$a = -\frac{m}{2} , \quad b = -\frac{1}{2}\sqrt{4k^2 - m^2} . \quad (5)$$

A and B are the amplitudes of the foreward and backward travelling waves respectively and k is the acoustic free space wavenumber (ω/c).

2.2 The Model

If a horn of arbitrary area profile is divided up into short sections or elements, each element can be considered to be approximately exponential in shape, allowing equations (4) & (5) to be applied to it in isolation. Given a knowledge of the length, flare rate and the acoustic impedance at one end of the element, equations (4) & (5) can be used to determine the sound field within the element and the acoustic impedance at the other end. As the horn is made up of a number of these elements, using the acoustic impedance at the mouth of the horn, the impedance of the "throat" of the first element can be found. This impedance then becomes the "mouth" impedance of the next element and so on until the throat of the horn is reached. This method forms the basis of a semi-numerical computer model which has been developed, the results from which are presented and discussed below.

2.3 Assumed Wavefront Shapes

Because of the one-dimensional form of equation (1), the area profile S(x) must refer to the area of surfaces which are independent of the other two dimensions of the chosen coordinate system. For example, for an exponential horn defined in cylindrical or Cartesian coordinates, with z as the dependant coordinate, S(x) defines flat surfaces which are normal to the z axis. These surfaces, by definition, define the shape of the wavefronts within the horn. As these wavefronts must be normal to the walls of the horn at the walls, and therefore curved for a flaring horn, the direct application of equation (1) to an exponential horn (or element of) using flat cross sectional areas would lead to errors with all but the most trivially low flare rates.

The splitting up of the horn into exponential elements alleviates this problem. Strictly speaking for an exponential element, equation (1) only requires definition of the area of the wavefronts at the throat and mouth of the element and the length of the element for an accurate solution, with no specific reference to any coordinate system. Horns with area profiles for which an exact solution of equation (1) is not possible, cannot contain a purely one-dimensional sound field. However, if careful assumptions about the shape of the wavefronts within the horn are made, using the area of these in equation (1) can give more accurate results than the assumption of flat cross-sections. This approach has been used by Benade [4] and others. The variations from an essentially one-dimensional sound field that occur with these horn shapes will be present in the form of cross modes, and although these modes could have a significant effect on the sound field within the horn, their effect on the throat impedance (which is one-dimensional) is largely accounted for in the curvature and hence areas of the assumed wavefronts. Measurements of the wavefront shapes (taking the definition of a wavefront to be an iso-phase surface) in a number of different horns have shown that for horns with a rectangular cross-section they can be approximated by circular arcs which cut the horn walls and the axis at ninety degrees. For axi-symmetric horns, a "flattened" spherical cap is observed which can be approximated by taking the average of the plane cross-sectional area and that of a spherical cap. Extending this "one-parameter" approximation to the mouth radiation of the horn requires the definition of "extra" mouth elements to link the horn mouth to a hemispherical surface to which is attached the known radiation impedance of a sphere (baffled horns only).

The theoretical results presented below have been calculated using this one-parameter, finite exponential element model with the above assumed wavefront shapes.

3. THROAT IMPEDANCE MEASUREMENT

In order to test the validity of the model described above and to obtain data for further horn research, measurements were taken of the throat impedance of a number of horns and compared to the model predictions.

Various methods available for impedance measurement were considered, including measuring the pressure near the throat of the horn whilst being driven by a diaphragm of known velocity, a similar method using acoustic reciprocity and another using far field power measurements in a reverberation chamber. The former two methods suffered from two problems. First, the throats of most of the horns available had a diameter of only one inch, so a very small microphone would be necessary for the throat pressure measurements. Second, it would be difficult to attach a diaphragm to the throat of the horns which could be relied upon to be purely piston-like over a range of frequencies. These methods would also require the accurate calibration of both velocity source and microphone. The latter method would only give results for the real part of the throat impedance and would suffer the diaphragm problems of the former methods. Because of these problems, the above methods were dismissed as either impractical or inaccurate. However a method similar to that used to measure the absorption coefficient of sound absorbing materials was considered.

This method involves mounting a sample of the material, the surface acoustic properties of which are required, at the end of a long tube containing a probe microphone. An axial standing wave pattern is generated in the tube by a loudspeaker mounted at the other end. This standing wave pattern is sampled using the probe at two known points along the axis of the tube and from these measurements the acoustic impedance of the sample can be calculated thus:

$$\hat{Z} = i\left\{\frac{\sin(kd_2) - \hat{T}\sin(kd_1)}{\hat{T}\cos(kd_2) - \cos(kd_1)}\right\} , \qquad (6)$$

where d_1 and d_2 are the distances from the microphone to the sample and T is the transfer function between the measured pressures at these positions. k is the acoustic free space wavenumber.

To measure the impedance at the throat of a horn using this method, the tube needs to be of the same diameter as the throat, so that when a horn is mounted at the end of the tube, the throat becomes a "virtual surface" and its impedance can be measured using exactly the same method as for an absorptive sample. Traditionally, these measurements are carried out at one frequency at a time. To build up a complete picture of the impedance over the frequency range of interest to compare with the 256 frequencies calculated in the model would obviously take a prohibitive amount of time so an adaptation, using a dual channel FFT analyser, developed by Fahy [5], was used. This adaptation involves exiting the tube with a pseudo-random binary signal (PRBS) possessing 500 discrete frequencies. Transfer functions are calculated between outputs of the probe at the two positions and the driving signal and these are fed into a computer which then calculates the impedance at all 500 frequencies. The complete process takes only minutes - a considerable saving in time over the traditional method. A diagram of the apparatus used and details of the tube are shown in figs. (1) and (2).

Various precautions were taken to minimise any errors in the measurements. The most important being the use of one microphone which is moved, instead of two fixed microphones to avoid matching and calibration problems, and the reference of each microphone output to the driving signal before calculation. The microphone positioning and spacing was carefully optimised to distances of 30 and 55 mm from the throat by testing on the open, unbaffled tube end (the impedance of which has been well researched) until good results were obtained over the frequency range of 250 to 5000 Hz. To minimise the effect of displacement of the probe on the sound field within the tube, the probe was made as small as was practically possible (3 mm). The upper frequency limit of such measurements is determined by the diameter of the tube and its corresponding plane-wave limit. With the one inch tube used this set an upper frequency limit of about 8000 Hz, so 5000 Hz was considered to be a "safe" maximum. The random errors associated with signal noise in the FFT analysis were reduced to an acceptable limit by allowing the analyser to take more than 500 averages of each measurement. The horn under test was mounted in a baffle "window" between two rooms, one containing the tube and measurement equipment and the other an anechoic recieving room into which the horn radiated. The wall containing the horn was left bare to closely approximate infinite baffle conditions.

4. DISCUSSION OF RESULTS

Figures 3 & 5 show the throat impedance of two horns calculated using the one-parameter model above and figures 4 & 6 the corresponding measured results. The first horn is an axi-symmetric design of 180 mm length, with a low flare rate terminating in a small mouth of 80 mm diameter. The second horn is rectangular in cross section with a total length of 280 mm. It has sides that flare almost to 180 degrees and a straight (as with conical horns) top and bottom terminating in short "lips". Both horns have throat diameters of one inch. Good agreement can be seen between the one-parameter model results and those of the measurements, with the various peaks and dips (indicating mouth reflections) occuring at the same frequencies and possessing similar amplitude.

The limitations of the present model became apparent when rectangular horns with large "lips" were measured. The agreement in these cases was less good, probably because the mouth condition could no longer be considered to be baffled. A different definition of mouth element however, gave good results and work is continuing on a more general element capable of modelling a wider variety of geometries. As a control, the horns were also modelled using the plane wavefront assumption. In the case of the axi-symmetric horn, with a baffled piston mouth impedance, this produced results close to those shown in figure 3 but for the rectangular horn, total disagreement between the predicted and measured results occured and the lips had to be totally ignored indicating the necessity for the "assumed wavefront" approach described above.

The "noise" seen in figures 4 & 6 at high and low frequencies is due to the limited bandwidth of the driver used. This has now been corrected for by the use of a filter on the input to the power amplifier (see fig. 1).

5.CONCLUSIONS

A one-parameter, finite exponential element model, using the area of the assumed wavefronts within a flare, has proved capable of the accurate prediction of the throat impedance of a number of baffled horns. Work on the extension to unbaffled horns is continuing. Measurements using a two-microphone-position impedance tube technique, giving good results over a frequency range of 250 to 5000 Hz were used to check the accuracy of the model..

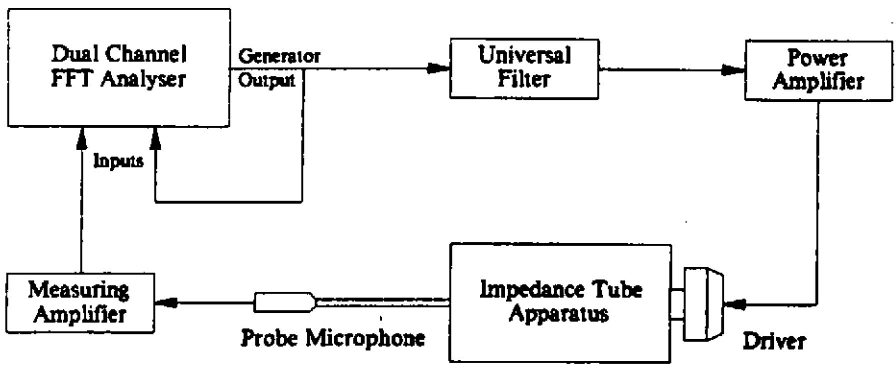

Fig.1 Impedance Measurement Set-up

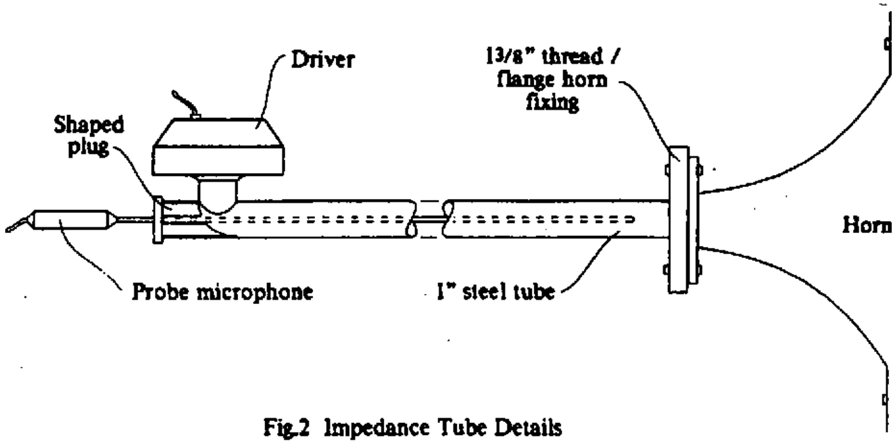

Fig.2 Impedance Tube Details

THROAT IMPEDANCE

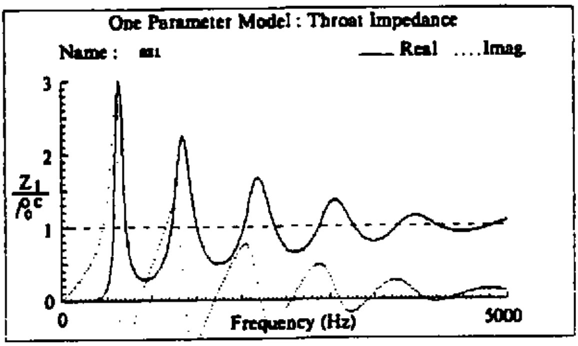

Fig.3 Axi-symmetric Horn : Predicted Result

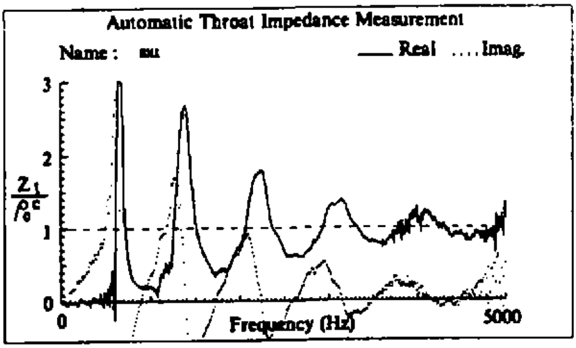

Fig.4 Axi-symmetric Horn : Measured Result

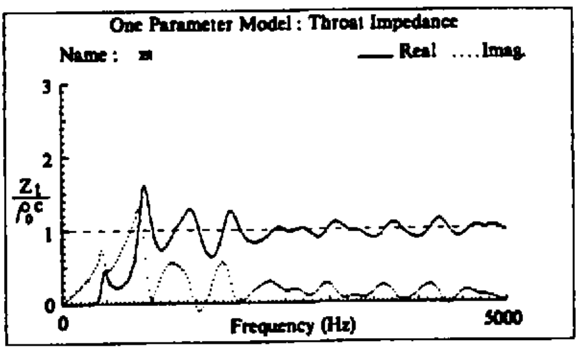

Fig.5 Rectangular Horn : Predicted Result

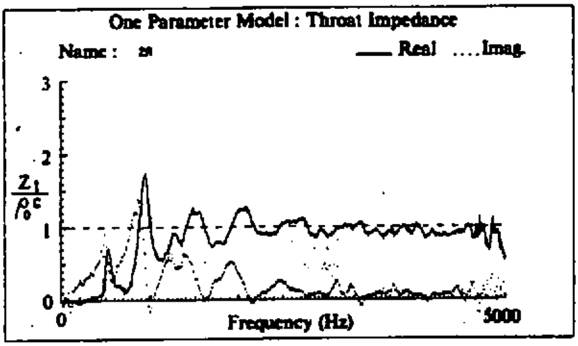

Fig.6 Rectangular Horn : Measured Result

6. REFERENCES

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