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THE USE OF CEPSTRAL ANALYSIS IN THE INTERPRETATION OF LOUDSPEAKER FREQUENCY RESPONSE MEASUREMENTS

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1. INTRODUCTION

Frequency response is generally regarded as the most important specification of the performance of a loudspeaker, and the pursuit of the 'ruler-flat' frequency response plot has occupied the minds of loudspeaker designers and engineers for many decades. It is unlikely that this ideal will ever be achieved due to the presence of resonances, reflections etc, both in the air and in the structure of the loudspeaker, which manifest themselves as peaks and dips in the response. The correct interpretation of these peaks and dips can help pin-point the physical causes for the departure of the measured response from the ideal.

Through the use of modern frequency response measurement techniques based on the Fourier transform, digital post-processing of measured responses can easily be carried out. One such post-processing technique is cepstral analysis, which can be used to detect the presence of reflections in a response and display them along a temporal scale. In order to usefully apply cepstral analysis to loudspeaker measurements, some pre-processing of the measured data is desirable.

In this paper, suggestions are put forward as to how cepstral analysis may be used to aid the interpretation of loudspeaker frequency responses, and some examples of its use are described and illustrated.

2. CEPSTRAL ANALYSIS

The presence of an echo in a measured signal (fig 1a) gives rise to comb-filtering of the frequency spectrum of the signal (fig 1b). If the logarithm of the spectrum is calculated, this comb filtering becomes a sinusoid along the frequency axis with a period equal to the reciprocal of the delay between the signal and its echo (fig 1c). If the resultant logarithmic spectrum is then treated as a time signal, the sinusoid will become a 'spike' in the spectrum of this signal, displaced along the horizontal axis by an amount proportional to the delay (fig 1d). The 'spectrum of the logarithmic spectrum' is known as the *cepstrum* of the original time signal, and has a horizontal axis with units of seconds.

Cepstral analysis was first defined by Bogert et al [1] as a means of detecting echoes in seismic signals, and it was they who named the process, and other associated terms, as anagrams of their time domain counterparts: cepstrum (spectrum), quefrency (frequency), liftering (filtering), rhamonics (harmonics), saphe (phase) etc. The original definition of the power cepstrum of a time signal is

$$C(\tau) = | \mathcal{F}\{ \log S(f) \} |^2 \quad , \tag{1}$$

where $\mathcal{F}$ denotes the Fourier transform, and

$$S(f) = | \mathcal{F}\{ a(t) \} |^2 \quad , \tag{2}$$

is the power spectrum of the time signal $a(t)$. The power cepstrum can also be defined as

$$C(\tau) = \mathcal{F}^{-1}\{ \log S(f) \} \quad , \tag{3}$$

where $\mathcal{F}^{-1}$ denotes the inverse Fourier transform; both give the same result but the latter method is reversible.

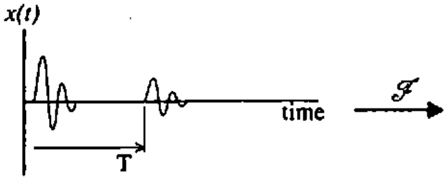

Figure 1a Time Signal with Echo

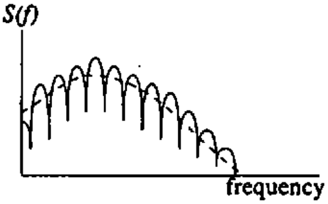

Figure 1b Spectrum

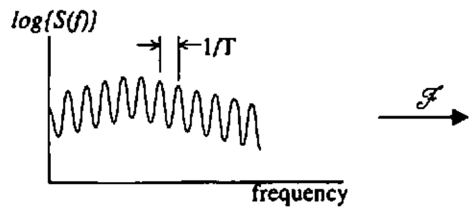

Figure 1c Log Spectrum

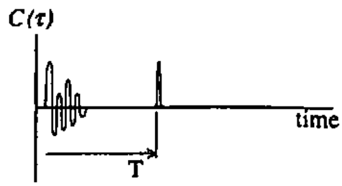

Figure 1d Cepstrum

As described above, cepstral analysis provides a means of displaying any repetitions of parts of a time signal as individual spikes along a time scale, thus the process can be used to locate these repetitions in time, almost regardless of the complexity of the time signal. By way of example, figure 2a shows an artificially generated time signal, and figure 2b, the same signal with an echo added 10ms later having one quarter of the magnitude of the time signal. Figures 2c and 2d show the power spectra, and figures 2e and 2f the power cepstra of the two time signals. Although small differences can be seen between the two time signals and their spectra, the presence of the echo can only clearly be seen in the cepstra; the two cepstra are identical except for a large spike at a quefrency of 10ms.

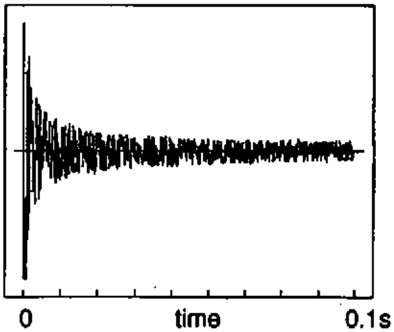

Figure 2a Time Signal

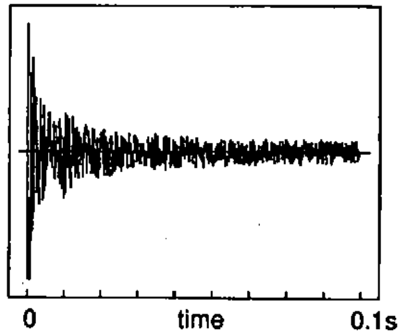

Figure 2b Time Signal with Echo

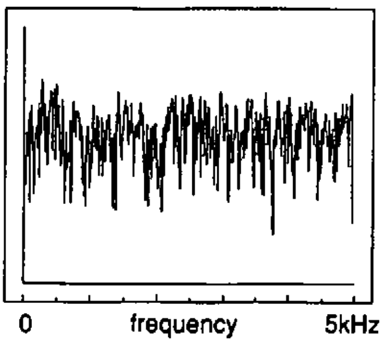

Figure 2c Power Spectrum of 2a

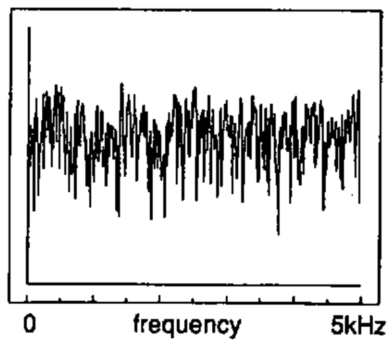

Figure 2d Power Spectrum of 2b

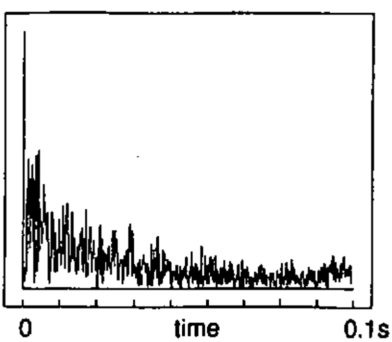

Figure 2e Power Cepstrum of 2a

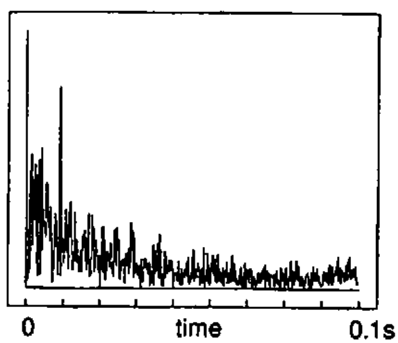

Figure 2f Power Cepstrum of 2b

3. THE APPLICATION OF CEPSTRAL ANALYSIS TO LOUDSPEAKER FREQUENCY RESPONSES

Ideally, for most applications, the frequency response function of a loudspeaker should be flat within a specified bandwidth. Any reflections of the signal applied to a loudspeaker which are present (from the back wall of a cabinet for example) will cause peaks and dips in the measured response and may be heard as coloration when the loudspeaker is auditioned. The detection of these reflections can improve interpretation of the measured frequency response, and may help pin-point the physical causes for any response irregularities.

The inverse Fourier transform of the frequency response function is the impulse response, which can be considered to be a time signal; the squared modulus of the frequency response function then becomes the power spectrum of the impulse response to which cepstral analysis may be applied. For example, the time signal shown in figure 2b could be the measured impulse response of a loudspeaker system in a room; the presence of the spike in the power cepstrum (figure 2f) at 10ms would indicate a reflection from a surface or object positioned such that the path length from the loudspeaker to the measurement point via the object is approximately 3.4m longer than the direct path.

The situation is a little more complicated when the response of real loudspeakers is considered. Gross frequency response errors such as low- and high-frequency bandwidth limitations give rise to very high values in the cepstrum at small time delays (low quefrencies) which can mask early reflections. To overcome this problem, an inverse filter, matched to the low- and high-frequency roll-offs can be applied to the measured frequency response function to remove the bandwidth limitations prior to calculation of the cepstrum. However, the measured frequency response is likely to be irregular outside the bandwidth of the loudspeaker (due to signal-to-noise problems for example), giving rise to unwanted information on the cepstrum, so further filtration of the spectrum is required. To remove the irregularities outside the loudspeaker bandwidth, the previously 'flattened' response can be normalised to an average value of zero and divided by the the 'envelope' of the inverse filter. The cepstrum of the resultant response will then contain only that information present in the bandwidth of interest. This process is demonstrated in figure 3. Figures 4a, 4b and 4c shows the cepstra for the measured response, the flattened response and the flattened / filtered response respectively for the loudspeaker in figure 3 (note different vertical scales). The bandwidth of this loudspeaker, being a mid-range unit, was limited to 1kHz to 6kHz, so the cepstrum of the measured response (figure 4a) is dominated by the low- and high-frequency roll-offs. Some irregularities can be seen in the flattened response at the frequency extremes which are of the same order as those within the bandwidth of the loudspeaker; a comparison between the cepstra for this response (figure 4b) and that of the flattened / filtered response (figure 4c) show that these out-of-band irregularities mask a great deal of the detail in the cepstrum that is due to irregularities within the loudspeaker bandwidth.

3.1 Choice of Filter.

The filter used in the flattening and filtering processes can be chosen by a number of different methods. If the bandwidth and roll-offs for the loudspeaker are known, a suitable filter can be designed quite easily; for example, in the case of a low-frequency drive-unit in an enclosure, for which the Theile-Small parameters are known, a mathematical model of the theoretical low-frequency roll-off can be used to generate the filter. For less well-defined systems, the filter coefficients can be found using a least-squares fit to the measured response, with the number of filter elements restricted to avoid losing smaller response irregularities. For the loudspeaker in the example (figures 3 and 4), the frequency response was measured through a band-pass filter (actually the mid-range part of an active cross-over network); the response of this filter was then measured and used in the pre-processing stages.

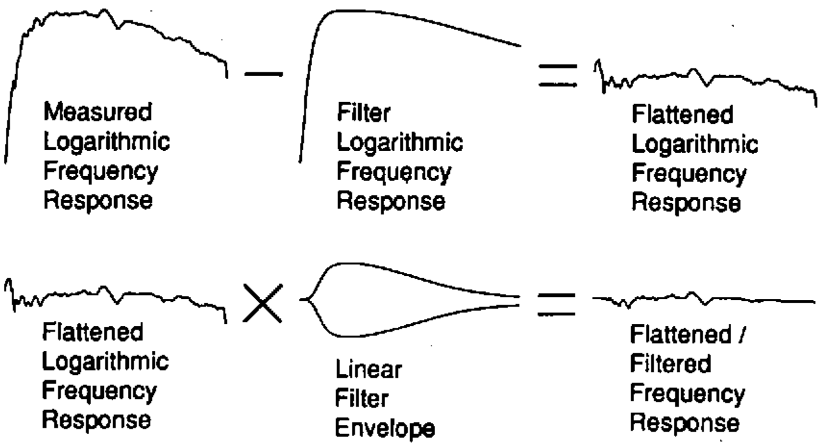

Figure 3 Pre-Processing of Measured Loudspeaker Frequency Response prior to Calculation of Cepstrum

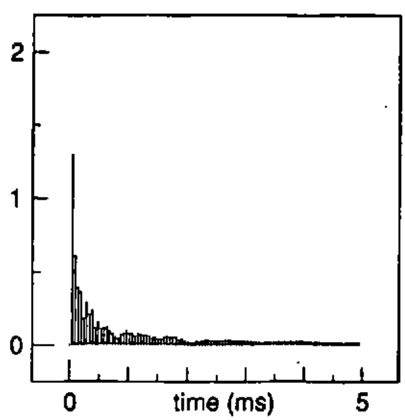

Figure 4a Cepstrum of Measured Frequency Response

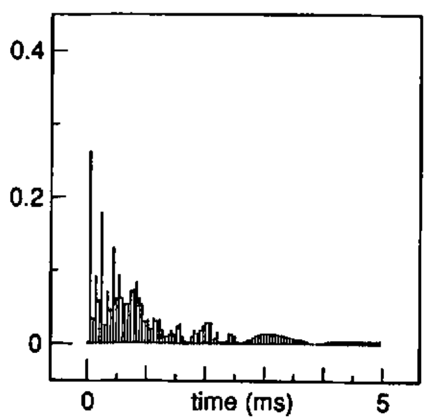

Figure 4b Cepstrum of Flattened Frequency Response

CEPSTRAL ANALYSIS

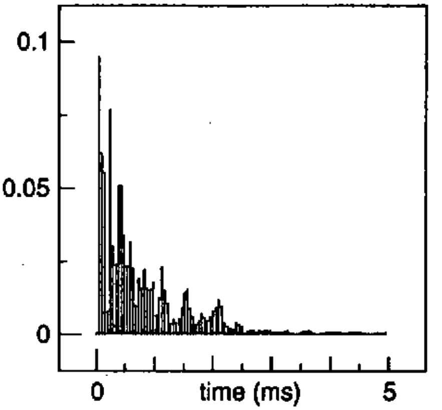

Figure 4c Cepstrum of Flattened and Filtered Frequency Response

3.2 Practical Uses of Cepstral Analysis

The example loudspeaker is one of twenty mid-range loudspeakers which formed the test examples in a listening test conducted as part of a three-year project on mid-range horn loudspeakers [2]. Some of the aims of the test were to find which, if any, of a range of horns sounded similar to example direct-radiating loudspeakers and what physical characteristics make a horn sound like a horn. Measurements were taken of the frequency response functions of all of the loudspeakers and comparisons were made between these measurements and the listening test results. The power cepstra of the loudspeakers were calculated as above and, when compared, were found to reveal many clues as to the similarity or otherwise of the sound of the different loudspeakers. For example, it was discovered that the shorter horns sound more like the direct-radiating loudspeakers than the long horns, and this was found to be due to the reflections from the mouths of the short horns occurring at around the same time as those in the cones of the direct-radiating loudspeakers; the reflections from the mouths of the long horns occur later and contribute to the characteristic 'horn sound' even though they are generally lower in level than those of the short horns.

During the same horn research, a flare miss-match at the throat of one of the horns when attached to a particular driver was found to be the cause of response irregularities. Cepstral analysis revealed quite strong reflections from the discontinuity and when the horn was attached to another driver both the spikes on the cepstrum and the response irregularities disappeared.

It may sometimes be desirable to remove a reflection from a response or signal. In order to use cepstral analysis for this application, the complex spectrum is used in the calculation of the cepstrum; which is then known as the complex cepstrum. This process is completely reversible, so that a time signal may be converted to a complex cepstrum, the offending reflection (spike) *liftered* out, and the cepstrum then converted back to a time signal.

4. REFERENCES

[1] B P Bogert, M J R Healy & J W Tukey, "The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo-Autocovariance, Cross-cepstrum and Saphe Cracking" in Proceedings of the Symposium on Time Series Analysis, by M Rosenblatt (Ed), Wiley, NY, 1963, pp 209-243.

[2] K R Holland, "A Study of the Physical Characteristics of Mid-Range Horn Loudspeakers and their Relationship to Perceived Sound Quality", PhD Thesis, University of Southampton, 1992.