Proceedings of The Institute of Acoustics

Parametric Acoustic Arrays in Temperature Stratified Shallow Water

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I. Introduction.

The application of parametric acoustic arrays to shallow water sound propagation has recently been given considerable interest [1]-[6]. The inherent beamwidth and bandwidth qualities for instance, of the parametric array make it particularly applicable to selective mode excitation in shallow water, and theoretical studies as well as model scale [1], [5], [6] and full scale [3], [4] tests have proven the superiority of parametric arrays over conventional linear sound sources for acoustic signal propagation in isovelocity shallow water, for instance under the influence of various seabed materials, seabed slopes etc. [6].
Due to the fact that a shallow water region as for instance the Baltic Sea most frequently shows a stratification due to temperature and salinity variations with depth and thus a sound velocity profile, frequently of a complicated geometry, it is of particular interest to know how sound velocity variations with water depth will influence the parametric beam formation and the mode excitation and thus the acoustic pressure variation with depth at greater distances from the parametric source.
This paper aims at an illumination of problems in relation to parametric arrays in temperature stratified shallow water by theoretical and experimental model studies of the subject.

II. Theory.

Let us consider a point source of source strength Q positioned in the depth y_o in temperature stratified shallow water of constant depth H. The source is transmitting a CW-signal of angular frequency ω into the water layer bounded above by air and below by a semi-infinite fluid of constant density ρ_2 and sound velocity c_2. The water density is assumed constant ρ_1 and the velocity of sound in the water layer varies as a function of depth $c_1(y)$. A receiver is located at a position (r,y), where r denotes the horizontal distance between the source and the receiver and y is the depth of the receiver, se figure 1.

Acording to [7] and [8] at long ranges (kr >> 1) the following velocity potential will be found:

$$\psi(r,\xi) = i\left(\frac{1}{8\pi r}\right)^{1/2}\frac{\rho_1}{H}\left[\sum_{n=1}^{m}\frac{U_n(\xi_o)\cdot U_n(\xi)}{k_n^{1/2}}\exp\left(i\left(k_n r-\frac{\pi}{4}\right)\right)\right]Q\exp(-i\omega t) \qquad (1)$$

where $\xi = y/H$ is the normalized depth, k_n is the horizontal component of the wave number of the n'th mode and m is the number of discrete modes excited.

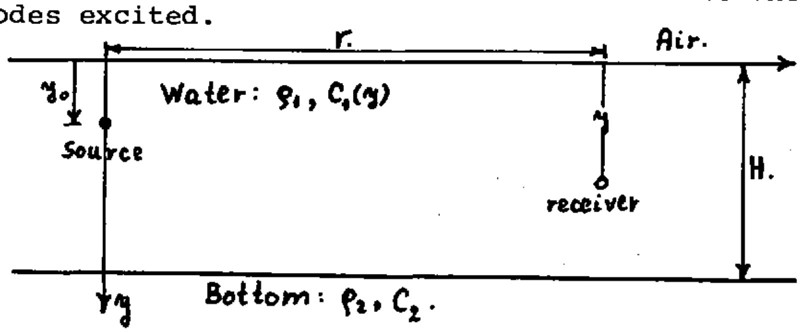

Figure 1. Coordinate system used in the model for point source excitation of modes.

The normalized depth functions $U_n(\xi)$ are defined through:

$$U_n(\xi) = \begin{cases} N_n Y_n^{(1)}(\xi) & \text{for} \quad 0 \le \xi \le 1 \\ N_n Y_n^{(2)}(\xi) & \text{for} \quad 1 \le \xi < \infty \end{cases} \tag{2}$$

where N_n are normalized constants given by:

$$N_n^2 \left\{ \int_0^1 \rho_1 [Y_n^{(1)}(\xi)]^2 d\xi + \int_1^\infty \rho_2 [Y_n^{(2)}(\xi)]^2 d\xi \right\} = 1 \tag{3}$$

and where Y_n and k_n are solutions to the differential equations:

$$\frac{d^2 Y_n^{(1)}}{d\xi^2} + H^2 \left(\frac{\omega^2}{C_1^2(\xi)} - k_n^2 \right) Y_n^{(1)} = 0 \quad \text{for} \quad 0 \le \xi < 1 \tag{4}$$

$$\frac{d^2 Y_n^{(2)}}{d\xi^2} + H^2 \left(\frac{\omega^2}{C_2^2} - k_n^2 \right) Y_n^{(2)} = 0 \quad \text{for} \quad 1 \le \xi < \infty \tag{5}$$

For trapped modes $k_n > \omega/c_2$, which from (5) gives:

$$Y_n^{(2)}(\xi) = B \exp(-H(k_n^2 - (\omega^2/C_2^2))^{1/2} \xi), \tag{6}$$

where B is an arbitrary constant which may be chosen as:

$$B = -H^{-1}\left(k_n^2 - (\omega^2/C_2^2)\right)^{-1/2} \cdot \exp\left(H\left(k_n^2 - (\omega^2/C_2^2)\right)^{1/2}\right) \qquad (7)$$

The normalized water depth is now divided into N horizontal layers of equal thickness, each layer having a constant temperature and sound velocity. These layers are numbered from the bottom (i = 0) towards the surface (i = N).

The differential equation (4) is approximated by a finite difference equation and the following boundary conditions are used:

$$Y_n^{(1)}(0) = 0 \qquad \text{at the surface} \qquad (8)$$

and

$$\rho_1 Y_n^{(1)}(1) = \rho_2 Y_n^{(2)}(1) \qquad (9)$$

$$\left(\frac{dY_n^{(1)}}{d\xi}\right)_{\xi=1} = \left(\frac{dY_n^{(2)}}{d\xi}\right)_{\xi=1} \qquad (10)$$

expressing continuity in acoustic pressure and the normal component of the particle velocity at the bottom boundary.

For the chosen value of B the boundary conditions (9) and (10) lead to:

$$Y_n^{(1)}(1) = -\frac{\rho_2}{\rho_1} H^{-1}\left(k_n^2 - (\omega^2/C_2^2)\right)^{-1/2}$$

and

$$\left(\frac{dY_n^{(1)}}{d\xi}\right)_{\xi=1} = 1.$$

The equation (1) represents the signal field at long ranges as the sum of a finite number of discrete terms, each term corresponding to one of the normal modes of the system given by the water layer and its boundaries.

A computer programme for calculation of k_n^2 and $(Y_n^1)_i$ for $0 \leq i \leq N$ has been worked out. The results thus found are inserted into Eq.(3) for determination of N_n which through expression (2) leads to $U_n(\xi)$.

Figure 2 gives an example on calculation of the eigenfunctions $U_n(y)$ - the mode amplitude - for the first mode in isovelocity water under the conditions:

A: (c_1 = 1481,4 m/s ; ρ_1 = 1000 kg/m^3 ; c_2 = 1650 m/s ;
ρ_2 = 1820 kg/m^3 ; H = 0,06 m ; $\omega = 2\pi\cdot400000$.
k_1 = 1695,81 m^{-1} (calculated))

and in a water layer having a constant temperature gradient characterized by:

B: (Surface temp. 21°C; bottom temp. 20°C , ρ_1 = 1000 kg/m^3
ρ_2 = 1820 kg/m^3 ; c_2 = 1650 m/s ; H = 0,06 m ,
$\omega = 2\pi\cdot400000$; k_1 = 1694,39 m^{-1} (calculated)).

Having calculated U_n the pressure variation $p(r,y)$ may be found through Eq.(1) by:

$$p(r,y) = -\rho_1 \frac{\partial \Psi}{\partial t} \tag{11}$$

No attenuation of the individual modes has been accounted for, but a simple mode attenuation coefficient for each individual mode may be introduced [7]. This coefficient comprising the effects of all loss sources influencing the propagation of a mode, may be determined through experiments only.

Introduction of the parametric array into the shallow water area in figure 1 determines the source strength density in complex form [9] by:

$$q = \frac{-i\beta\,\omega_d}{\rho_1^2\, c_0^4}\cdot D(k_1 a \sin\phi)\cdot D(k_2 a \sin\phi)\cdot\frac{r_0^2}{r^2}\,p_1\,p_2\,\cdot$$

$$\cdot\exp(-i(\omega_d t - r(k_d + i(\alpha_1+\alpha_2)))) \tag{12}$$

If rotational symmetry in source strength density is assumed around the acoustic axis, - even if the acoustic axis due to the velocity gradient is no longer a straight line it may through a ray-theoretical transformation be transformed back to the isovelocity straight line case -, then an annulus shaped incremental volume dV with its center on the acoustic axis and having a constant source strength density overall in the same annulus is assumed.

The velocity potential arising from the incremental volume dV may now be written using Eq.1:

$$d\Psi(R,\zeta) = i(8\pi(R-r\cos\phi))^{1/2}\,\frac{\rho_1}{H}\sum_{n=M_e}^{M_h}\Big[\frac{U_n(\zeta_o(r))\cdot U_n(\zeta)}{k_n^{1/2}}\,\cdot$$

$$\cdot\exp(i(k_n(R-r\cos\theta)-\tfrac{\pi}{4}))\Big]\,q\,dV \tag{13}$$

which inserted into (11) and after some coordinate transformations leads to the pressure variation as a function of space and time in a vertical plane through the acoustic axis of the array given by:

$$p(R,\xi,t) = i\cdot A\exp(-i\tfrac{\pi}{4})\cdot\exp(-i\omega_d t)\int_{r_1}^{r_2}\sum_{n=M_\ell}^{M_h}\Big[\frac{U_n(\xi_o(r))\cdot U_n(\xi)}{k_n^{1/2}}\cdot$$

$$\cdot\exp(i k_n R)\cdot\exp(r[(k_d-k_n)i-(\alpha_1+\alpha_2)])\Big](R-r)^{-1/2}dr\cdot$$

$$\cdot\int_0^{\phi_{-3dB}}\sin\phi\cdot D(k_1 a\sin\phi)\cdot D(k_2 a\sin\phi)\,d\phi. \qquad (14)$$

with A is a real constant given by:

$$A = \frac{\sqrt{\frac{\pi}{2}}\,\beta\,\omega_d\,r_o^2\,p_1\,p_2}{H\,C_o^4}$$

A computer programme has been worked out for solving expression (14) for a measured sound velocity profile approximately consisting of two constant gradients.
In (14) M_ℓ and M_h are the lowest and the highest mode over which the summation takes place. R is the horizontal distance between the transducer and the observation point, k_d is the free field difference frequency wave number, α_1 and α_2 are absorption coefficients of the primaries.

The integration limit for the angle ϕ between the acoustic axis and radius vector r to the volume element (for the non-isovelocity case the ray through the transducer and the volume element) has been chosen equal to the 3dB angle for the lowest frequency (3.8 MHz) primary wave.

All interaction is assumed to take place in the farfield of the primaries and the horizontal integration limits $r_1 = r_o$ (Rayleigh distance) and

$$r_2 = (\alpha_1+\alpha_2)^{-1}$$

are used.
The original tilting of the array in order to avoid any reflection by the shallow water boundaries before $r > r_2$ is comprised by the angle ϕ.

Several normalized depth profiles showing $p(\xi)$ for R as a parameter have been calculated for single modes and mode combinations, see figures 8 - 10.

III. Experimental equipment and results.

The experiments were performed in a shallow water test basin, 12 m long and 1.6 m in width. The waterdepth was kept constant on 0.06 m over a medium sand seabed. Experimental procedures used and transmitting and receiving transducers are described in detail in Ref.[6]. The mean primary wave frequency was 4 MHz being 100% amplitude modulated by 200 kHz leading to sideband interaction (suppressed carrier) giving f_d = 400 kHz. Primary source level 195 dB re 1 μPa·m.

Various quasi-steady temperature profiles were generated through slow mixing of hot and cold water using a thin layer of polystyrene foam plates as a "buffer" between the hot and the cold water. Simultaneous measurement of temperature profile and pressure variation with depth was performed at various times during the heat transmission caused change of the temperature gradients. Measured temperature profiles and pressure-depth variations are given in the figures 3 - 7 for various R-positions. The measurements reported in the figures 3 - 7 were all performed using the tilt angle zero.

IV. Discussions.

Compared to isovelocity sound propagation the introduction of a temperature (velocity) profile seems in general to lead to a more complicated distribution, more modes are simultaneously exited and superposed at the R-positions. Changes in tilt angle did not alter this general course of the pressure-depth functions, where a smaller temperature gradient seems to lead to the excitation of fewer modes and thus to more smooth pressure-depth functions.

The figures 8 - 10 show a comparison between measured and calculated pressure-depth functions. Figure 8 represents isovelocity sound propagation with the measured pressure profile a) showing a peak sound pressure level of 128 dB re 1 μPa. Profile b) is calculated using the superposition of the 5 lowest modes and taking into account the individual phase contribution from each source. Profile c) represents a modification to the computer programme where all contributions from sources are arriving at the observation point in phase. Only one mode (the lowest) is considered. Profile d) represents the superposition of 5 modes using the modified computer programme.
A good qualitative agreement between the profiles a) and b) and c) are observed, while d) deviates strongly from a) in shape, but not in amplitude.
In figure 9 a comparison between measured e) and calculated f) profiles for an approximate temperature profile g) is given at the position R = 4 m. Max. sound pressure level is 135 dB re 1 μPa. The profile f) is calculated using a superposition of the 8 lowest modes taking into account the phase differences between the individual sources. A fair qualitative agreement, but not quantitative (SPL_{max} = 110 dB re 1 μPa), has been found.

Also in figure 10 the superposition of the 8 lowest modes has been used for the pressure-depth profile calculation based on the temperature profile k). Profile i) has been calculated taking into account the phase differences between the sources, while j) assumes all contributions to arrive in phase at the observation point. A more correct peak sound pressure level (131 dB compared to measured 135 dB re 1 μPa) is obtained in j), but strong qualitative deviations exist.

In relation to the validity of the computer model used some essential points are still shrouded in mist and much more work has to be done before any far reaching conclusions can be drawn. These points are for instance:

a. The theory demands long ranges in order appropriately to establish the modes. The limited tank length and the lack of a useable gating device have prevented us from long range experiments.

b. The energy distribution on modes is still not clear. The assumption of equal energy distribution on modes is questionable. The qualitative deviations observed in profile d) and j) may among other things be due to this problem.

c. The transformation from virtual to real sources needs some more studies, and the transition from the use of the free field difference frequency wave number in the array to the mode wave number close to or at some distance from the interaction region should also be studied more closely.

d. No nearfield interaction is considered and the termination of the array after $r_2 = (\alpha_1+\alpha_2)^{-1}$ puts limitations on the interaction field, which according to some preliminary calculations have shown that several dB in sound pressure level without changes in profile shape may be obtained by increasing the interaction region at both ends. Also an increase in ϕ_{eff} to more than the 3 dB limit leads to a gain of several dB.

e. An evaluation of the applicability of the assumption of a constant phase in planes perpendicular to the (non-straight) acoustic axis, in particular by strong temperature gradients, is needed.

f. No attenuation due to absorption or transmission (and dissipation) of sound into the seabed has been accounted far. Some calculations based on [10] seem to indicate that for weak temperature gradients (i.e. 0.1 - 1.0 °C over o.06 m) the bottom will reflect almost all the incident acoustic energy and no phase change will occur, but this problem also have to be studied more closely. This list of still open questions is not pretending to be exhaustive.

Conclusions.

In spite of some encouraging qualitative results obtained a number of questions are still open and have to find their replies before a clear picture is formed of the operation of a parametric array in a temperature (and salinity) stratified shallow water region.

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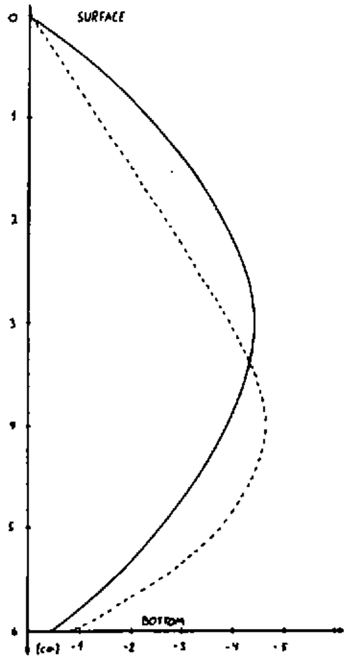

Figure 2. 1'st Mode.
: Isovelocity.
: Temp. gradient.

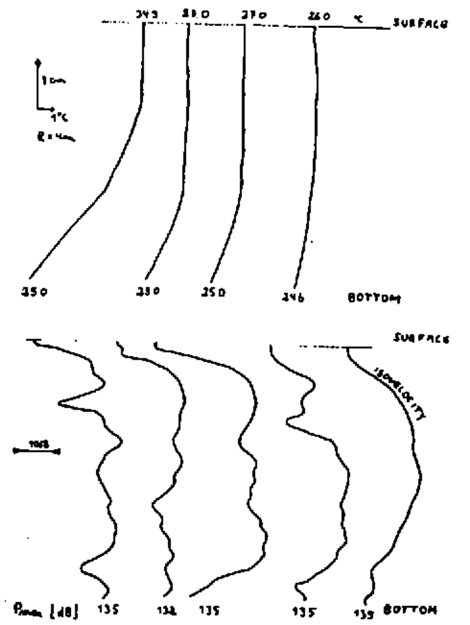

Figure 3. R = 4 m.
a. Temp. profiles.
b. p(y) dB re 1 μPa

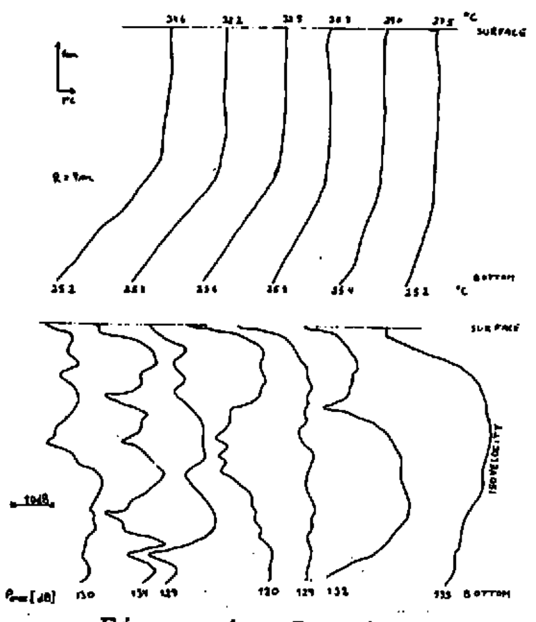

Figure 4. R = 4 m.
a. Temp. profiles.
b. p(y) dB re 1 μPa

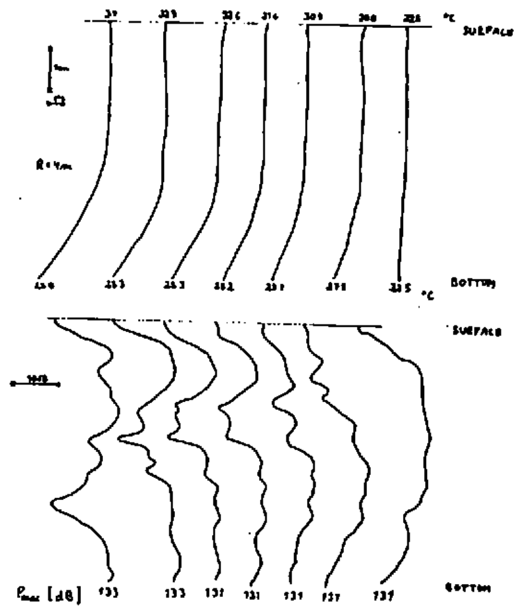

Figure 5. R = 4 m.
a. Temp. profiles.
b. p(y) dB re 1 μPa.

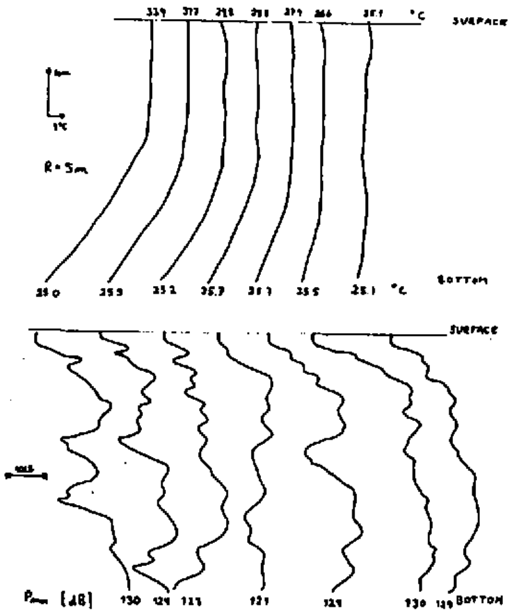

Figure 6. R = 5 m.

a. Temp. profiles.
b. p(y) dB re 1μPa

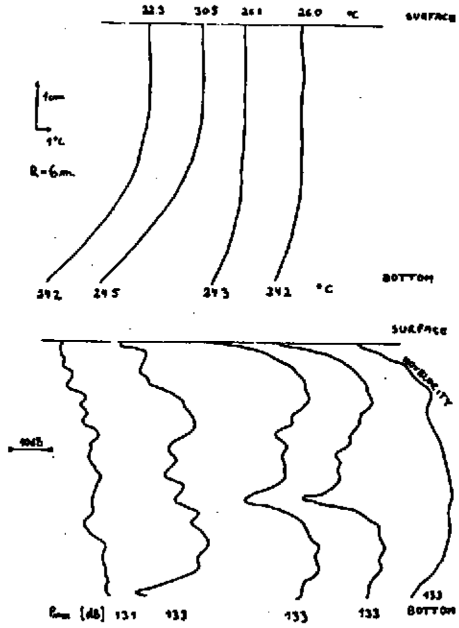

Figure 7. R = 6 m.

a. Temp. profiles.
b. p(y) dB re 1 μPa

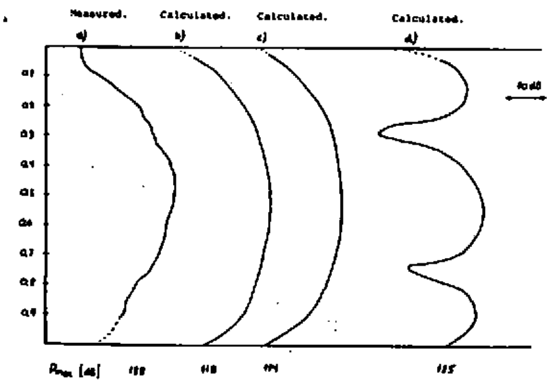

Figure 8. R = 4 m.

Isovelocity water.
Meas. & calc. p(y).

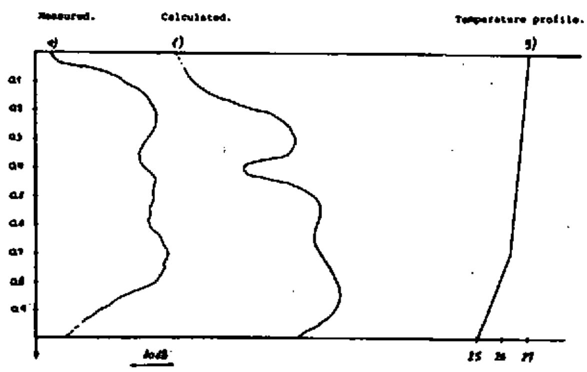

Figure 9. R = 4 m.

Temp. gradient. Meas.& calc. p(y).

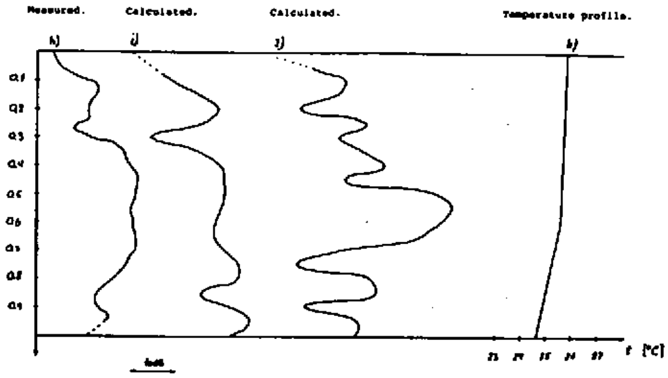

Figure 10. R = 4 m.

Temperature gradient.
Meas.& calc. p(y).