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A MODEL FOR ESTIMATING ACOUSTIC EMISSION AMPLITUDES

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A model for calculating and describing the signal amplitudes for acoustic emission stress waves is presented. The model is based on Kosevich theory for moving dislocations and takes into account the transfer function of the measurement system, see figure. The aim of this work is to work out a complete model which gives a possibility to correlate the amplitude of the measured electrical voltage to the physical event within the material.

So for however, the proposed model don't takes into consideration the influence of the material damping on the signal amplitude. This might be the next step to improve the model.

The usual used measurement techniques, ring-down connting and distribution analysis, don't give any direct correlation between the absolute magnitude of the acoustic emission event and the recorded parameters.

1 *Introduction*

There exist several, both simple and extensive models for describing the AE-mechanism. To-day one cannot say that one model is better than the other.

This paper presents a model based upon a dynamical dislocation theory stated by Kosevich [I].

This model shows how the physical process can be related to the AE-signal. Most of this is, however, pure mathematics, which is presented in [II]. The last part of the work is not yet published, but will be in the near future.

2 *Stress Field around a Time-varying Inelastic Strain*

Acoustic emission usually occurs due to a sudden change in the internal strain distribution. To describe this AE-source we consider an inelastic distribution

$$\beta^*(\underline{r},t) \qquad (1)$$

We evaluate the stress-field around this time-varying inelastic strain

Equilibrium:

$$\rho\ddot{U}_i = \sigma_{ij,j} = C_{ijk\ell}\beta_{k\ell,j} \qquad (2)$$

Note: U is the total deformation

$$U_{m,n} = \beta_{nm} + \beta^*_{nm} \qquad (3)$$

(2) and (3) give

$$\rho\ddot{U}_i - C_{ijk\ell} \cdot U_{k,\ell j} = - C_{ijk\ell}\, \beta^*_{k\ell,j} \qquad (4)$$

This differential equation can be solved using Green's Function.

Green's function represents the displacement U_i $(\underline{r},t)$ due to a unit impulse force in the m-direction at $\bar{r}'$ and t', figure 1.

The notation for Green's function is G_{im} $(\underline{r} - \underline{r}', t - t')$

This means

$$\rho\ddot{G}_{im} - C_{ijk\ell}\, G_{km,\ell j} = \delta_{im}\, \delta(\underline{r} - \underline{r}')\ \ \delta(t - t') \qquad (5)$$

For an isotropic material G is known, see [II].

The solution to eq (4) is thus

$$U_i(\underline{r},t) = - C_{mjk\ell} \int d\underline{r}' \int_{-\infty}^{\infty} dt' \cdot \frac{\partial}{\partial x_j} \left[G_{im}(\underline{r} - \underline{r}', t-t')\right] \beta^*_{k\ell}\ (\underline{r}', t) \qquad (6)$$

Hookes law gives the stress

$$\sigma_{ij} = C_{ijk\ell}\ \beta_{k\ell} \qquad (7)$$

Note:

$$\beta^{*}_{k\ell}(\underline{r}', t') = 0 \text{ for } t'>t,$$

since we can have a response at time t only from events before t

We also consicer that $\beta^{*}_{k\ell}(\underline{r}, t) = 0$ for $t = \pm\infty$

In this way and after some mathematical manipulations one gets the following expression for the stress

$$\sigma_{st}(\underline{r},t) = -\int d\underline{r}' \int_{-\infty}^{\infty}\Big\{\mu[G_{sk,t\ell}(\underline{r}-\underline{r}',t-t') + G_{tk,s\ell}(\underline{r}-\underline{r}',t-t')] + + \lambda\delta_{st}G_{qk,q\ell}(\underline{r}-\underline{r}',t-t')\Big\}\Big\{\mu[\beta^{*}_{\ell k}(\underline{r}',t') + \beta^{*}_{k\ell}(\underline{r}',t')] + + \lambda\delta_{k\ell}\beta^{*}_{nn}(\underline{r}',t')\Big\}dt' , \quad (8)$$

We are interested in the stress in a certain frequency range, because we measure the stress with some transducer which has a certain frequency response.

By transforming the stress from the t-plane to the ω-plane we can take in to the calculations the frequency.

We use the following expression for the Fourier-transform:

$$F[\omega] = \int_{-\infty}^{\infty} f(t)e^{i\omega t}\, dt \quad (9)$$

and the inverse

$$f[t] = \frac{1}{2\pi}\int_{-\infty}^{\infty} F[\omega]e^{-i\omega t}d\omega$$

If we take into account that $\underline{r} >> \underline{r}'$ we got an expression for σ_{st} in this form

$$\sigma_{st}(\underline{r},\omega) = R_{stk\ell}(\underline{r},\omega) \cdot S_k(\omega) \qquad (10)$$

where the response of the medium is represented by $\underline{R}$ and the source by $\underline{S}$.

With

$\frac{\omega r}{a} \simeq \frac{\omega r}{c} \gg 1$ and $a \approx c$

c = shear wave velocity
(transverse)
(distorsion)

a = longitudinal wave velocity
(dilatation)

and ignoring the orientation dependence we find

$$R(r,\omega) \simeq \frac{\omega^2}{4\pi r^2 c} \cdot e^{\frac{i\omega r}{c}} \qquad (11)$$

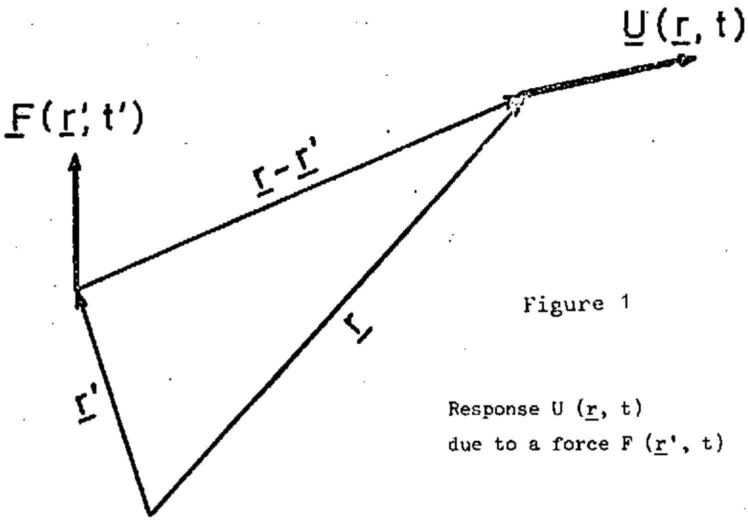

Figure 1

Response U ($\underline{r}$, t)
due to a force F ($\underline{r}$', t)

2 *The Source*

In order to describe the source we assume a change in the inelastic distorsion $\Delta\ \beta^*_{ij}$ at time t = 0, see figure 2. As a first approximation we use a step function. The displacement function has of course a certain rise time, but this is very short. Thus for practical cases we can use the step function. This matter is further discussed in ref [II]. $S_{k\ell}$ can be written

$$S_{k\ell}(\omega) = \left[2\mu\Delta\varepsilon^*_{\ell k}\ V^* + \lambda\delta_{k\ell}\Delta\varepsilon^*_{nn}\ V^*\right] \cdot f(\omega) \qquad (12)$$

We have here assumed that the distorsion is homogenously spread out over the small volume V^*

$$\Delta\varepsilon^*_{\ell k} = \frac{1}{2}(\Delta\beta^*_{\ell k} + \Delta\beta^*_{k\ell}) \qquad (13)$$

For the step function we have

$$f(\omega) = -\frac{1}{i\omega} + \pi\delta(\omega) \qquad (14)$$

Let us simplify the expression for the source in this form

$$S(\omega) = D \cdot f(\omega) \qquad (15)$$

Summing up the discussion of the stress field in a point $\underline{r}$ se get

$$\sigma(\underline{r},\ \omega) \simeq \frac{\omega^2}{4\pi r^2 c} \cdot e^{\frac{i\omega r}{c}} \cdot D \cdot f(\omega) \qquad (16)$$

By retransforming this expression we can get the $\sigma(\underline{r},\ t)$.

3 *Measuring system*

Let us now have a look at a practical case where we measure the above calculated stress. This can be done with a piezo-electrical transducer which transforms the mechanical stress or mechanical pressure to an electrical voltage. Because of the very broadbanded stress pulse the transducer normally is energized at resonance.

We can describe the transfer functions by figure 3.

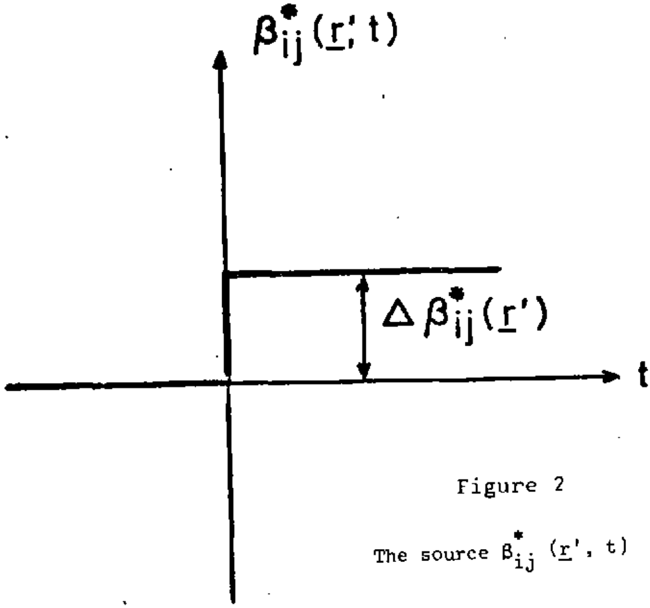

Figure 2

The source $\beta^*_{ij}(\underline{r}', t)$

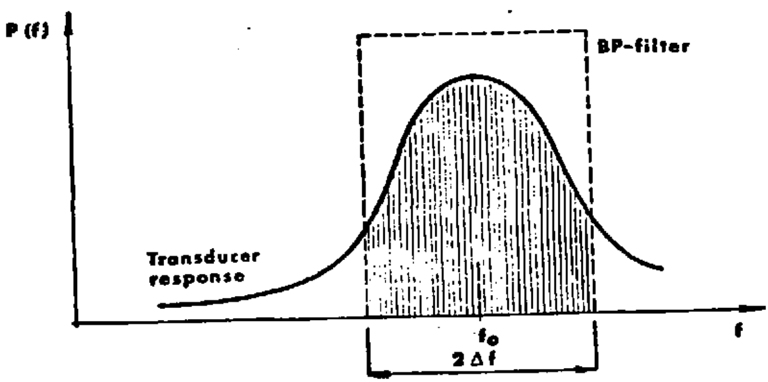

Figure 3 Transfer functions

If we call the transducer output $V(\underline{r},\omega)$, we can write

$$V(\underline{r},\omega) = R(r,\omega) \cdot S(\omega) \cdot P(\omega) \qquad (17)$$

In eq (15) we have $\pi\delta(\omega)$ which vanishes for every ω except $\omega = 0$. Because of the filter we can drop this term in the following.

We also can see that because of the BP-filter we can change the limits in eq (9) from $-\infty, \infty$ to $\omega_o - \Delta\omega, \omega_n + \Delta\omega$.

If we are able to describe the transducer response in mathematical form we can do the retransformation. This results in an expression for $V(\underline{r},t)$, which can be studied on an oscilloscope or xy-recorder. From an amplitude reading we can evaluate D, see eq (12).

4 *Dislocation movement and V(t)*

Let us now study D, to see the meaning of it.

Recalling eq (12) we get

$$D = 2\mu\, \Delta\, \varepsilon^*_{\ell k}\, V^* + \lambda\delta_{k\ell}\Delta\, \varepsilon^*_{nn} V^* \qquad (12)$$

For a plastic strain

$$\Delta\varepsilon^*_{nn} = 0$$

thus

$$D = 2\mu\, \Delta\, \varepsilon^*_{\ell k}\, V^* \qquad (18)$$

This can be related to dislocation by

$$\Delta\varepsilon^* V^* \simeq b\, \Delta x\, N_m V^* \qquad (19)$$

where b = Burgers vector, Δx the distance the dislocations have moved, and N_m the density of moving dislocations.

Thus D is related to the strain. A numerical calculation based upon date from James-Carpenter [IV] gives

$$\Delta\varepsilon^{*}V^{*} \simeq 3 \cdot 10^{-14}\ m^{3}$$

With the model presented in this paper and a Dunegan/Endevco-transducer D-140 the peak amplitude should be 200 µV. A signal with this amplitude is fully detectable.

5 *Conclusions and Summary*

The model is schematically summarized in figure 4.

Here the model and the evaluations are simplified and give only a first step to get a complete knowledge of how the measured voltage is connected with the physical event.

When working with ring down counting or distribution analysis the observed parameters have no direct correspondence with the physical process within the material. To be a "*complete*" NDT-method, the AE-technique should give answer not only to the questions *where* and *when*, but also *what* has taken place.

References

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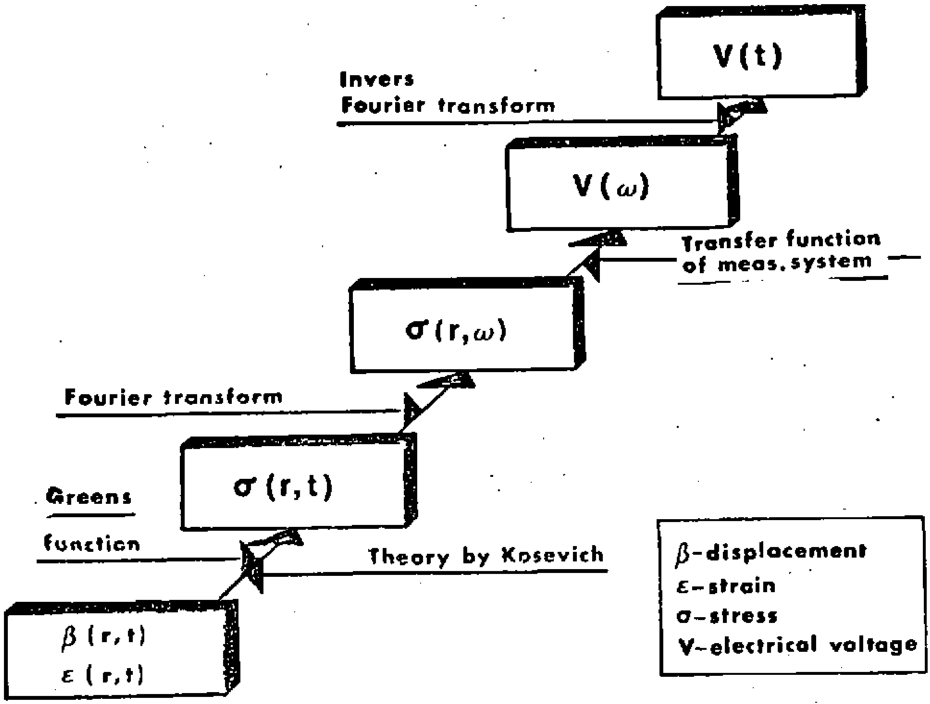

Figure 4 Block diagram of the model