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DETERMINATION OF THE ACOUSTICAL IMPEDANCE OF COMPLEX GEOMETRY SLOTS USED IN AUTOMOTIVE EXHAUST SYSTEM BY MEANS OF A MEASUREMENT PROCEDURE IN A TUBE

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INTRODUCTION

The mathematical models of the acoustical behaviour of exhaust mufflers require to know the acoustical properties of the walls of the ducts which a gas mean flow with a superimposed noise propagates within.
Perforated tubes facing to resonating chambers, in particular, show an acoustical behaviour which depends on the perforation distribution and on the shape of the slots. The evaluation of the slot acoustical impedance is particularly difficult if it shows a complex geometry as in the frequently used shapes of Fig. 2a-2c.
This paper deals with an experimental investigation, based on an advanced technique of acoustical impedance measurement in a plane wave tube with random excitation. Such a technique, introduced by Seyber and Ross [1] and further developed by Chung and Blaser [2], allows the determination of the complex reflection coefficient R on the basis of the transfer function between two wall-mounted microphones. This measurement procedure avoids many drawbacks of traditional techniques, being much faster and apt to be controlled by a computer procedure.

SLOT IMPEDANCE MEASUREMENT

The test bench for the determination of the acoustical impedances developed at the Acoustical Laboratory of IEN "Galileo Ferraris" (Fig. 1), is made of a 50 mm diameter duct connected to a fan-muffler system and a compression driver noise source. The sample under test (Fig. 2) consistes of a cylindrical Helmholtz resonator whose volume may be varied by a movable piston. The cylindrical cavity communicates with the plane-wave duct-wall trough the slot whose characteristics are to be

determined.
The plane wave duct is terminated on free space by a circular exponential horn 950 mm long and 1800 mm (diameter) wide, which faces an anechoich room. The cut-off frequency of the system is 200 Hz and its acoustical impedance may be considered constant from 800 to 4000 Hz. Two B. & K. 1/4' microphones, 26 mm spaced, face the duct wall in proximity of the sample on a rotating support which allows to interchange the microphone positions in order to calibrate the measuring chains [2].
With reference to Fig. 1, the reflection coefficient measured at the section of the duct which pertains to microphone 1, is:

$$R_1(f) = S_{1i1r}(f)/S_{1i1i}(f) \tag{1}$$

where: S_{1i1i} = power spectrum of the incident acoustic pressure; S_{1i1r}= = cross spectrum between incident and reflected acoustic pressures.
$R_1(f)$ may be expressed as a function of three transfer functions, two (H_i, H_r) dependent on the duct geometry and the remaining one (H_{12}), measured between the two microphones:

$$R_1(f) = [H_{12}(f) - H_i(f)]/[H_r(f) - H_{12}(f)] \tag{2}$$

where: $H_i(f) = \exp(-jk_i d)$, transfer function pertaining to forward propagation between microphones; $H_r(f) = \exp(-jk_r d)$, transfer function pertaining to reverse propagation between microphones; d, microphone spacing; $k_i = 2\pi f/(c\ (1 + M))$, wave number for forward propagation; k_r = $= 2\pi f/(c\ (1 - M))$, wave number for reverse propagation; c, velocity of sound; M, Mach number (mean flow velocity/c).
From R_1, the reflection coefficient at the sample section may be evaluated as:

$$R(f) = R_1(f)\ \exp(j2k_i s) \tag{3}$$

where: s, spacing between microphone 1 and test sample.
and consequently the normalised acoustical impedance Z_o of the sample:

$$Z_o = (1+R)/(1-R) \tag{4}$$

The measurement technique is quite sensitive to errors in the evaluation of H_{12} and so it is necessary to eliminate the amplitude and phase non-linearities of the two microphone chains. This is accomplished by executing two estimates of H_{12} interchanging the microphones and using the geometrical mean of the two estimates as H_{12} [2].
The measured acoustical impedance Z_o is the shunt of the termination impedance Z_T and the resonator impedance Z_A. The resonator impedance may be expressed as:

$$Z_A = S_A Z_T Z_o/(S_o\ (Z_T - Z_o)) \tag{5}$$

where: S_A, cross section of the slot; S_o, cross section of the duct.
In terms of the lumped parameter circuit:

$$Z_A = R+1/sC+sL \tag{6}$$

For example, an enclosure around a power transformer must be especially effective at a frequency of 120 Hz (that is, twice the 60-cycle power frequency). Or the absorptive lining for the compressor inlet or exhaust in a jet engine should be most efficient at the blade passage frequency of the rotor, about 2000 Hz.

One of the great advantages of perforated metal is that it can be used as an element in a "tuned sound absorber" to provide remarkably high absorption in the targeted frequency range, though it sacrifices high efficiency in other frequency ranges.

An example was shown in the second sketch on page 1. It works as follows:

All resonant devices have a preferred frequency of operation. For example, when disturbed, a ball suspended on a rubber band oscillates at one frequency only; that frequency is determined by the mass of the ball and the springiness of the rubber band.

In a resonant sound absorber, the oscillation involves the air motion in and out of the holes in the metal sheet, in response to an incident sound wave. The preferred frequency of oscillation is determined by the mass of the air in the perforations and the springiness of the trapped air layer. At that frequency, the air moves violently in and out of the holes, and also back and forth in the sound-absorptive layer; there, the acoustic energy is converted by friction into heat and is thereby removed from the acoustical scene.

It is especially a problem to achieve enough sound absorption at low frequencies, for example in the transformer problem mentioned above.

We have seen that if there is room for a six inch layer of sound absorptive material, it is possible to achieve high values of sound absorption even at low frequencies. Generally, the thicker the layer, the more low-frequency absorption.

The use of perforated metal in a resonant sound absorber permits efficient absorption at low frequencies without requiring so much space.

The first step is to determine the frequency f_R at which maximum sound absorption is desired, relying on sound measurements of the source or on manufacturer's information.

4. Calculation of the Dimensions of the Absorber to Give the Desired Resonance Frequency

The next step in the design of a resonant sound absorber, once the frequency of desired maximum absorption is known, is to choose suitable dimensions for the various elements, in order to make the absorber resonate at that frequency.

The resonance frequency can be determined with the help of the following nomogram, where:

f_R = resonance frequency (Hz);

h = distance between the perforated plate and the solid wall (see sketch, p. 1), in inches;

e = effective "throat length" of the holes; it is given by: e = t + 0.8 d, where t is the sheet thickness and d is the hole diameter, in inches;

P = percent open area of sheet:

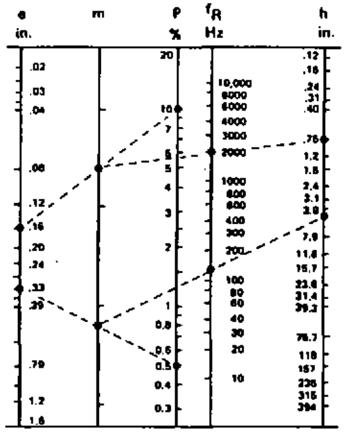
For round holes, staggered:

$$P = 0.9\,(d/b)^2 \times 100\%.$$

For round holes, straight:

$$P = 0.785\,(d/b)^2 \times 100\%.$$

Here, b is the on-center spacing of the holes, with diameter d, both in inches.



EXAMPLE A: TO CALCULATE THE RESONANCE FREQUENCY OF A GIVEN TUNED SOUND ABSORBER.

Suppose a sheet of 16 gauge sheet metal is perforated with 1/8-in. holes, staggered at 3/8 in. on center (about 8 holes/sq in.) and is used as facing for a glass wool blanket 3/4-in. thick, against a solid wall. What is the resonance frequency?

SOLUTION. The thickness of 16 gauge sheet is about 0.06 in. So, for this problem,

t = 0.06 in.
d = 0.125 in.
h = 0.75 in.
b = 0.375 in.

e = 0.06 + 0.8 × 0.125
= 0.16 in.

P = 0.9 (0.125 in./0.375 in.)^2 × 100 = 10%.

Locate the points on the nomogram corresponding to e = 0.16 in. and P = 10% and connect these points with a straight line; mark the point where this line crosses the unnumbered "m"-scale. Now connect that point with the point on the h-scale corresponding to the absorber depth, h = 0.75 in. Read the resonance frequency where this line crosses the f_R-scale: 2000 Hz.

This would be a suitable structure for the jet engine duct lining mentioned above.

EXAMPLE B: TO DESIGN A TUNED ABSORBER TO RESONATE AT A GIVEN FREQUENCY.
Suppose that we want a structure with a resonance frequency of 120 Hz, to be used as an absorptive enclosure for a large power transformer. The available space behind the perforated metal is 4 in. and the sheet is 0.125-in. thick. Determine the required hole size and percent open area.

SOLUTION: What we must do is, given the values for f_R, t, and h, to choose a combination of d and P that satisfies the nomogram.

Connect the points corresponding to h = 4 in. and f_R = 120 Hz with a straight line, continuing it across to intersect the m-scale. Now let us try perforations 1/4 in. in diameter. With t = 0.125 in. and d = 0.250 in., we have:

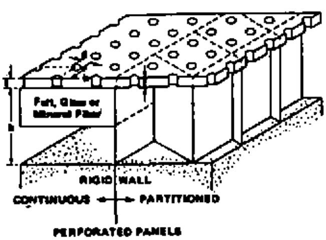
$$e = 0.125 + 0.8 \times 0.250$$

$$e = 0.33 \text{ in.}$$

Connect the point corresponding to e = 0.33 in. to the point found above on the m-scale, to intersect the P-scale at 0.5% (about 0.1 holes/sq in.).

5. Qualitative Guidelines
Once the dimensions are chosen to produce the desired resonance frequency, certain other precautions will help to maximize the absorption achieved. As suggested in the sketch on p. 1, the absorptive material is most effective if it is placed near the perforated plate, rather than near the solid wall. However, if the material is to be wrapped in a thin plastic film for protection, it must not lie directly against the sheet, but about one hole diameter away.

Also, the air layer trapped behind the facing should be subdivided into cells, by means of partitions, rather than being continuous, as shown in this sketch:



When the airspace is continuous, the behavior of the absorber changes greatly at different angles of incidence of sound, becoming less efficient and changing the resonance frequency away from the design value. By contrast, with the partitioned structure, not only does the resonance frequency remain the same as the angle of incidence increases, but the bandwidth for high absorption actually becomes broader, so that the structure is a more efficient sound absorber.*

Finally, there is the effect of the density of the fibrous material in the airspace. If it is too loose, the sound passes right through without being absorbed. If it is too dense, the sound is reflected from the surface, and cannot penetrate the material to be absorbed. Suitable materials would be OCF 705, JM Spintex, or Gustin-Bacon Ultralite, with a density of about 5 lb/ft³ and a thickness of one inch.

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*Note that one way to get the advantages of both the absorptive material and the partitions is to subdivide the half of the airspace near the wall into cells and then fill the half near the sheet with absorptive material.