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IMPROVED LEAST-MEAN-SQUARE ALGORITHM USING A BLOCK-BASED STRATEGY FOR ACTIVE NOISE CONTROL

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This paper presents a novel adaptive algorithm for active noise control that distributes the adaptive filter coefficients between two blocks. The aim of the proposed algorithm is to improve the performance of a single adaptive algorithm that contains all the coefficients in a unique single block.

Distribution is performed depending on the energy of the optimal coefficients that leads to different degrees of sparsity on the coefficient blocks and to the use of different adaptation rules as well. One of the blocks will contain the coefficients that bring a high sparsity degree, that is, the vast majority of the coefficients are zero or with little energy, and a few coefficients exhibit higher energy. The other block contains coefficients with intermediate energy values that provide a sparsity degree close to zero.

Taking into account all the above, we propose an adaptive filter that uses two independent least-mean-square (LMS) based algorithms with the filtered-x scheme embedded to update each block of coefficients. For the first block, characterized by a high degree of sparsity, we will use a filtered-x improved proportionate normalized LMS (Fx-IPNLMS) algorithm that allows to benefit from different coefficient update speeds by setting a configuration parameter κ. Furthermore, a filtered-x normalized LMS (Fx-NLMS) algorithm is proposed for the second block of coefficients, which is characterized by a dispersive nature.

Results show that the proposed approach improves the convergence speed in terms of mean square error with regard to the Fx-NLMS or the Fx-IPNLMS algorithms, while maintaining the steady-state behavior. Moreover, the computational cost of the two-block algorithm is smaller than that of the conventional Fx-IPNLMS, since some of the coefficients are updated with the Fx-NLMS algorithm. Finally, this strategy can be straightforwardly applied not only to active noise control scenarios but to other sound control applications.
Keywords: active noise control, block-based, proportionate algorithms.

1. Introduction

Active noise control (ANC) is a field of growing interest that combines digital signal processing techniques with traditional acoustics. The use of adaptive algorithms for ANC [1, 2] has been subject of continuous study and research since the 1980s. Fig. 1 shows a typical configuration of an ANC system. The aim of this system is to cancel the undesired signal $d(n)$ in the sensor location by processing a reference signal ($x(n)$) correlated with the previous one in order to generate an antinoise that cancels out the primary noise. The error signal $e(n)$ is obtained as the acoustical combination of the undesired signal and the adaptive filter output.

In the context of adaptive filtering, it is well known that the LMS algorithm exhibits a good performance in terms of convergence speed, computational complexity and final residual error [3, 4]. It is important the choice of the step size μ. A high value accelerates the initial convergence, but

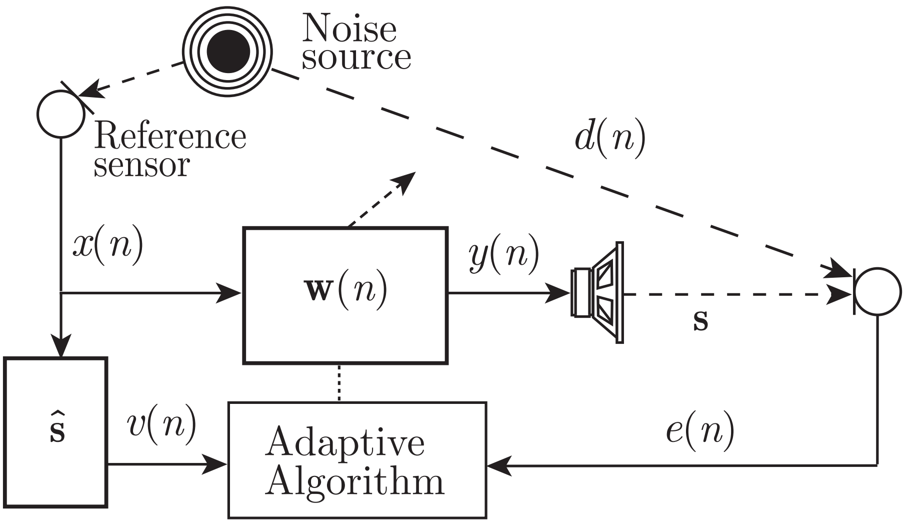


Figure 1: Block diagram of an adaptive ANC system.

it should not be too high to keep the algorithm stability. On the contrary, a small value leads to a slow convergence, but allows to reduce the final residual error. On the other hand, and in order to do the algorithm independent from the input signal statistics, the normalized LMS (NLMS) algorithm (see [5]) uses a time-varying adaptation speed normalized with the estimated instantaneous power of the input signal $x(n)$. The filter coefficients of the NLMS algorithm suitable for ANC and based on the filtered-x scheme (Fx-NLMS) are updated as:

$$w_l(n) = w_l(n-1) - \frac{\mu e(n) v(n-l)}{\delta_{\mathrm{N}} + \sum_{k=0}^{L-1} v^2(n-k)}, \tag{1}$$

for $l = 0, \ldots, L-1$ and where $w_l(n)$ is the lth coefficient of the L-length vector $\mathbf{w}(n)$ and $v(n)$ corresponds to the input signal $x(n)$ filtered through the L_h-length estimated secondary path $\hat{\mathbf{s}}$. δ_{N} is a small regularization value to avoid division by zero.

Regarding LMS-type algorithms (including the Fx-NLMS algorithm), one of their main drawbacks is that they distribute the adaptation energy equally among all filter coefficients due to the fact that the step size μ is common to all the coefficients. Thus, they suffer from slow convergence speed for sparse solutions, that is, for solutions where the vast majority of the coefficients are zero or with little energy, and a few coefficients exhibit higher energy. In order to speed up filter convergence, proportionate adaptation [6] has been proposed for the identification of sparse or quasi-sparse solution as in the case of ANC solutions. The proportionate adaptive filter (PNLMS) algorithm [6] was introduced to accelerate filter convergence in scenarios where the optimal solution presents a high degree of sparsity, as it spends more energy on adapting the active coefficients. The sparsity degree gives an idea of how many coefficients of the adaptive filter have a significant magnitude, whereas the rest of them are zero or small. If the sparseness measure gives a value close to 1, the filter is very sparse; on the contrary, the closer the measure to 0, the denser or more dispersive the filter coefficients are. This sparseness characteristic can be used to take advantage on the knowledge of the coefficient energy distribution of the filter. For instance, the performance of the PNLMS algorithm and its filtered-x version (Fx-PNLMS) degrades significantly when the optimal filter is not so sparse. The improved proportionate Fx-NLMS (Fx-IPNLMS) [7] tries to alleviate this problem improving filter convergence for different degrees of sparsity using a κ variable, where $\kappa \in [-1, 1]$ arranges from the Fx-NLMS algorithm (for a value of $\kappa = -1$) to $\kappa = 1$ for the Fx-PNLMS algorithm. However, one of its drawbacks is that it requires to know the degree of sparsity of the optimal solution, which rarely occurs in practical systems, and an inappropriate choice may result in a bad performance.

The Fx-IPNLMS assigns a different adaptation speed $\mu_l(n)$ to each coefficient according to,

$$w_l(n) = w_l(n-1) - \mu_l(n)e(n)v(n-l), \tag{2}$$

$$\mu_l(n) = \frac{\mu g_l(n-1)}{\delta_{\mathrm{IP}} + \sum_{k=0}^{L-1} g_k(n-1)v^2(n-k)}, \tag{3}$$

$$g_l(n-1) = (1-\kappa)\frac{1}{2L} + (1+\kappa)\frac{|w_l(n-1)|}{\varepsilon + 2\sum_k |w_k(n-1)|}, \tag{4}$$

where $g_l(n)$ is the adaptation gain factor of the lth filter coefficient, δ_{IP} and ε are small values to avoid division by zero.

Recently, new schemes relying on block-based adaptive filters have been proposed with the aim of improving the performance when the energy distribution of the filter solution is not uniform [8, 9], as in the case of ANC systems. Specifically, the block-based adaptive strategy introduced in the context of adaptive equalization [8] uses different settings for the different blocks of an adaptive filter. The design of the block filter location was dependent on the energy distribution of the filter solution. For instance, in a two-block approach, the central coefficients were grouped in the first block and a second block contained the remaining coefficients even if they were not contiguous.

In this paper, we present a block-based strategy focused on the sparseness degree of each block rather than just the energy distribution of the coefficients, in order to improve the ANC system performance in terms of both convergence speed and computational complexity compared to the Fx-IPNLMS performance.

The working principle of the new scheme is to use two independent filtered-x LMS-based adaptive filters to update each block of coefficients. For the first block, characterized by a high degree of sparsity, we will use the Fx-IPNLMS algorithm that allows to benefit from different coefficient update speeds. For the second block, characterized by a dispersive nature, a Fx-NLMS algorithm is considered. The rest of the paper is organized as follows. The proposed scheme (called fitted-block Fx-LMS algorithm) is presented in Section 2. An experimental evaluation of our proposal is provided in Section 3. The paper finishes with the main conclusions of our work.

2. Fitted-block Fx-LMS approach

The novel algorithm presented here tries to improve the filter performance in terms of convergence speed independently of the sparsity degree of the filter. For that purpose, the filter coefficients are split into two block filters. One of them is build with a small number of coefficients with significant magnitude and the rest of them with small energy, thus it has a sparse response. The remainder, with intermediate values, are located at the other filter. Thereby, this second filter has a dispersive behavior. The new algorithm will be obtained as the addition of a Fx-IPNLMS algorithm applied to the sparse filter and a Fx-NLMS algorithm for the dispersive response.

The advantages of using this strategy are twofold: it splits the filter into two blocks that provide a known degree of sparsity, which allows to apply a proper κ value for each block. The second advantage is that it reduces the computational cost with regard to the Fx-IPNLMS algorithm, since one of the filters uses the Fx-NLMS approach to update the coefficients.

Fig. 2 shows the block diagram of this block-based adaptive algorithm, where the adaptive filter has been divided into two blocks. The adaptive block $\mathbf{w}_{\mathrm{N}}(n)$ has a length of L_{N} coefficients, whereas the Fx-IPNLMS algorithm is updated with the L_{IP}-length vector $\mathbf{w}_{\mathrm{IP}}(n)$. The sum of L_{N} and L_{IP} corresponds to the whole filter of length L.

The adaptation of the coefficients of each block can be rewritten as:

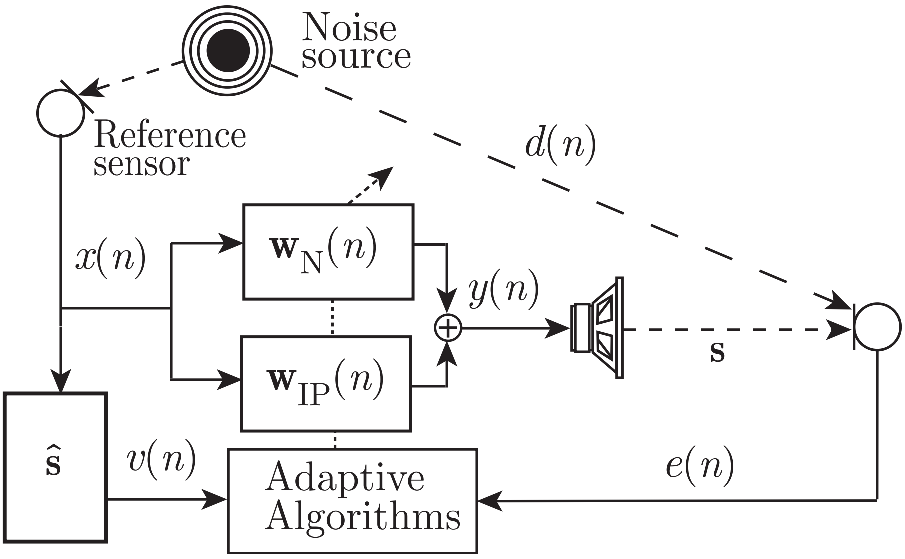


Figure 2: Block diagram of an ANC system with 2 adaptive filter blocks.

$$w_{\mathrm{N},l}(n) = w_{\mathrm{N},l}(n-1) - \frac{\mu e(n) v_\mathrm{N}(n-l)}{\delta_\mathrm{N} + \sum_{k=0}^{L_\mathrm{N}-1} v_\mathrm{N}^2(n-k)}, \tag{5}$$

for $l = 0, \ldots, L_\mathrm{N}-1$ and $w_\mathrm{N}(n)$ refers to the block, whose coefficients are updated with the Fx-NLMS algorithm.

$$w_{\mathrm{IP},l}(n) = w_{\mathrm{IP},l}(n-1) - \mu_l(n) e(n) v_\mathrm{IP}(n-l), \tag{6}$$

$$\mu_l(n) = \frac{\mu g_{\mathrm{IP},l}(n-1)}{\delta_\mathrm{IP} + \sum_{k=0}^{L_\mathrm{IP}-1} g_{\mathrm{IP},k}(n-1) v_\mathrm{IP}^2(n-k)}, \tag{7}$$

$$g_{\mathrm{IP},l}(n-1) = (1-\kappa)\frac{1}{2L_\mathrm{IP}} + (1+\kappa)\frac{|w_{\mathrm{IP},l}(n-1)|}{\varepsilon + 2\sum_k |w_{\mathrm{IP},k}(n-1)|}, \tag{8}$$

for $l = 0, \ldots, L_\mathrm{IP} - 1$ and $w_\mathrm{IP}(n)$ refers to the block whose coefficients are updated with the Fx-IPNLMS algorithm.

3. Simulation results

In this section, we carry out a series of experiments in a single channel ANC setup to illustrate the performance of the proposed approach that has been introduced when a proper distribution of the coefficients is applied. In this regard, the fitted-block algorithm is compared with the standard Fx-NLMS and Fx-IPNLMS solutions.

For these simulation results, the required acoustic channels have been measured at a sampling rate of $\mathrm{f_s} = 44.1$ kHz in a real listening room and decimated to 256 samples that leads to a $\mathrm{f_s} = 2$ kHz to simulate the primary and secondary channels of the ANC system. The *a priori* knowledge of the acoustic channels allows to obtain the optimal filter and therefore to perform a smart distribution of the coefficients between the two block filters.

Fig. 3 represents the optimal filter $\mathbf{w}_0$ of 128 coefficients, that has been obtained with the least-squares estimation method, [10]. For ANC applications, it is obtained as the inverse filter of the secondary path with a minus sign, convolved with the primary path of the ANC system in the time

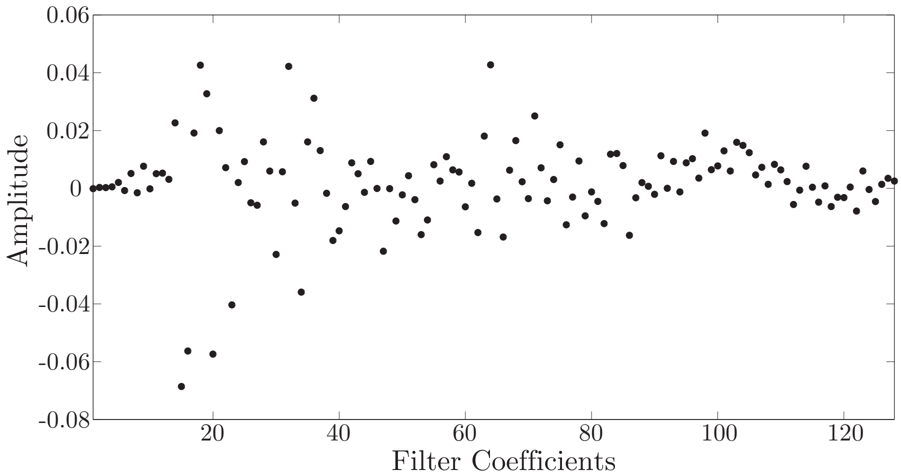


Figure 3: Coefficient values of the adaptive filter.

domain. This optimal filter has been used to distribute the coefficients between the two blocks. In particular, three lengths of the block filters have been considered, which provide different sparsity degrees.

Fig. 4 shows the coefficient distribution between the two blocks. The black square markers correspond to the coefficients assigned to the Fx-IPNLMS whereas the blue circle markers belong to the Fx-NLMS block. The red dashed lines are used to split the coefficients. To obtain the Fx-IPNLMS block coefficients, a 2% of the coefficients with the higher absolute value has been chosen. For the first algorithm in Fig. 4 (a), the Fx-IPLNMS block is completed until one third of the filter length with the 31% of the coefficients with lower absolute values. The remaining intermediate (two thirds or 2/3) values are assigned to the Fx-NLMS block. In contrast, Fig. 4 (b) represents an equal distribution between the two blocks (64 coefficients each). In this case, the central red lines are modified to select the 48% of the coefficients closer to zero for the Fx-IPNLMS algorithm. Finally, Fig. 4 (c) shows a wider Fx-IPNLMS block with 96 coefficients (3/4) and a 32 (1/4) Fx-NLMS filter.

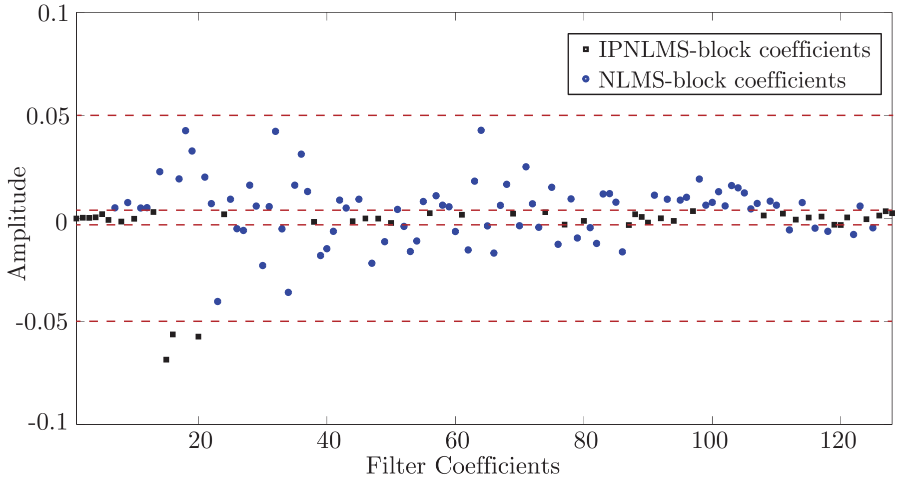
The sparsity degree of the different filters has been computed as in [11],

$$\xi(\mathbf{w}) = \frac{L}{L-\sqrt{L}}\left(1-\frac{\|\mathbf{w}\|_1}{\sqrt{L}\|\mathbf{w}\|_2}\right), \tag{9}$$

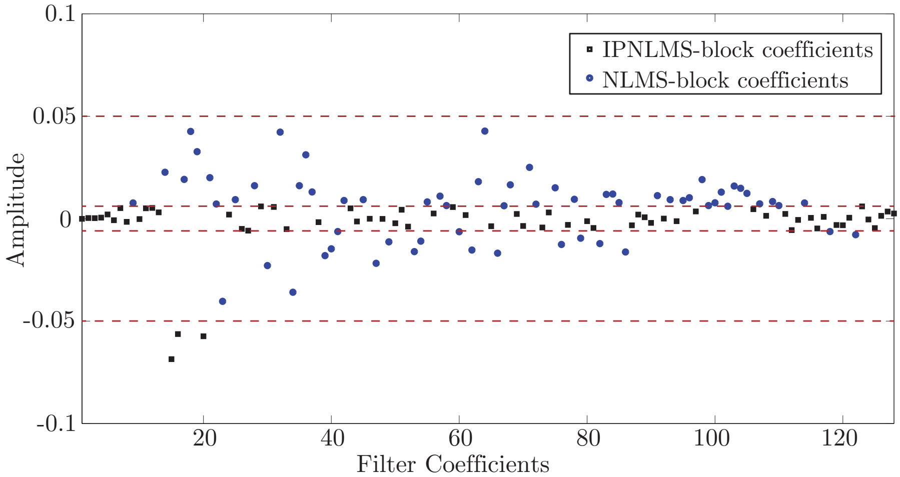
where L refers to the filter length and $\|\mathbf{w}\|_p$ refers to the p-norm of vector $\mathbf{w}$, for $p = 1, 2$.

Table 1 shows the sparsity values and computational cost of the whole filter of Fig. 3 and for each block filter of the different configurations of Fig. 4. The first column refers to the coefficient distribution, whereas the second and third columns gives the sparsity degree of the Fx-IPNLMS and Fx-NLMS blocks, respectively. As can be seen, a proper distribution of the coefficients of the filter between two blocks allows to increase the sparsity of the Fx-IPNLMS block and to reduce the value of the Fx-NLMS block with regard to the whole filter. For the last row, the energy distribution of the coefficients has not been taken into account and only the coefficient location has been consider. Thus, the first coefficients are allocated in the first block and updated by using the Fx-IPNLMS. The remaining coefficients has been updated with the Fx-NLMS. Note that in this case, the sparsity degree of the Fx-IPNLMS block has not increased. The last column contains the computational complexity in number of multiplications per iteration. It arranges from the Fx-NLMS approach to the Fx-IPNLMS algorithm. For the block-based schemes, the longer the Fx-IPNLMS block, the higher its computational cost. The last two approaches have the same computational cost, but different sparsity degrees that affect the algorithm performance, as it will be seen next.

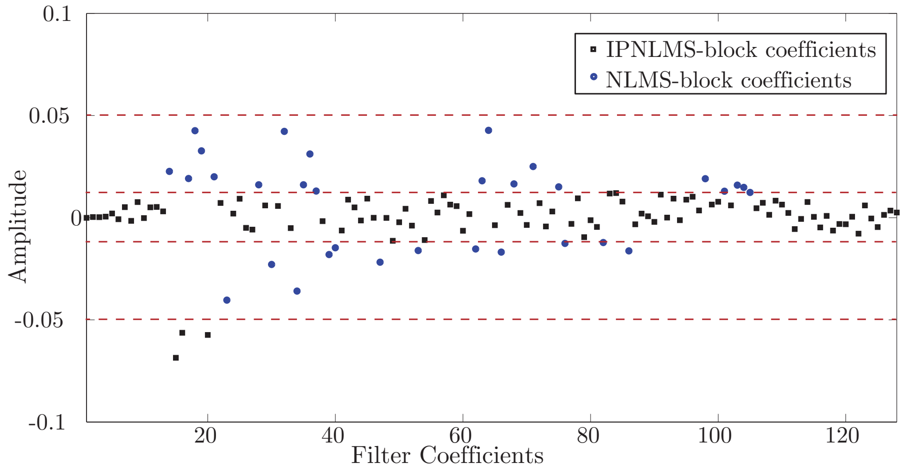
The figure of merit to evaluate the performance of the different algorithms will be the instantaneous Noise Reduction, NR(n), defined as the ratio in dB between the estimated error power with



(a)



(b)



(c)

Figure 4: Coefficient distribution of each block, limited by the red dashed lines, black square markers for the Fx-IPNLMS block and blue circle markers for the Fx-NLMS algorithm. (a) Fx-IPNLMS block: 1/3 (43) coefficients and Fx-NLMS block: 2/3 (85) coefficients. (b) 1/2 (64 coefficients) for each algorithm. (a) Fx-IPNLMS block: 3/4 (96) coefficients and Fx-NLMS block: 1/4 (32) coefficients.

and without the application of the active noise controller, which will be estimated by averaging 1000

Table 1: Sparsity degree and computational cost in number of multiplications per iteration.

Filter	Fx-IPNLMS block	Fx-NLMS block	Computational cost
Whole Fx-NLMS block (128)	-	$0,388$	514
Whole Fx-IPNLMS block (128)	$0,388$	-	1027
Fx-IPNLMS $1/3$ block & Fx-NLMS $2/3$ block	$0,765$	$0,222$	689
Fx-IPNLMS $1/2$ block & Fx-NLMS $1/2$ block	$0,685$	$0,177$	773
Fx-IPNLMS $3/4$ block & Fx-NLMS $1/4$ block	$0,523$	$0,104$	901
Fx-IPNLMS (first) $3/4$ block & Fx-NLMS (last) $1/4$ block	$0,380$	$0,265$	901

independent runs of the algorithms.

$$\text{NR}(n) = 10 \times \log_{10} \frac{e^2(n)}{d^2(n)} \quad (10)$$

Fig. 5 shows the noise reduction for the ANC system for the different algorithms with $\mu = 0.05$ and $\kappa = -0.5$ for the Fx-IPNLMS filters. As expected, the Fx-IPNLMS in dashed red line has convergence faster than that of the Fx-NLMS (solid blue line). On the other hand, the proposed block-based algorithms with a proper coefficient distribution also present a faster convergence speed. They have been computed for the three configurations of Fig. 4, where a longer Fx-IPNLMS block provides a better convergence speed provided they exhibit a proper sparsity degree. This fact can be checked for the unfitted-block coefficient distribution algorithm whose details are shown in the last row of Table 1. Fig. 4 illustrates its behavior in dashed-dotted orange line and diagonal markers. It presents an initial instability that leads to a worst convergence.

4. Conclusions

This paper presents a novel algorithm for ANC. For ANC applications, adaptive filter coefficients usually present both sparse and dispersive sections, which may imply the use of Fx-IPNLMS algorithms as a proper solution. However, its computational cost could be decreased if only the sparse section of the filter is updated with the Fx-IPNLMS algorithm, whereas the dispersive part uses a Fx-NLMS approach, which implies a lower computational cost. The *a priori* knowledge of the coefficient distribution between the two blocks is the key to achieve the optimum algorithm performance. Assuming a proper distribution, it has been proved that the proposed algorithm provides a convergence speed faster than that of the Fx-NLMS and Fx-IPNLMS approaches as well as a computational cost lower than the lFx-IPNLMS. Regarding the block length, simulation results show that longer Fx-NLMS blocks require lower computational complexity but a worst initial convergence speed is obtained. On the other hand, a too shorter Fx-NLMS block implies a higher computational cost and even may result in a bad convergence speed, as it could lead to a unfitted sparsity degree of the blocks. In conclusion, it has been proved, that the proposed algorithm, with a proper setup, reduces the computational cost in contrast to the Fx-IPNLMS algorithm and improves the convergence speed with regard to both the Fx-NLMS and Fx-IPNLMS approaches.

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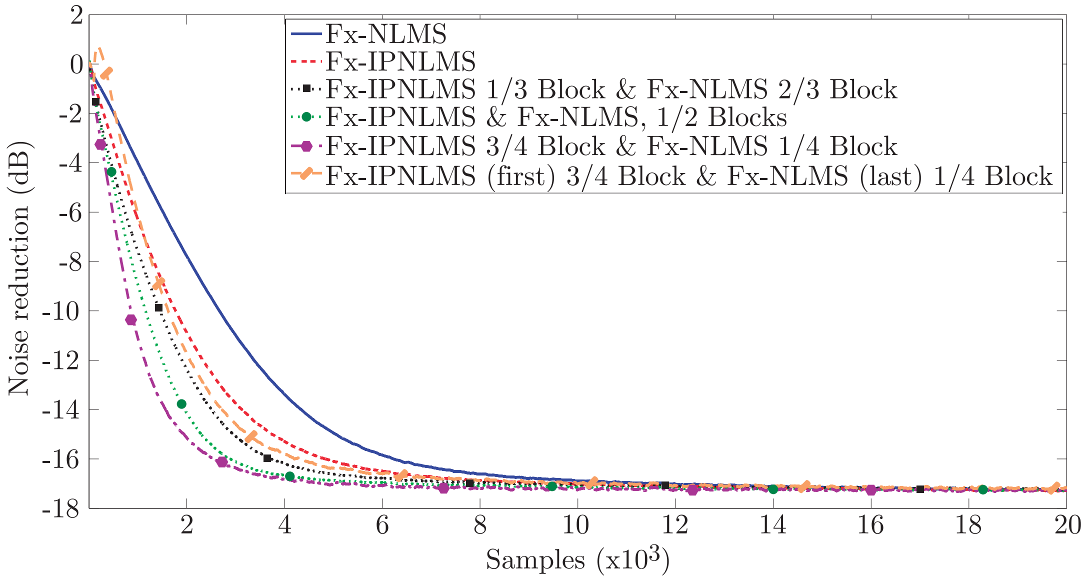

Figure 5: Noise reduction of the algorithms compared in Table 1. Fx-NLMS algorithm in solid blue line. Fx-IPNLMS algorithm in dashed red line. Fx-IPNLMS $1/3$ Block & Fx-NLMS $2/3$ Block, in dotted black line with square markers. Fx-IPNLMS & Fx-NLMS, $1/2$ Blocks, in dotted green line with circle markers. Fx-IPNLMS $3/4$ Block & Fx-NLMS $1/4$ Block, in magenta dashed-dotted line and hexagon markers. And its unfitted-blocked version, Fx-IPNLMS 3/4 Block & Fx-NLMS 1/4 Block, in orange dashed-dotted line and diagonal markers.

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