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OVERVIEW OF TECHNIQUES FOR MEASURING IMPULSE RESPONSE IN ROOM ACOUSTICS

L.OTSHUDI (1),J.P GUILHOT (1), J.L CHARLES (2)

(1) U.P.S-LAMI 38, rue des 36 Ponts 31062 TOULOUSE (FRANCE)
(2) THOMSON -CSF- BP 53 06802 CAGNES-SUR-MER (FRANCE)

ABSTRACT
We examine the principal methods for measuring impulse response in room acoustics. The first part of the paper is a synthesis of conventional techniques ; the second discusses modern methods. A theoretical and a practical approach is presented for each method, together with the main advantages and disadvantages of the technique. It is shown in the second part that all new techniques in our signal processing are based on convolution and correlation theory.

INTRODUCTION

The major difficulty encountered in the field of room acoustics measurement is due to the fact that there is no mathematical description (model) characterizing the behaviour of the systems. The approach generally used by acousticians is one of direct experiment on the system to determine its acoustic properties. This experimentation usually leads to measurement of the impulse response,which is the time operator proper to the system. This function is determined by studying the response of the system to particular excitation signals. In this study we shall see that the measurement techniques can be distinguished principally by the nature of the test signal. This article also discusses the main advantages and disadvantages connected with the use of each technique. The mathematical approaches given here are based on the operational hypothesis of linear acoustics for the case of a one-dimensional stationary, ergodic variable.

I CONVENTIONAL TECHNIQUES

I.1 Mathematical form

All these conventional measurement techniques are based on spectral analysis theory. The impulse response H (t) of an acoustic channel (room) in Fig.1.1 is given by the integral expression :

$$H(t) = \int_{D(\nu)} h(\nu)\, e^{2j\pi\nu t} d\nu \qquad (1)$$

or, symbolically, $H(t) \leftrightarrows h(\nu)$

(H,h) make up a Fourier pair. The integral formula (1)relates the two representations, time and frequency, of the functional characteristics of a room by means of a Fourier transforms, which is considered as the fundamental mathematical tool of frequency metrology. Certain two-channel analysers, such as the HP 5420, enable the impulse response to be measured directly from a measurement of interaction spectral density (ISD), commonly known as the "interspectrum". If $x(\nu)$ and $y(\nu)$ are the frequency representations of the input,$X(t)$, and output, $Y(t)$, signals respectively of the system to be measured (room),we obtain the following spectral relationship :

$$y(\nu) = h(\nu)\, x(\nu) \qquad (2)$$

or $x^*(\nu)\, y(\nu) = h(\nu)\, x^*(\nu)\, x(\nu)$
$x^*(\nu)$ being the complex conjugate of $x(\nu)$.

Let $S_{12}(\nu) = h(\nu)\, S_{11}(\nu)$

Since in practice we take

$$g(\nu) = \begin{cases} 2s(\nu) & \text{for } \nu > 0 \\ s(\nu) & \text{for } \nu = 0 \\ 0 & \text{for } \nu < 0 \end{cases}$$

we can also write $g_{12}(\nu) = h(\nu)\, g_{11}(\nu)$,

whence we find $h(\nu) = \dfrac{g_{12}(\nu)}{g_{11}(\nu)}$ (3)

To take into account the additional noise N(t) on the output signal Y(t), equation (3) must be corrected as follows :

$$g'_{12}(\nu) = x(\nu)\left[y(\nu) + n(\nu)\right]$$

where $n(\nu)$ is the frequency representation of the noise N(t).

Finally, we obtain :

$$h'(\nu) = \frac{g'_{12}(\nu)}{g_{11}(\nu)} = h(\nu) + \frac{g_{13}(\nu)}{g_{11}(\nu)} \qquad (4)$$

The noise contribution is represented here by the term g_{13}/g_{11}, this term generally being made very small by increasing the number of realizations used in the averaging operation.

1.2 Practical aspects of the methodology

1.2.1 Analysis with an impulse source (impact,pistol shot)

This is the most direct approach for the practical measurement of the impulse response of a room. It consists of exciting the room with an impulse source of this type and directly observing the response. To this simplicity of implementation of measurement must however be added two disadvantages :

1- irregular energy distribution at all usefull frequencies,
2- mediocre dynamics and reproducibility of the results.

1.2.2 Analysis with a source of white noise

In practice the interpretation of equation (4) is based on the Fourier pair (H,h) which permits us to write :

$$H'(t) = \int_{D(\nu)} h'(\nu)\, e^{2j\pi\nu t}\, d\nu = \int_{D(\nu)} \left[h(\nu) + \frac{g_{13}(\nu)}{g_{11}(\nu)}\right] e^{2j\pi\nu t}\, d\nu$$

$$= \int_{D(\nu)} \frac{g_{12}(\nu)}{g_{11}(\nu)}\, e^{2j\pi\nu t}\, d\nu + \int_{D(\nu)} \frac{g_{13}(\nu)}{g_{11}(\nu)}\, e^{2j\pi\nu t}\, d\nu \qquad (5)$$

As the input signal X(t) is a microscopic correlation process and thus a constant power spectral density (PSD) process, i.e. here $g_{11}(\nu) = \frac{1}{2}N_0$, equation (5) becomes :

$$H'(t) = \frac{2}{N_0}\int_{D(\nu)} g_{12}(\nu)\, e^{2j\pi\nu t}\, d\nu + \frac{2}{N_0}\int_{D(\nu)} g_{13}(\nu)\, e^{2j\pi t\nu}\, d\nu \qquad (6)$$

And, if we consider the crosscorrelations and interspectra, (G_{12}, g_{12}) and (G_{13}, g_{13}) constitute two Fourier pairs.

Whence finally :

$$H'(t) = \frac{2}{N_0}\left[G_{12}(t) + G_{13}(t)\right] \qquad (7)$$

Equations (6) and (7) show two ways of performing the measurement of an impulse response.

Firstly : by measuring the interspectrum, i.e. the energy interspectral density (EID), to which a Fourier transform is applied. This procedure is the oldest and the most widespread in signal processing spectral analysers.

Secondly : by measuring the crosscorrelation function. There are two main advantages to the use of microscopic correlation sources ; suitable dynamics and the possibility of ensuring a uniform energy distribution with time (use of an omnidirectionally emitting loud speaker). On the other hand, experience shows that the main drawback in this approach is that a fairly large number of averages (50 upwards) are needed before a measurement result can be validated (random excitation).

II-MODERN TECHNIQUES

II-1 Mathematical conceptualization

The various modern metrological approaches to impulse response measurement are at present essentially based on two signal processing operators, convolution and correlation. If we start by assuming that the noise, N(t), does not act on the room (Fig.2.1), the response of the room is given by VASCHY's formula, as follows :

$$Y'(t) = (H*X)(t) \qquad (8)$$

where the symbol* represents the convolution product.

If we accept the influence of noise N(t) in the room, its contribution to the output can also be deduced :

$$Y''(t) = (H*N)(t) \qquad (9)$$

Taking account of the linearity hypothesis put forward in the introduction, we obtain the overall response of the room by additivity :

$$Y(t) = Y'(t) + Y''(t)$$

$$= (H*X)(t) + (H*N)(t)$$

Whence the convolution integral : $$Y(t) = \int_0^{\infty}\left[X(t-\theta) + N(t-\theta)\right]H(\theta)\,d\theta \qquad (10)$$

Furthermore, considering the second order statistics of the signals X(t) and Y(t) in correlation, we can also write :

$$S_{21}(r) = \lim_{T \to \infty} \frac{1}{T} \int_0^T Y(t) X(t-r) dt \qquad (11)$$

$$S_{11}(r) = \lim_{T \to \infty} \frac{1}{T} \int_0^T X(t) X(t-r) dt \qquad (12)$$

Introducing into (11) the expression for Y(t) given by the convolution integral yields the following development :

$$S_{21}(r) = \int_0^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_0^T \left[X(t-\theta) + N(t-\theta) \right] X(t-r) H(\theta) dt d\theta$$

Taking the hypothesis of statistical decorrelation between the signals X(t) and N(t), we have

$$S_{13}(r-\theta) = \lim_{T \to \infty} \frac{1}{T} \int_0^T N(t-\theta) X(t-r) dt = 0$$

Whence, finally

$$S_{21}(r) = \int_0^{\infty} S_{11}(r-\theta) H(\theta) d\theta \qquad (13)$$

We observe in fact that the concept of a "convolution and correlation" time doublet is made explicit here by the relationship between the autocorrelation function of the input signal and the crosscorrelation function of the input and output signals of the system analysed.

$S_{21} = S_{11} * H$

Fig. 2.2 shows the impulse response (cross-correlogram)

It should be pointed out that equation (13) is a general integral expression for the measurement of an impulse response. The distinction between modern methods depends mainly on two aspects, the nature of the source of excitation used and the operational approach for the calculation of the impulse response from this expression.

II.2 Typology of test signals and operational approaches

The brief description of the test signals given here is the one which makes it possible to solve the convolution integral (13) in order to extract the impulse response. Microscopic correlation sources are also in common use in modern acoustic metrology. This is justified above all by the impulse nature of their autocorrelation function.

$$S_{11}(r) = \frac{1}{2}N_0\,\delta(r)$$

In this case, equation (13) becomes

$$S_{21}(r) = \frac{1}{2}N_0\int_0^{\infty}\delta(r-\theta)H(\theta)d\theta$$

Let $S_{21}(r) = \frac{1}{2}N_0 H(r)$ (14)

Thus we see that, to within 1/2. No, the crosscorrelation function is equal to the impulse response sought. Pseudo-random binary sequences are also used as modern sources of excitation. A PRBS, also commonly known as a "maximum length sequence" or "M-sequence" is a source made from an nth degree primitive polynomial $P_n(X)$ configuration (with $N = 2^n - 1$, the sequence length) which makes it possible to specify the required loops of a binary shift register [12].
The autocorrelation function of such a sequence having a maximum time duration.

$T = (2^n - 1).\ \Delta T$ is given by :

$$S_{PRBS}(r) = S_{11}(r) = \begin{cases} 1 \text{ for } r = 0 \\ -1/T \text{ for } 1 \leq r \leq 2^n - 1 \end{cases}$$

With $\Delta T = 1/F$ where F is the register-operation clock-frequency.
The form of this autocorrelation function is similar to that of the physical approach of the Dirac distribution, which allows it to be expressed in distribution form as follows :

$$S_{11}(r) = \left(1 + \frac{1}{T}\right)\delta(r) - \frac{1}{T} \simeq \delta(r) - \frac{1}{T}$$

In the case where the excitation is a PRBS, equation (13) becomes :

$$S_{21}(r) = \int_0^{\infty}\delta(r-\theta)H(\theta)d\theta - \frac{1}{T}\int_0^{\infty}H(\theta)d\theta$$

or finally

$$S_{21}(r) = H(r) - H_{moy}$$

where H_{moy} is the continuous component of the impulse response, generally made small by good ajustment of the measurement system.

II.3 Numerical processing

Given that, in both of the cases mentioned above (microscopic correlation and PRBS excitation), the final solutions of the convolution integral (13) given by (14) and (15) demonstrate the proportional character of the crosscorrelation function and the impulse response, the numerical processing performed consisted globally of evaluating the discrete time estimator of correlation $\hat{S}_{21}$ given by

$$\hat{S}_{21}(i) = \frac{1}{N}\sum_{k=0}^{N-1} Y(k)\,X(k-i)$$

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For an analysis with microscopic correlation excitation, the practical procedure is the acquisition then storage in files of the input and output signals sampled, X(i) and Y(i). The operations involved in this processing, namely the piloting of the analog-to-digital converter, acquisition, storage, calculation proper of the estimator, and the averaging were performed using an HP 9000 computer. The main drawback of this experimental metrological approach is the relatively long time taken for the estimator caculation. With a PRBS, the calculation of the impulse response by correlation is described in terms of matrix multiplication by the relationship :

$$\hat{S}_{21} = \frac{1}{N} X_N Y_N$$

where X_N is a signal represented by a matrix (dimension = 2^n-1) made up of elements obtained by (right) circular shifting of the sequence, made symmetrical by assigning the values $\pm$ 1 to the two binary states (1 set to -1, and 0 to 1). The case where a room is excited by a process of this type has particular practical importance as it makes it possible to minimize the operations required for correlation estimator evaluation by using Hadamard's fast transform algorithm. It is worth recalling here that Hadamard's matrix is a square matrix of dimension 2^n , of which the elements are +1 or -1, and the rows (or columns) are mutually orthogonal. To carry out the processing relative to this part, we referred in particular to the works given in references [5] and [10].
Four program modules were thus created :

- Module for acquisition of the response, Y(i), of the room ;
- Sequence generation module
- Module for calculating permutation transforms
- Modules for calculating Hadamard's fast transform itself

For the module calculating the permutation transforms (P1 and P2), our approach is no longer based on a matrix expression of the sequence as put forward in [5] and [10] but on a "series-sequential" one, which allows us to store simply a vector of length $N = 2^n-1$ instead of a table of dimensions (N xN). This possibility of reducing the matrix to one dimension is due to the deterministic character of the pseudo-random binary sequence.

CONCLUSION

This overview has shown the range of operational possibilities available to acousticians for measuring an impulse response. All methods of measurement in this field stem from frequency metrology or time metrology. However, it can be observed at present that the theoretical concepts underlying these techniques are essentially based on time expressions, which partly explains the development of time metrology in acoustics. In present-day room acoustics, the impulse response, function proper to the channel, can be considered as the starting point for any study leading to the characterization of such systems. It is for this reason that acousticians have fixed their attention on this operator and that it is usua-ly recommended to take special care when determining it. But the problem is that most of the theoretical models which form the basis of these measurement techniques are either too idealized or too simplified, and this naturally poses the problem of the "sensitivity of representativity" of this operator. In this respect the methodological approach suggested by Berkhout, de Vries and Boone 7 appears to be of great interest as it enables a "filtered" impulse response to be obtained.

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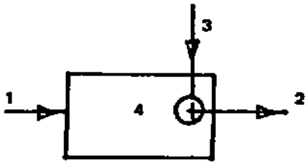

Fig 1.1 Representation of an acoustic channel (room):
1- excitation signal X(t),
2- output signal Y (t)
3- added noise N (t)
4- acoustic channel H(t)

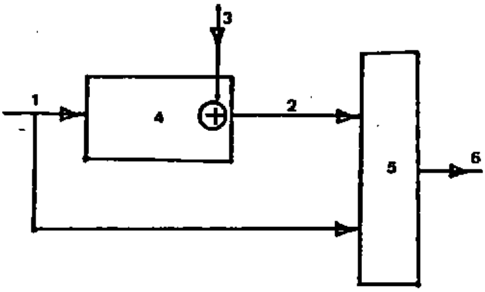

Fig 2.1 Block diagram of impulse response measurement by correlation:
1- excitation signal
2- room response
3- added noise
4- channel (room)
5- correlator
6- impulse response

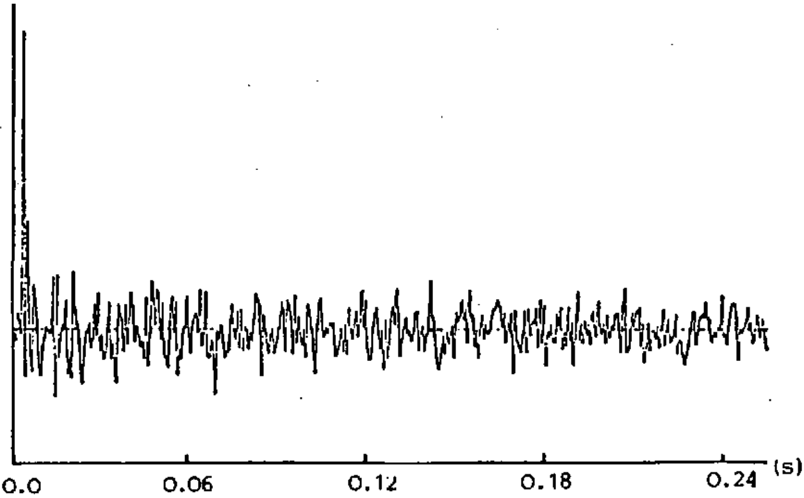

Fig 2.2 Impulse response (cross-correlogram)