Experimental study on normal mode coupling in range-dependent shallow water Environments

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1. Introduction

This paper presents the results of an experimental study of the adiabatic [1]-[5] approximation of the normal mode method. The purpose of the study is to provide a simple criterion that may be used to estimate the limits of validity of the adiabatic approximation in range-dependent environments.

In order to study mode coupling, experimental work has been carried out to measure the mode propagation in shallow water channels with variations of water depth. Mode coupling is introduced by deforming the water surface and the mode coupling process is shown by measuring the sound field as a function of range and depth. Separating the individual modes in the channel by means of mode orthogonality is particularly convincing.

The sound fields in the channels are predicted numerically by using IFD (implicit finite difference) method [12]-[14]. It is found the agreements between the experimental and the numerical results are very good especially when the mode angles become smaller due to the change of water depth.

2. Theory of adiabatic approximation

The wave equation in a water channel with cylindrical symmetry is written as

$$\nabla^2\psi(r,z)+k^2(r,z)\psi(r,z)=0 \tag{1}$$

Following the theories of Pierce [1] and Milder [3], the sound field ψ can be expressed as

$$\psi(r,z)=\sum_n \phi_n(r)u_n(r,z) \tag{2}$$

The function $u_n(r,z)$ satisfies the following differential equation at range r:

$$\frac{\partial^2 u_n(r,z)}{\partial z^2}+[k^2(r,z)-k_n^2(r)]u_n(r,z)=0 \tag{3}$$

where $k_n(r)$ is the eigenvalue of the solution of Eq. (3) at range r, and $u_n(r,z)$ is orthonormal, *i.e.*

$$\int \rho u_n(r,z)u_m(r,z)dz=\delta_{nm} \tag{4}$$

where ρ is the density of the medium.

Substituting Eq. (2) into (1), and using the orthonormality of function $u_m(r,z)$, one may obtain the coupled normal mode equation as

$$\frac{d^2\phi_m}{dr^2}+\frac{1}{r}\frac{d\phi_m}{dr}+k_m^2(r)\phi_m=-\sum_n\left[A_{mn}\phi_n+B_{mn}\left(\frac{\phi_n}{r}+2\frac{d\phi_n}{dr}\right)\right] \tag{5}$$

where

$$A_{mn}(r)=\int\rho u_m\frac{\partial^2 u_n}{\partial r^2}dz \tag{6}$$

and

$$B_{mn}(r)=\int\rho u_m\frac{\partial u_n}{\partial r}dz \tag{7}$$

Eqs. (6) and (7) are the coupling coefficients. It can be seen that the mode coupling is proportional to the first and the second derivatives of the mode amplitude with respect to range.

When mode couplings are very small, the right hand side of Eq. (5) can be omitted, and it is simplified as

$$\frac{d^2\phi_m}{dr^2}+\frac{1}{r}\frac{d\phi_m}{dr}+k_m^2(r)\phi_m=0 \tag{8}$$

The adiabatic theory suggests that normal modes can propagate adiabatically in range-dependent environments, provided that the environmental changes are very small within a certain scale. The adiabatically propagating modes adapt themselves to suit the local environment without coupling between the modes.

Concentrating on the discrete modes, The conditions for validity of the adiabatic approximation given by Brekhovskikh [5] and by Rutherford and Hawker [4] are effectively the same but obtained in different ways. Follow the analysis of Brekhovskikh [5], the conditions are written as

$$D/M \ll 1 \tag{9}$$

where D is the cycle range of a corresponding ray, which is determined by $k_m-k_n=2\pi/D$, and M is a scale of the horizontal variation of the medium. Since the coupling is the strongest between the contiguous modes, in a water channel with a sloping bottom, we have

$$D=2\pi/(k_m-k_{m+1})\approx 2\pi k_o/(k_m^2-k_{m+1}^2)\approx\frac{h^2k_o}{m\pi}=\frac{h}{\sin\theta} \tag{10}$$

where $m\pi=k_oh\sin\theta$, θ is the ray angle, and

$$M=a/\tan\phi \tag{11}$$

where a is a constant, and ϕ is the angle of local slope with respect to the horizontal plane. So that Eq. (9) becomes

$$\tan\phi\ll a\frac{\sin\theta}{h} \tag{12}$$

Since most slopes in the oceans have small angles, and the angles of the rays which have the major contributions to the sound field are also small, Eq. (12) can be simplified as

$$\phi \ll a\frac{\theta}{h} \tag{13}$$

Eq. (13) demonstrates that mode coupling is dependent on the water depth, the shallower the water, the better the adiabatic approximation holds. However, the water depth has to be greater than the wave length in order to sustain normal modes.Equation (13) also implies that adiabatic approximation improves with decreasing frequency as for any given mode the corresponding grazing angle becomes greater as frequency decreases. The criterion can be relaxed from "No mode coupling is allowed" to "Mode coupling is only allowed through angles smaller than θ." So long as we are not interested in detailed interference on the finest scale, the requirement is that the grazing angle be large compared with the bottom slope [6].

To summarise, we have now turned the condition of the validity of the adiabatic approximation into a simple geometrical requirement in terms of angles. Coupling (between contiguous modes) will still increase with h/λ, but the effects are only noticeable at low frequency when only a few modes are present. In the experimental measurements to follow later, the water depth is such that only three or so modes can co-exist and we therefore expect mode coupling to increase with water depth.

3. EXPERIMENTAL FACILITIES AND EXPERIMENTAL WORK

The experimental work has been carried out in a laboratory tank. The water tank for the experiments is made of 6mm thick white polypropylene. Its inner dimensions are 1.98m long, 1.73m wide, and 0.25m deep. A sand layer of 0.11m thick is used as a penetrable bottom. The average diameter of the sand particles is about 50μm [7]. The shear wave in the bottom is assumed to be negligible, therefore the sand bottom can be considered as a fluid bottom. The sound velocities in the water and in the sand are 1477m/s and 1665m/s. The densities of the water and the sand are 1.0g/cm^3 and 1.9g/cm^3. The attenuation coefficient of the sand bottom is 0.68dB/m/kHz. A sophisticated experimental system which is capable of measuring sound pressure in three dimensions has been used to investigate sound propagation in shallow waters with range-independent and range-dependent environments. The configuration of the experiment is shown in Fig. 1. The aperture of the source is 20mm in horizontal plane and 3mm in vertical plane. The size of the hydrophone is 0.7mm in height and 0.6mm in diameter. The signal frequency is 500kHz. At such a high frequency, there is almost no reflection from the bottom of the sand layer, so the model can be seen as a water column overlay on a semi-infinite fluid bottom. Pulsed signals were used to avoid the interference from the signals reflected by the walls of the tank. Two kinds of measurements were carried out; one is to measure the sound pressure as a function of range, *i.e.* the propagation loss, and the other one is to measure the sound signals at different water depths. The water depth is 11mm at the source position. The water depth at the source can sustain three normal modes. However, it will be very easy to demonstrate mode coupling if one mode appears from being absent after passing the deformed water surface; the source was put so that the second mode was not excited. The depth of the acoustic centre of the source should be 6.35mm according to normal mode calculation. It was quite difficult to put the source at the exact depth in order not to excite the second mode at such a high frequency. The actual depth of the source in the experiment was determined by reducing the second mode to the minimum. In order to make absolute comparisons between the experimental and the theoretical results, the measurements were calibrated [8].

Different bottom shapes were formed in order to model range-independent and gradual range-dependent environments. The sound pressure as a function of the range has been measured in three shallow water channels. The channels are shown in Fig. 2, one with a flat bottom, one with a shallow basin, and one with a deep basin. The water depth at the source is h. The basins are constructed by five segments. Each segment is 200mm long. The depths of the basins are 5mm for the shallow one and 10mm for the deep one. The angles of the segments are 0.95° for the steep slopes and 0.473° for the gentle slopes in the shallow basin-shaped channel, 1.92° for the steeper slopes and 0.95° for the gentle slope in the deep basin-shaped channel. The slopes in these channels are very small, the change of the water depth is gradual. So that there is no mode coupling caused by the bottom slopes in these channels when the source is deployed at the most shallow part of water column. Results for the flat bottom channel only are reported here.

To generate abrupt changes of water depth in the channel, a plastic tube coated with neoprene rubber was put on the water surface as shown in Fig. 3. The part of the tube immersed in the water is referred to as a scatterer here. The radius of the tube is 60mm. The neoprene rubber is full of air bubbles, so it can be considered as perfect reflector. The depth of the tube immersed in the water is adjustable. Supported by two bench jacks, the tube is put perpendicular to the acoustic axis of the source. The centre of the tube is 0.3m away from the source. The tube is long enough to separate the diffractions at the ends of the tube from the interesting signals in time.

The effect of the tube is to cause scatterering of sound energy in the channel. It is obvious that the scatterering is dependent on the angle ϕ between the water surface and the tangent of the tube at the crossing point with the water surface. From ray point of view, it is obvious that the larger the angle ϕ, the stronger the scattering. The modal equivalent of this is strong mode coupling. As it can be seen that angle ϕ is a function of the scatterer's depth d in this configuration, increasing d will result in increased ϕ. In the experiment, d is increased step by step from 0mm to 7mm, and the sound field was measured for each d. The relation between the immersed depth d and the angle ϕ can be seen from Fig. 3 that

$$\cos\phi = 1 - \frac{d}{\rho} \tag{14}$$

where ρ is the radius of the tube. Table 1 shows the angles corresponding the depth d,

Table 1 Angle ϕ vs depth of the scatterer

Depth d	0mm	1mm	2mm	3mm	4mm	5mm	6mm	7mm
ϕ in degrees	0°	10.48°	14.84°	18.19°	21.04°	23.56°	25.84°	27.95°
ϕ in radians	0	0.183	0.259	0.317	0.367	0.411	0.45	0.488

NORMAL MODE COUPLING IN RANGE DEPENDENT ENVIRONMENTS

In order to observe the mode coupling process directly, normal modes in the flat bottom channel were separated from the measured data. The technique of mode separation is based on mode orthogonolity, it has been used by Ingenito [9] and Tindle *et al* [10] to study the properties of normal modes. To separate normal modes in a shallow water channel with a flat bottom, sound pressure as a function of depth has been measured. Short pulse signals of 20μs were used. The sample frequency was 20MHz. The hydrophone was put at the range of 0.55m from the source, and it was moved from the water surface to the bottom interface to measure the sound pressure at different depths with an equal depth increment of 1mm. Signal transmitting and receiving are triggered by the pulse generator in order to obtain a precise arrival time for all the received signals at the different depths. The maximum errors are only one half of a sample period after the adjustments.

4. Experimental results

The results of mode separation are shown in Figs. 4 and 5 (the mode amplitudes are on an arbitrary scale). Since the channel can sustain three modes, the amplitudes of the modes in the channel without the tube on the water surface are shown in Fig.4. It can be seen that all the three modes are present in the channel. The first mode is dominant in the channel, and the third mode is about 6dB smaller than the first mode, however the second mode was not totally eliminated because it is very difficult to put the source exactly at the null of the mode at such a frequency.

When the tube is put on the water surface with depth less than 1.0mm, there is almost no change on the amplitudes of the three modes, so that mode coupling is negligible. The second mode overtakes the third mode when the depth of the tube is 1.5mm. This evidence indicates that the mode coupling is taking place between angles $\phi = 10.48°$ and $\phi = 12.83°$. Most energy of the third mode is coupled into the lower modes (mainly to the second mode) when the depth of the tube exceeds 2.5mm. The first mode is still the strongest, but the second mode becomes important and the third mode is small compared with the first and second modes. Fig. 6 shows the results of measured sound pressure and IFD predicted propagation loss as a function of range at a constant depth in the flat bottom channel. Without the scatterer, the measured results shows a sinusoidal pattern as expected. However, small fluctuations can be seen in the result. The reason for this is mainly due to the inaccurate source depth. Comparison of the measured result and the IFD prediction clearly shows the phase error associated with the latter.

When the scatterer is positioned at 3mm depth (1.02λ), the depth of the water at the shallowest point is only 8mm (2.71λ). This water depth can not sustain the third mode any more. The sound field is dominated by the first and the second modes shown in Fig. 7. However, the measured result indicates that the third mode is still present. When the scatterer was immersed at the depths from 4mm (1.35λ) to 7mm (2.37λ), the sound fields are dominated by the first and the second modes. Although the third mode should be cut off as the water at scatterer is too shallow to sustain it, there is enough evidence indicate the existence of the third mode. It is the same for the second mode when the scatterer is immersed at the depth of 7mm (2.37λ), the water is so shallow even the second mode should be cutoff at the scatterer. However, the experimental results indicate that the second mode is still very strong as shown in Fig. 8. In our case, the third mode couples its most energy into the second mode and a small amount into the higher modes----the continuous modes. For the deepest immersed scatterer, all the energy of the third mode coupled into the first mode and the continuous modes at the shallowest water depth, then the energy in the continuous modes coupled back to the second and the third modes when the water depth increased rapidly, and most the energy was kept by the first and the second modes.

5. Discussions on the experimental results

It is found that the mode coupling gradually occurs when the immersed depth of the scatterer is increased from about 1mm to 2mm. The corresponding angles are $\phi = 10.48^\circ$, and $\phi = 14.84^\circ$. The grazing angles for the first mode at the place where the scatterer is are 6.596° for the flat bottom. Experimental work not reported here give figures of 5.829° for the shallow basin, and 5.226° for the deep basin. As given by Eq. (13), the condition for mode coupling can be written as

$$\phi \sim A\theta$$

where ϕ is the bottom slope, θ is the ray grazing angle. Choosing 14.84° as the upper limit of slope angle for mode coupling, it is found that

$1.6 \le A \le 2.2$	for the flat bottom
$1.8 \le A \le 2.5$	for the shallow basin
$2.0 \le A \le 2.8$	for the deep basin

Roughly, we can choose $1.5 \le A \le 3.0$ to estimate mode coupling in the range-dependent environments.

6. CONCLUSIONS

The following conclusions can be obtained according to the work we have done.

The limitation of the adiabatic approximation can be written in the form $\phi < A\theta$ where ϕ is the bottom slope and θ is the ray grazing angle. The conclusion is that the constant A is between the following limits $1.5 \le A \le 3.0$ in a channel with changing bottom slope and that coupling between the contiguous modes is seen to increase with h/λ as predicted.

References

1. A. D. Pierce, "Extension of the Method of Normal Mode to Sound Propagation in an Almost-stratified medium," J. Acoust. Soc. Am. **37**, 19-27 (1965).
2. R. D. Graves, Anton Nagl, H. Uberall, and G. L. Zarur, "Range-dependent normal mode in underwater sound propagation: Application to the wedge-shaped ocean," J. Acoust. Soc. Am. **58**, 1171-1177 (1975).
3. D. M. Milder, "Ray and wave invariants for SOFAR channel propagation," J. Acoust. Soc. Am. **46**, 1259-1263 (1969).
4. R. Rutherford and K.E.Hawker "An examination of the influence of the range dependence of the ocean bottom on the adiabatiic approximation",J. Acoust.Soc.Am **66**,1145-1151,(1979)
5. L. Brekhovskikh and I. Lysanov, *Fundamentals of Ocean Acoustics* (Berlin: Springer-Verlag, 1982).

6. C. H. Harrison, " Three-dimensional Ray Paths in Basins, Troughs, and near Seamounts by Use of Ray-invariants," J. Acoust. Soc. Am. **62**, 1382-1388 (1977).
7. P. R. Thomas and N. G. Pace, "Broadband measurements of acoustic attenuation in water-saturated sands," ULTRASONICS pp. 13-17. January 1980.
8. L. Wang and N. G. Pace, "Physical Modelling of Propagation Loss in range Dependent Environments", Final report to YARD ltd. (1990).
9. F. Ingenito, "Measurements mode attenuation coefficients in shallow water," J. Acoust. Soc. Am. **53**, 858-863 (1973).
10. C. T. Tindle, H. Hobaek, and T. G. Muir, "Downslope propagation of normal modes in a shallow water wedge," J. Acoust. Soc. Am. **81**, 275-286 (1986).
11. C. T. Tindle, K. M. Guthrie, G. E. J. Bold, M. D. Johns, D. Jones, K. O. Dixon, and T. G. Birdsall, "Measurements of the frequency dependence of normal modes," J. Acoust. Soc. Am. **64**, 1178-1185 (1978).
12. Ding Lee and G. Botseas, "Finite-difference solution to the parabolic wave equation," J. Acoust. Soc. Am. **70**, 795-800 (1981).
13. S. T. McDaniel and D. Lee, "A finite-difference treatment of interface conditions for the parabolic wave equation: The horizontal interface," J. Acoust. Soc. Am. **71**, 855-858 (1982).
14. D. Lee and S. T. McDaniel, "A finite-difference treatment of interface conditions for the parabolic wave equation: The irregular interface," J. Acoust. Soc. Am. **73**, 1441-1447 (1983).

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NORMAL MODE COUPLING IN RANGE DEPENDENT ENVIRONMENTS

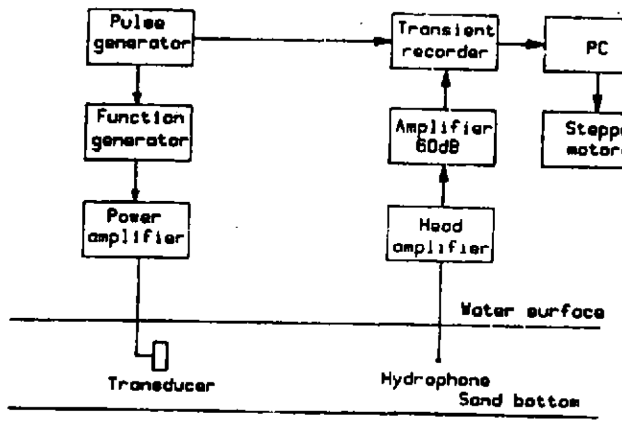

Fig. 1. Configuration of the experimental devices

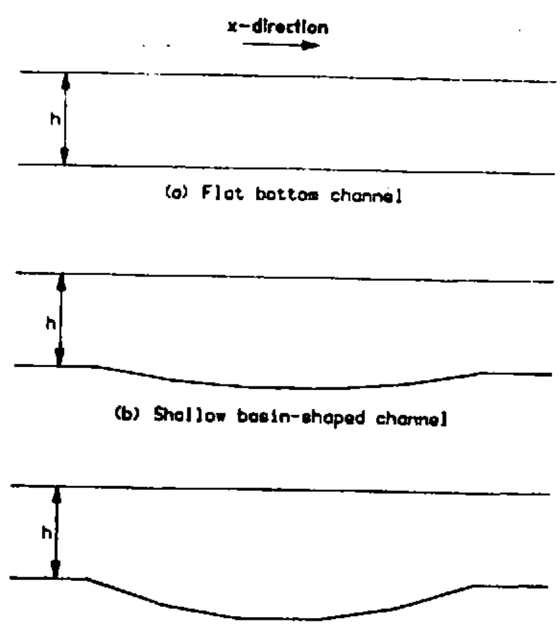

Fig. 2. Modelled shallow water channels in the experiments

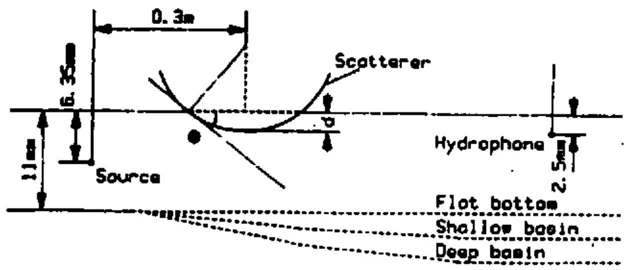

Fig. 3.
Measurements of sound pressure as a function of range in flat bottom, shallow basin, and deep basin channels with a scatterer on the water surface

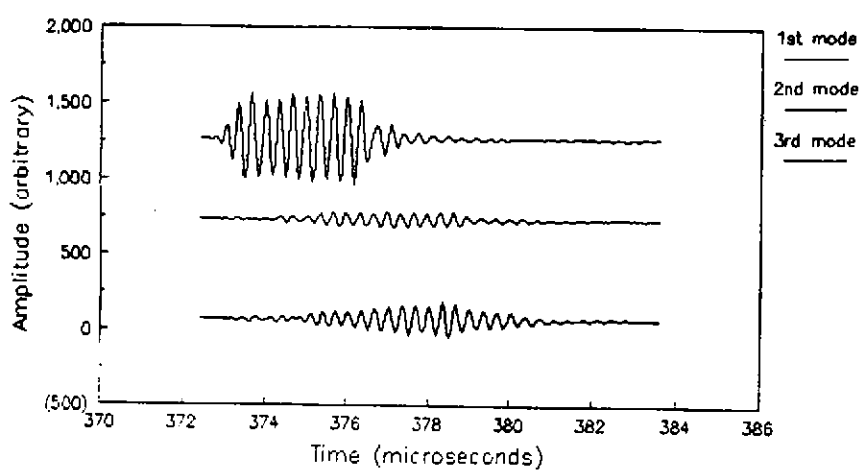

Fig. 4 The first three modes in the channel

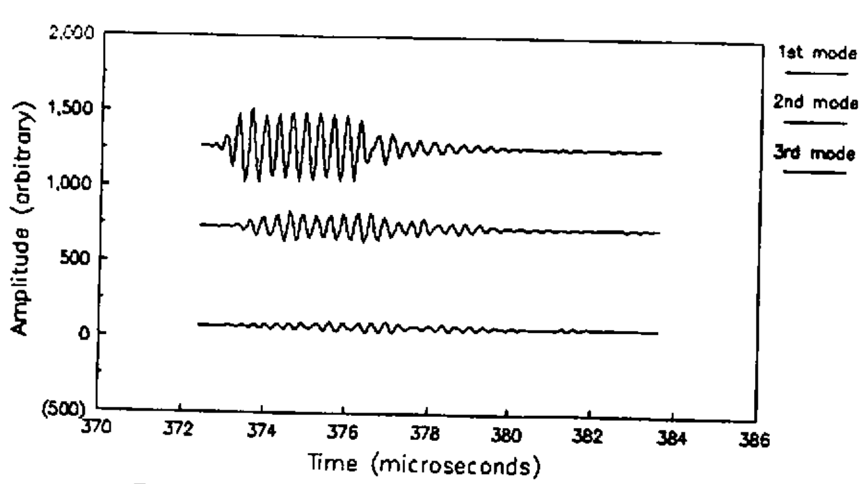

Fig. 5 The first three modes in the channel with the cylinder depth 3.0mm

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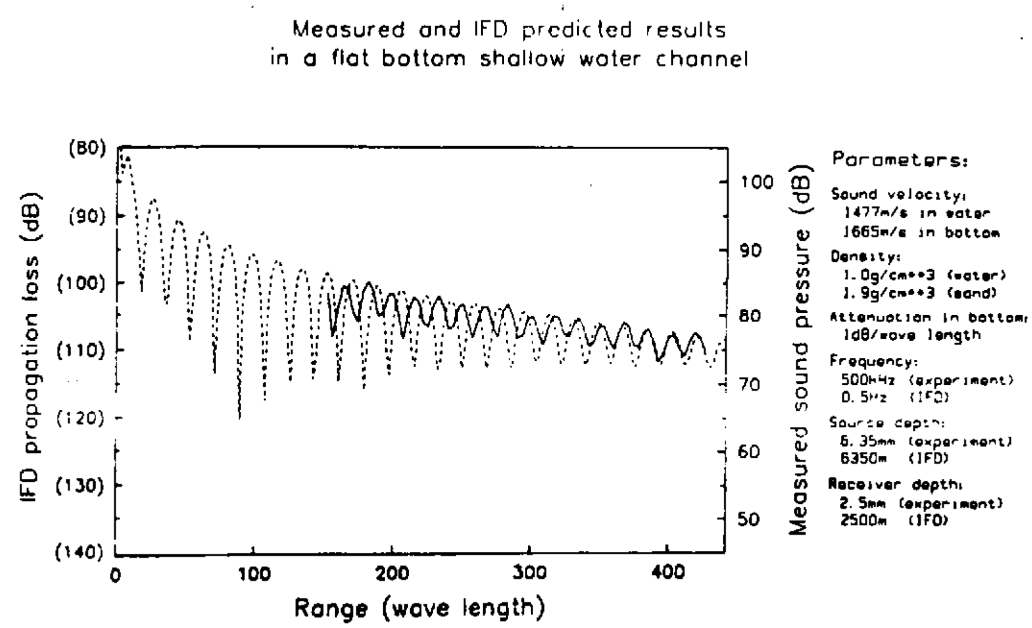

Fig. 6

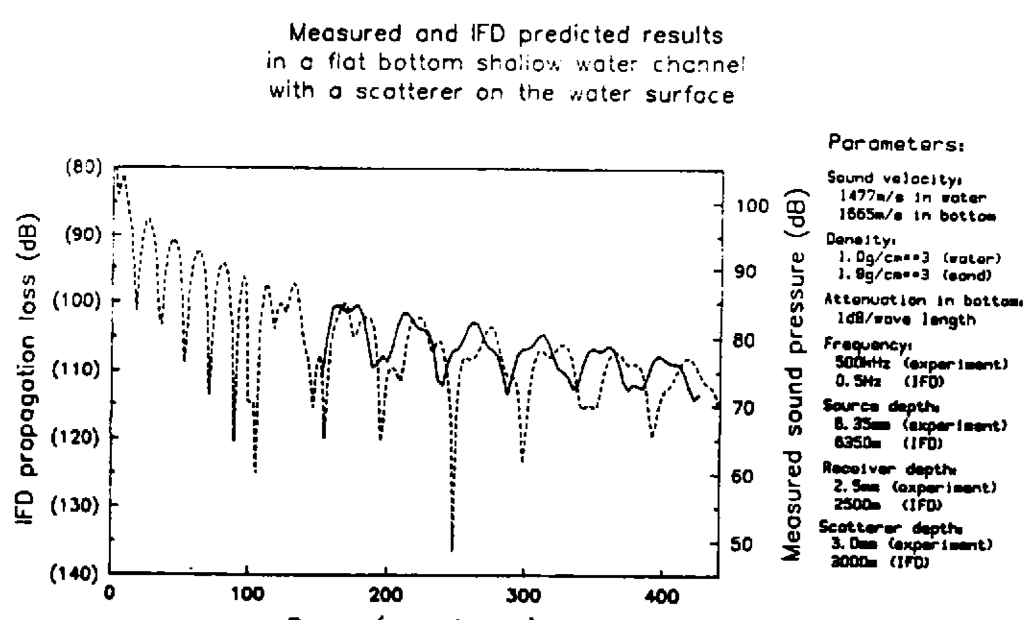

Fig. 7

NORMAL MODE COUPLING IN RANGE DEPENDENT ENVIRONMENTS

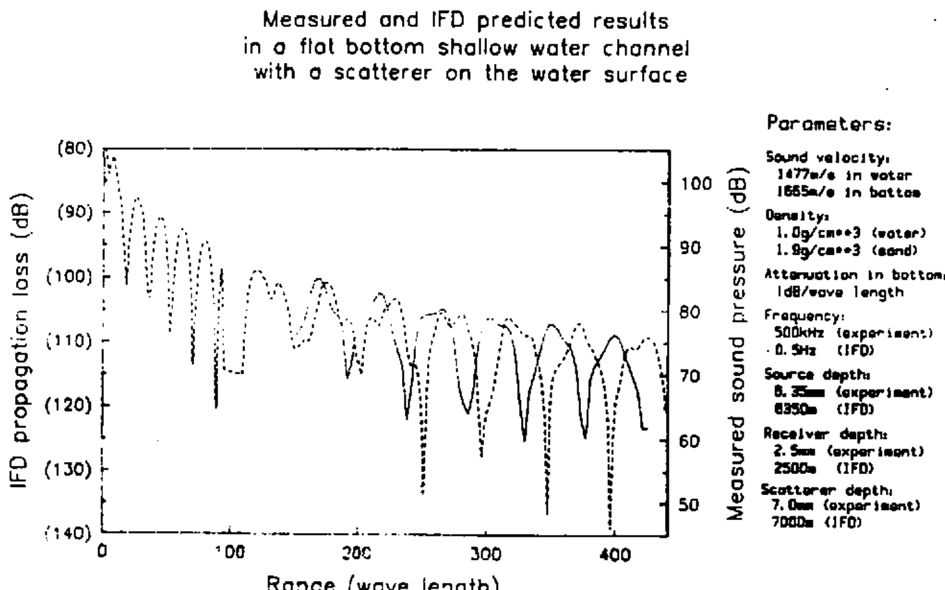

Fig. 8