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DETERMINATION OF THE UNCERTAINTY CONTRIBUTION OF ACOUSTIC FRONT ENDS ON SOUND PRESSURE LEVEL MEASUREMENTS

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Sound pressure level measurements are key points to determine whether there is a breach of noise limits. Any measurement made without the knowledge of its uncertainty lacks significance. For this reason, an open measurement system has been developed at the National Research Council Canada for sound pressure level measurements with the focus on measurement uncertainties. For uncertainty evaluation of such a system, the most difficult task is determining the uncertainty contribution of its acoustic front end, or its microphone and preamplifier assembly. The acoustic frond end can be modelled as a linear time-invariant system. Once the frequency response of the acoustic front end has been measured, its effect on an arbitrary signal can be analyzed. However, measured microphone frequency response does not include phase information, as no country in the world yet has Calibration and Measurement Capability (CMC) for microphone pressure sensitivity phase. With the completion of key comparison CCAUV.A-K5, this situation will change in the near future. Now is the time to determine the uncertainty contribution of acoustic front end using the phase information available, such as that in CCAUV.A-K5. In this paper, a method for the evaluation of the uncertainty contribution of acoustic front ends is presented. The acoustic front end is first modelled as a linear time-invariant system. Its measured frequency response at discrete frequencies is then curve-fit to obtain the response covering the entire frequency domain. The output of the acoustic front end is simply the convolution of the input with the acoustic front end's impulse response. The uncertainty contribution is finally calculated by propagating the uncertainties of the acoustic front end output at every time instance. Examples are given for typical acoustic front ends with various types of acoustic signals.
Keywords: sound pressure level, measurement uncertainty

1. Introduction

Sound pressure level (SPL) measurements are key points to determine whether there is a breach of noise limits. Any measurement made without the knowledge of its uncertainty is completely meaningless. For this reason, an open measurement system has been developed at the National Research Council Canada for sound pressure level measurements with the focus on measurement uncertainties [1]. The sound pressure level measurement system developed is an open system that includes the exposure of the data at every stage. The uncertainty contribution at each stage can then be assessed. For uncertainty evaluation of such a system, the most difficult task is to determine the uncertainty contribution of its acoustic front end, or its microphone and preamplifier assembly. A common practice is to use the worst-case scenario, a concept in risk management wherein the planner considers the most severe possible outcome. This method would use the largest measurement uncertainty of the measured frequency response of a microphone, normally occurring at the low or high frequency end, as the uncertainty contribution of acoustic frond end [2]. This may lead to an overestimation of the uncertainty contribution if the acoustic signal occupies a mid-frequency band, as the worst uncertainty may be five times larger than the best one for most countries around the world [3]. Overestimation of uncertainty may result in unnecessary measures to reduce noise that can be very expensive. To avoid the overestimation of uncertainty, an accurate estimation of the uncertainty contribution of acoustic frond end is needed. The acoustic frond end can be modelled as a linear time-invariant system. Once the frequency response of the acoustic front end has been measured, its effect on an arbitrary acoustic signal can be analyzed. Frequency response is a measure of the magnitude and phase of the output as a function of frequency, in comparison to the input. However, measured microphone frequency response does not include phase information, as no country in the world has Calibration and Measurement Capability (CMC) for the microphone pressure sensitivity phase [3]. With the completion of key comparison CCAUV.A-K5 [4], this situation will change in the near future. Now it is the time to determine the uncertainty contribution of acoustic front end accurately using the phase information available, such as that generated in CCAUV.A-K5.

In this paper, a method for the evaluation of the uncertainty contribution of acoustic front ends is presented. The acoustic front end is first modelled as a linear time-invariant system. The measured frequency response of the acoustic front end at discrete frequencies is then curve-fit to obtain the frequency response covering the entire frequency domain. The output of the acoustic front end is simply the convolution of the input with the acoustic front end's impulse response. The uncertainty contribution is finally calculated by propagating the uncertainties of the acoustic front end output at every time instance. Examples are given for typical acoustic front ends with various types of acoustic signals.

2. Acoustic front end

The acoustic front end of the sound pressure level measurement system developed is a microphone and preamplifier assembly. The simplified schematic diagram of the sound pressure level measurement system is shown in Fig. 1a. Figure 1b shows the modelling of the system with the focus on the uncertainty contribution of the acoustic front end.

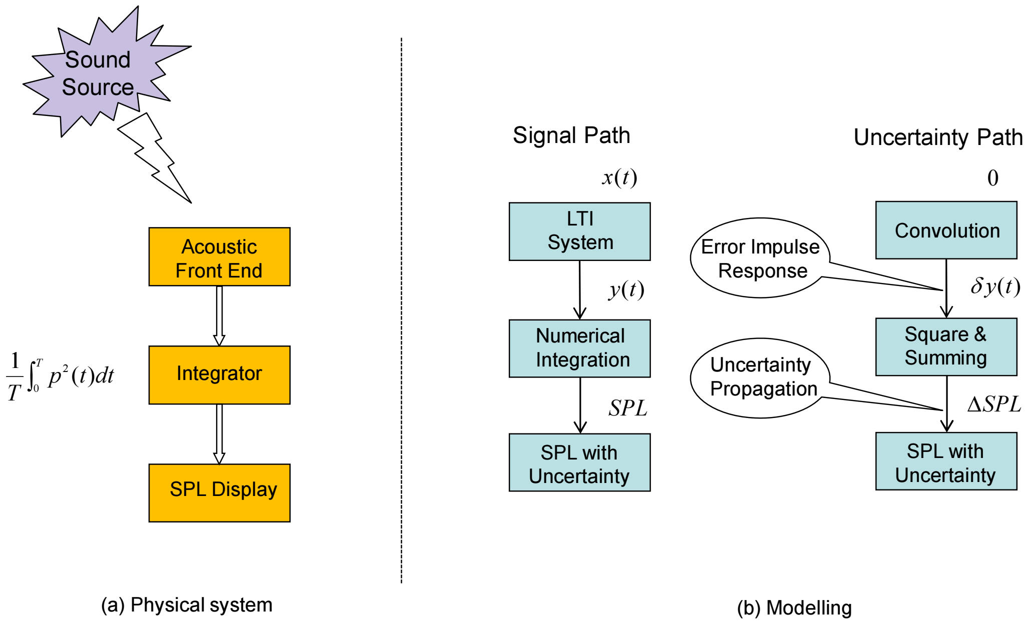


Figure 1: A measurement system for sound pressure level measurements. (a) Simplified physical system. (b) System modelling with the focus on acoustic front end.

2.1 Acoustic front end and its modelling

Acoustic front ends have two main functions. Firstly, these units provide power to microphone/preamplifier assemblies, including microphone polarization voltages. Secondly, active circuitries inside the units amplify the output of preamplifiers to a level that is suitable for integrator or digital recording. Most acoustic systems may be regarded as linear and time-invariant [5]. Therefore, acoustic front ends can be modelled by linear time-invariant systems. The system is linear if the total input signal is a sum of signals. The total output signal is then the sum of output signals generated by passing each individual input signal through the system. Condenser measurement microphones are most commonly used in acoustic front ends for precision SPL measurements. For amplitudes that are not too high (up to 140 dB), there is a linear relationship between sound pressure and sound-induced voltage. Time-invariant implies that if an output signal is generated by an input signal through the system then any time shifted input signal results in an output signal with an identical time shift. In other words, physical parameters of acoustic front end do not vary with time. This may not be true for acoustic front ends as microphone sensitivities vary with the change of environmental conditions, especially barometric pressure [6]. However, most sound level meters update measured sound pressure levels every second or so. Within such a short period the estimated microphone sensitivity change due to environmental changes for LS2P microphones is 0.000015 dB, which is not significant. Therefore, acoustic front ends for sound pressure level measurements are linear time-invariant systems.

Figure 2a sketches a linear time-invariant (LTI) system with single input and single output, where $h(t)$ is the impulse response of the system and $H(\omega)$ is its Fourier transform. Any LTI system can be characterized by the system's impulse response. The output $y(t)$ of the system is simply the convolution of the input $x(t)$ to the system with the system's impulse response $h(t)$.

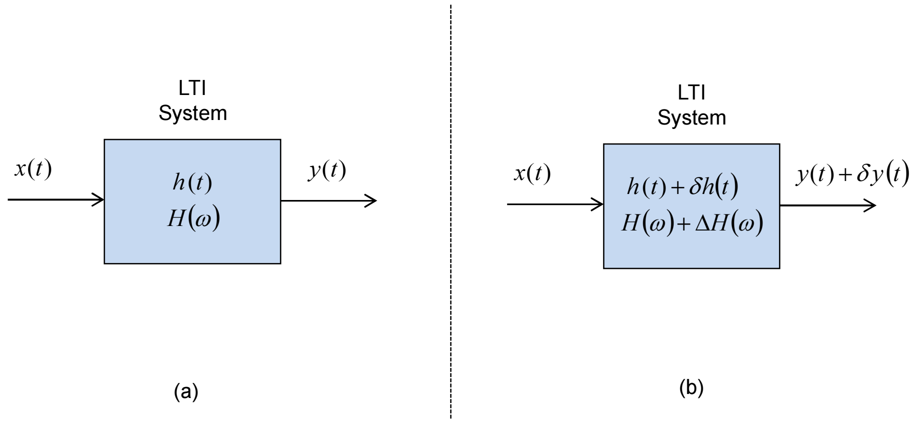


Figure 2: A linear time-invariant system with single input and single output. (a) Ideal system. (b) System with measurement uncertainties.

2.2 LTI system with measurement uncertainties

Figure 2b illustrates a LTI system that includes measurement uncertainties. The uncertainty is treated as a random variable $\delta h(t)$ added to the system's impulse response $h(t)$. The Fourier transform of the system's impulse response is then given by $H(\omega) + \Delta H(\omega)$, where $\Delta H(\omega)$ is the Fourier transform of $\delta h(t)$ and is also a random variable. The distribution of $\Delta H(\omega)$ can be derived from the distribution of $\delta h(t)$. In other words, the distribution of $\delta h(t)$ can be derived from the distribution of $\Delta H(\omega)$ via the inverse Fourier transform. The latter is a more practical approach as $\Delta H(\omega)$ can be related to the measurement uncertainty of the frequency response of an acoustic front end. Since $h(t)$ is a real-valued function then $H(\omega) = H^*(-\omega)$, where the superscript * denotes complex conjugate. The same applies to $\delta h(t)$ and $\Delta H(\omega)$. With this equation, the measured frequency response of acoustic front end can be extended to entire frequency domain from $-\infty$ to $+\infty$.

2.3 Effect of measurement uncertainties on LTI system output

As illustrated in Fig. 2b the output $y(t) + \delta y(t)$ of the system is the convolution of the input $x(t)$ to the system with the system's impulse response $h(t) + \delta h(t)$. That is, $y(t) = x(t) * h(t)$ and $\delta y(t) = x(t) * \delta h(t)$, where the asterisk * denotes convolution. In other words, the output of the system consists of a deterministic component $y(t)$ and a stochastic component $\delta y(t)$. It is more convenient to work in frequency domain as $H(\omega)$ and $\Delta H(\omega)$ are easily available from frequency response measurements. Thus, the Fourier transform of the output $y(t) + \delta y(t)$ of a LTI system including measurement uncertainties can be expressed as $Y(\omega) + \Delta Y(\omega) = X(\omega)H(\omega) + X(\omega)\Delta H(\omega)$, where $X(\omega)$ is the Fourier transform of the input $x(t)$ and $\Delta Y(\omega) = X(\omega)\Delta H(\omega)$ is the Fourier transform of the stochastic component $\delta y(t)$.

The random variable $\Delta H(\omega)$ has a zero mean and its variance can be obtained from the standard uncertainty of frequency response measurements. This implies that the stochastic component $\delta y(t)$ of the system's output has a zero mean since

$$\begin{aligned} \mathrm{E}[\delta y(t)] &= \mathrm{E}[F^{-1}\{\Delta Y(\omega)\}] = F^{-1}\{\mathrm{E}[X(\omega)\Delta H(\omega)]\} \\ &= F^{-1}\{X(\omega)\mathrm{E}[\Delta H(\omega)]\} = F^{-1}\{0\} = 0 \end{aligned} \tag{1}$$

where $\mathrm{E}[\cdot]$ denotes expectation and $F^{-1}\{\cdot\}$ denotes inverse Fourier transform. Thus, the variance of $\delta y(t)$ equals to $\mathrm{E}[\delta y^2(t)]$. Its relationship with the input to the system and the system's impulse response can be derived as follows:

$$\begin{aligned} \mathrm{E}[\delta y^2(t)] &= \mathrm{E}[F^{-1}\{\Delta Y(\omega)\}F^{-1}\{\Delta Y(\omega)\}] \\ &= \mathrm{E}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\Delta H(\omega)e^{-i\omega t}d\omega \; \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\Delta H(\omega)e^{-i\omega t}d\omega\right] \\ &= \mathrm{E}\left[\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} X(\theta)\Delta H(\theta)X(\varphi)\Delta H(\varphi)e^{-i(\theta+\varphi)t}d\theta\, d\varphi\right] \\ &= \frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} X(\theta)X(\varphi)\mathrm{E}[\Delta H(\theta)\Delta H(\varphi)]e^{-i(\theta+\varphi)t}d\theta\, d\varphi \end{aligned} \tag{2}$$

Note that $\mathrm{E}[\Delta H(\theta)\Delta H(\varphi)]$ has the following property:

$$\mathrm{E}[\Delta H(\theta)\Delta H(\varphi)] = \begin{cases} \mathrm{E}[\Delta H(\theta)]\mathrm{E}[\Delta H(\varphi)] = 0, & \text{if } \theta \neq \varphi \\ \mathrm{E}[\Delta H^2(\theta)], & \text{if } \theta = \varphi \end{cases} \tag{3}$$

since $\Delta H(\theta)$ and $\Delta H(\varphi)$ are independent random variables for different frequencies. The two-dimensional integral in Eq. (2) is thus reduced to an one-dimensional integral, that is, a line integral along a straight line, $\theta = \varphi$. Equation (2) can then be rewritten as

$$\begin{aligned} \mathrm{E}[\delta y^2(t)] &= \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} X^2(\theta)\mathrm{E}[\Delta H^2(\theta)]e^{-i(2\theta)t}d\theta \\ &= \frac{1}{4\pi}\left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} X^2\left(\frac{\omega}{2}\right)\mathrm{E}\left[\Delta H^2\left(\frac{\omega}{2}\right)\right]e^{-i\omega t}d\omega\right] \\ &= \frac{1}{4\pi}F^{-1}\left\{X^2\left(\frac{\omega}{2}\right)\mathrm{E}\left[\Delta H^2\left(\frac{\omega}{2}\right)\right]\right\} \end{aligned} \tag{4}$$

With Eqs. (1) and (4) the effect of measurement uncertainties of a LTI system's frequency response on its output can be analyzed.

3. Sound pressure level

Sound pressure level is calculated according to its definition given in ANSI/ASA S1.1-2013, the current American National Standard on Acoustical Terminology, published by the American National Standards Institute. It is calculated by the definite integral

$$SPL = \frac{1}{T}\int_0^T p^2\,(t)dt \tag{5}$$

where $p^2(t)$ is the squared instantaneous sound signal that may be frequency-weighted and/or time-weighted and *T* is a stated time interval. The frequency weighting may include the correction for frequency response of acoustic front end.

3.1 Numerical integration

Equation 5 is implemented by numerical integration. That is, an approximate solution to a definite integral $\int_a^b f(x)\,dx$ is computed for a given accuracy. With over sampling, the uncertain contribution due to numerical integration is not significant to the overall measurement uncertainty [1]. Thus, Eq. (5) can be replaced by

$$SPL = \frac{1}{n}\sum_{i=1}^{n} p^2(t_i) = \frac{1}{n}\sum_{i=1}^{n} p_i^2 = \frac{1}{n}\sum_{i=1}^{n} y_i^2 \tag{6}$$

where p_i is the sound pressure at the time instance t_i, or the output y_i of the LTI system assuming $H(\omega) = 1$ (the acoustic front end's frequency response is corrected) and denoting $y_i = y(t_i)$ and $\delta y_i = \delta y(t_i)$.

3.2 Uncertainty propagation

Uncertainty propagation occurs when mathematical operations are performed on measured quantities. The output of a LTI system sampled at the time instance t_i can be considered as such a measured quantity with the measurement uncertainty given by Eqs. (1) and (4). The mathematical operations (square and sum) in Eq. (6) are then performed on the measured quantities y_i with the uncertainties of δy_i to obtain the measurement sound pressure level *SPL*. This results in the uncertainty propagation from δy_i to ΔSPL, the uncertainty of the measured *SPL*. For the case of noise measurements, all input quantities (instant measured sound pressure signals) of Eq. (6) are independent. The combined standard uncertainty ΔSPL is the positive square root of the combined variance ΔSPL^2 that is given by [7]

$$\begin{aligned}\Delta SPL^2 &= \sum_{i=1}^{n}\left(\frac{\partial f}{\partial y_i}\right)^2 \mathrm{E}[\delta y_i^2] \\ &= \frac{4}{n^2}\sum_{i=1}^{n} y_i^2\,\mathrm{E}[\delta y_i^2]\end{aligned} \tag{7}$$

where *f* is the function given in Eq. (6). , For the case where two or more input quantities are related, that is, are interdependent or correlated, the treatment is discussed in 5.2 of JCGM 100:2008 [7].

3.3 Biased estimator

Equation 6 can be considered as an estimator for calculating a SPL approximately based on observed data, or output values of the acoustic front end. The difference between this estimator's expected value and the true value (SPL is not affected by the measured frequency response of acoustic front end) of the parameter being estimated, the bias of the estimator, can be calculated as follows:

$$\begin{aligned}\mathrm{E}[\Delta SPL] &= \frac{1}{n}\sum_{i=1}^{n}\mathrm{E}[(y_i+\delta y_i)^2] - \frac{1}{n}\sum_{i=1}^{n} y_i^2 \\ &= \frac{1}{n}\sum_{i=1}^{n}\mathrm{E}[\delta y_i^2] \neq 0\end{aligned} \tag{8}$$

Thus, Eq. (6) leads to a biased estimator. The discussion of the reduction of bias, however, is beyond the scope of this paper. A review of methods that have been developed to reduce bias in parametric estimation can be found in [8].

4. Applications

Examples are given here for typical acoustic front ends with various types of acoustic signals to demonstrate the uncertainty contribution of the acoustic front end.

4.1 Sinusoidal signals

For sinusoidal signals exact solutions can be obtained rather than using the first order approximation in Eq. (7). Let the signal be $A_0\cos(\omega_0 t)$ with $\omega_0 = 2\pi f_0$. Let $\Delta H(\omega_0)/H(\omega_0) = \delta$ be a real random variable. (The phase of $H(\omega_0)$ or $\Delta H(\omega_0)$ does not play any role in Eq. (5) as long as Tf_0 is equal to an integer.) Then, $\Delta SPL/SPL = 2\delta + \delta^2$ with $\mathrm{E}[\Delta SPL/SPL] = \mathrm{E}[\delta^2]$ and $\mathrm{Var}[\Delta SPL/SPL] = 4\mathrm{E}[\delta^2] + 4\mathrm{E}[\delta^3] + \mathrm{E}[\delta^4] + (E[\delta^2])^2$. $\mathrm{E}[\delta^n]$ can be calculated through the moment-generating function $M_\delta(t)$ of the random variable δ. That is, $\mathrm{E}[\delta^n] = \frac{d^n}{dt} M_\delta(0)$. For normal distribution, or $\delta \sim N(0, \sigma^2)$, $\mathrm{E}[\Delta SPL/SPL] = \sigma^2$ and $\mathrm{Var}[\Delta SPL/SPL] = 4\sigma^2(1 + \sigma^2)$. The standard uncertainty of the frequency response at ω_0 is propagated to the standard uncertainty of the measured SPL through a coefficient of 2 and a factor of $\sqrt{1+\sigma^2}$.

4.2 Toneburst signals

When testing the measurement capability of a SPL measurement system for transient signals, a 4 kHz toneburst signal is applied [9]. Deviations of measured toneburst responses from the corresponding reference toneburst responses given in IEC 61672-1:2013 shall be extended by the actual expanded uncertainties of measurement [9]. Each extended deviation shall be within the applicable tolerance limits given in IEC 61672-1:2013. Therefore, the uncertainty plays an important role in determining the acceptance of a test device. Because of technical difficulties, the 4 kHz toneburst signal is applied to the electrical input of the test device, bypassing the acoustic front end to exclude its uncertainty contribution as specified in IEC 61672-1:2013. This practice is in conflict with the test purpose as transient signals are wideband signals, while acoustic front ends are band limited. The measurement capability for transient signals is limited by the bandwidth of the acoustic front end, rather than the electronic circuitry inside the device that can have any desired bandwidth. With the method discussed in Sections 2 and 3, the uncertainty contribution of acoustic front end for toneburst signals can be calculated. Thus, acoustic front ends can be included in tests.

Let tonebursts, extracted from a steady signal $\sin(\omega_0 t)$ with $\omega_0 = 2\pi f_0$ and the durations T_b specified in IEC 61672-1:2013, be the input signal $x(t)$. The Fourier transform of $x(t)$ is

$$X(\omega) = \frac{1}{2i}\int_{-T_b/2}^{T_b/2}\left[e^{-i(\omega-\omega_0)t} - e^{-i(\omega+\omega_0)t}\right]dt \\ = -i\left\{\frac{\sin[(\omega-\omega_0)T_b/2]}{(\omega-\omega_0)} - \frac{\sin[(\omega+\omega_0)T_b/2]}{(\omega+\omega_0)}\right\} \quad (9)$$

where the sinusoid is expanded using the Euler formula. This is the sum of a pair of shifted imaginary $\sin(x)/x$ functions, centered at frequencies $\pm\omega_0$. Since $|\sin(x)| \le 1$, $\sin(x)/x$ decreases quickly as x increases. Thus, the integral for the inverse Fourier transform in Eq. (4) can be calculated over a finite frequency interval.

Consider a simple case such that $\mathrm{E}\left[\Delta H^2\left(\frac{\omega}{2}\right)\right] = \Delta H^2$ is a constant over the entire frequency range. Equation 4 can then be rewritten as

$$\mathrm{E}[\delta y^2(t)] = \frac{\Delta H^2}{\pi}F^{-1}\left\{\left[\frac{1}{2}X\left(\frac{\omega}{2}\right)\right]\left[\frac{1}{2}X\left(\frac{\omega}{2}\right)\right]\right\} = \frac{\Delta H^2}{\pi}x(2t)\otimes x(2t) \quad (10)$$

where $\otimes$ denotes convolution. For $x(2t) = \cos(2\omega_0 t)$, the convolution of $x(2t)\otimes x(2t)$ is equal to $\begin{cases} 1/2\{(T_b - |t|)\cos(2\omega_0 t) + \sin[2\omega_0(T_b - |t|)]/(2\omega_0)\}, & \text{for } |t| \le T_b, \\ 0, & \text{otherwize.}\end{cases}$ As an example, Table 1 lists the uncertainty contributions of an acoustic front end with a constant standard uncertainty (being excluded in the calculation) of its frequency response for various toneburst signals. The toneburst durations and the reference frequency of 4 kHz are taken from Table 4 in IEC 61672-1:2013 [9]. The sampling frequency $f_s = kf_0$, where k is the over sampling factor (OSF). It is interesting to see from the table that the standard uncertainty of measured SPL is proportional to $1/\sqrt{k}$.

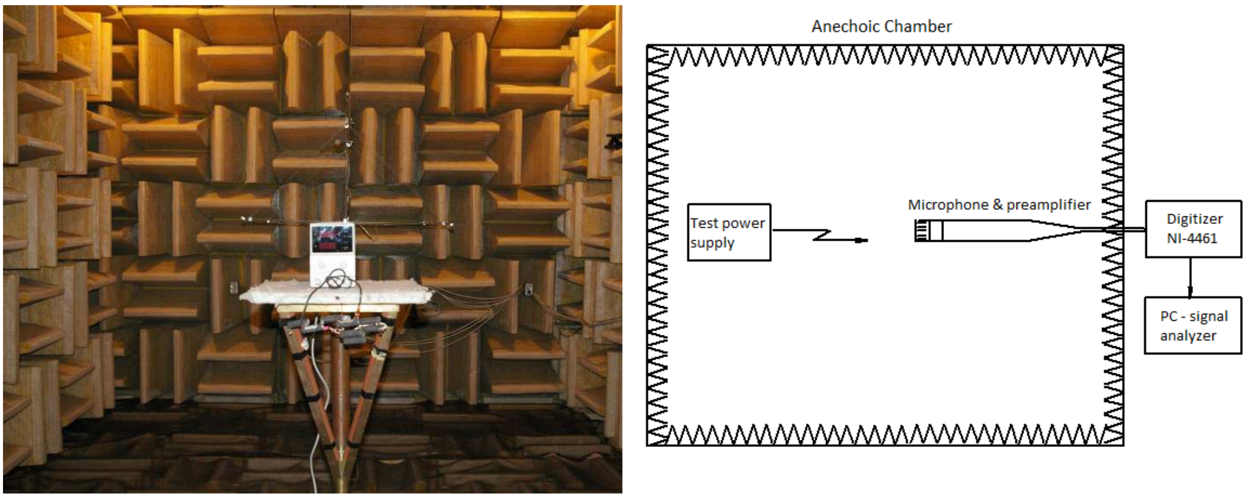
Table 1: Standard uncertainty (in dB) of measured SPL for toneturst signals with different durations

OSF, k	Toneburst duration, T_b ms											
	1000	500	200	100	50	20	10	5	2	1	0.5	0.25
5	0.019	0.028	0.044	0.061	0.087	0.136	0.191	0.268	0.417	0.578	0.797	1.088
10	0.014	0.019	0.031	0.043	0.061	0.097	0.136	0.191	0.299	0.417	0.578	0.797
100	0.004	0.006	0.010	0.014	0.020	0.031	0.043	0.061	0.097	0.136	0.191	0.268
1000	0.001	0.002	0.003	0.004	0.006	0.010	0.014	0.020	0.031	0.043	0.061	0.087

4.3 General signals

The unweighted sound pressure levels of a DC power supply with a variable speed fan were measured in the NRC anechoic chamber using the measurement system developed at NRC. The acoustic front end of the system consists of a measurement microphone, Brüel and Kjær Type 4190, and a microphone preamplifier, Brüel and Kjær type 2619. The measurement set up is shown in Fig. 3.

The sound pressure levels were measured in a preselected direction at a distance of 1.00 m from the fan centre. The fan centre is defined as the dimensional centre of the fan. The power supply was mounted on a table with the fan centre facing the microphone. The sound pressure signals were recorded over 60 seconds with a sampling frequency of 50 kHz to simulate common sound level meters. Then 60 sound pressure level samples were calculated, corresponding to an integration time interval of T = 1 s, similar to those of sound level meters. The signal analyzer was set to calculate both the sound pressure level and the associated uncertainty contribution from the acoustic front end at every second. The measurement was repeated without remounting the power supply for different current loads.



(a) (b)

Figure 3: Measurement set up. (a) Photo of the measurement set up. (b) Schematic diagram of the set up.

Table 2 lists the measured sound pressure levels with the associated measurement uncertainties. The uncertainty varies slightly between different measured SPLs for same operating condition. This is expected as the spectrum of the input signal has little change. However, the uncertainty varies significantly between different measured SPLs for different operating conditions as the spectrum of the input signal changes considerably for different loads. The spectrums plotted in Fig. 4 show a 5.8 dB difference between the major frequency components for 60 % and 80 % of full loads.

Table 2: Standard uncertainty (in dB) of measured SPL (in dB) for fan noise with different loads

Load	60 % of full load					80 % of full load				
Measured SPL	52.40	52.70	52.74	52.40	52.45	53.18	53.21	53.12	53.11	53.13
Uncertainty	0.59	0.60	0.60	0.62	0.61	0.91	0.93	0.94	0.93	0.93

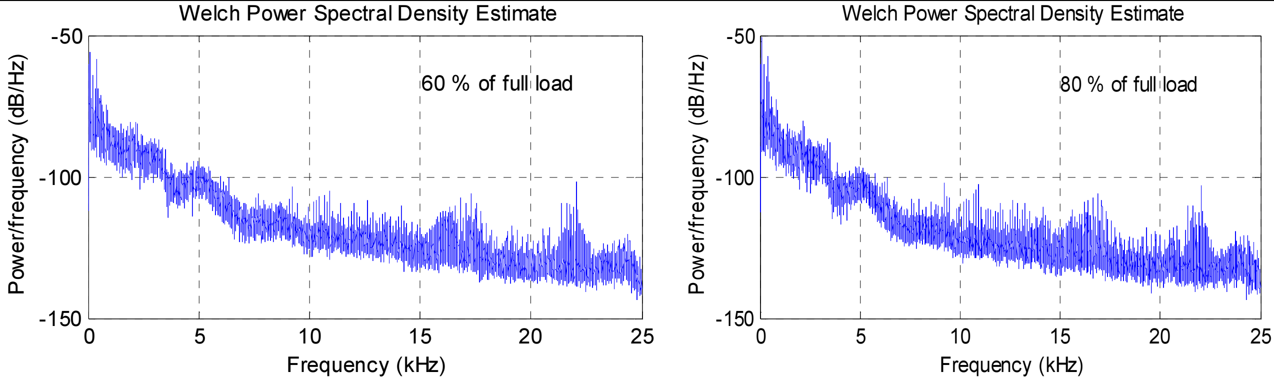


Figure 4: Spectrums of the acoustic input signals for SPL measurements

5. Conclusions

This work has contributed two advances to the evaluation of sound pressure level measurement uncertainty: 1) an expression for calculating the uncertainty contribution of acoustic front ends, and 2) applications demonstrating how to use the expression for various signals. The exact solution for sinusoidal signals shows that the traditional approach neglects high order terms and thus leads users to underestimate uncertainty. The results of toneburst signals can be used in the revision of IEC 61672-1:2013, specifically tolerance limits, to include acoustic tests. The results of general signals demonstrate that the uncertainty contribution of acoustic front end depends on the input acoustic signal considerably.

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