

# THE NONLINEAR GRNN MODEL OF MAGNETO-RHEOLOGICAL ENGINE MOUNTS AND FUZZY LOGIC CONTROL

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A novel flow mode MR mount is designed and fabricated in which the radial and annular flow paths are applied together to increase the adjustability of MR mount. A series of tests was conducted on the prototype MR mount in order to characterize MR mount. Generalized regression neural network (GRNN) method is proposed and used to establish the direct and inverse model of MR mount based on experimental data. Specially, the inverse model of MR mount determines the quality of feedback control system. The accuracy of the proposed identification method is analysed and evaluated. The full vehicle dynamic model with MR mounts is derived and a fuzzy logic controller is designed and fabricated to isolate the vibration of engine significantly. The fuzzy logic rules are analysed and discussed. In order to demonstrate the effectiveness of the proposed semi-active system with MR mounts, the control responses such as acceleration in frequency domain are evaluated and presented. The results show that the prototype MR mount demonstrates good vibration isolation performance over a wide frequency range. The proposed GRNN models can describe the nonlinear of MR mount accurately. The vibration of engine can be attenuated significantly by using the proposed fuzzy logic controller. Therefore, a potential application of the proposed semi-active MR engine mount system is demonstrated in the near future.

Keywords: MR mount, GRNN generalized regression neural network, Fuzzy logic control

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## 1. Introduction

Magneto-rheological mount is an intelligent engine vibration isolation device. The rheological behaviour of MR fluids change instantaneously depending on the application of a magnetic field. The change from a free-flowing liquid state to a solid-like state is reversible [1]. The magneto-rheological mount technology provides an important way to isolate the broadband vibration from the engine to the chassis so to guarantee the comfort and enhance the life of the vehicle [2].

MR mount has highly nonlinear and hysteresis characteristics [3], a reasonable direct and inverse model of MR mount is the precondition for accurate control of mounting system [4]. Parameterized Bingham model was simple and easy to analyse, which describe the relationship between force and displacement, but it can't describe the hysteresis characteristics of MR damper [5]. As a non-parameterized model, neural network model is one of the most application prospect of model identification method due to its nonlinear approximation capability [6]. Wang [7] built the neural network model to describe nonlinearity characteristics of the magneto-rheological damper using the recursive neural network (RNN). However, the convergence speed of RNN model was slow due to complex network structure.

Currently, lots of studies were carried on semi-active control of MR mount. Li Rui designed vertical fuzzy controller which can modify the parameters by the imitation of human's intelligent

control [8]. S. R. Hong developed a linear quadratic Gaussian (LQG) controller for MR mount system. The Acceleration levels of structure and the force transmitted to the base are experimentally proved to be reduced effectively [9].

In this paper, generalized regression neural network (GRNN) method is proposed and used to establish the direct and inverse model of MR mount based on experimental date. The full vehicle dynamic model with MR mounts is derived and a fuzzy controller is designed. The control responses such as acceleration and displacement of engine in time and frequency domain are evaluated and presented.

## 2. Design and experiment

The MR mount is designed in the flow mode in which the radial and annular flow paths are applied together to increase the adjustability of MR mount. Fig 1 is a longitudinal cross-sectional schematic view of the MR mount. Magnetic core assembly was connected by a long bolt through the central axis. The radial flow path formed by upper core outside and upper core inside and the annular flow path formed by upper core outside and inner core are filled with MR fluid. The magnetic coil imposes a magnetic field on the magnetic core assembly to control the shear properties of the MR fluid in the paths.

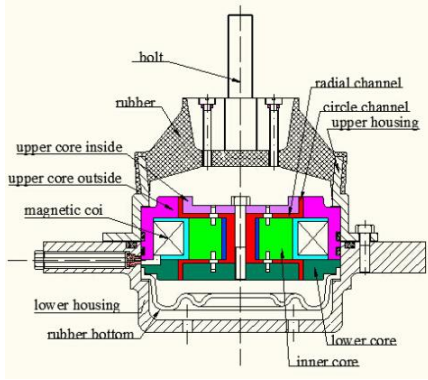


Figure 1: MR mount

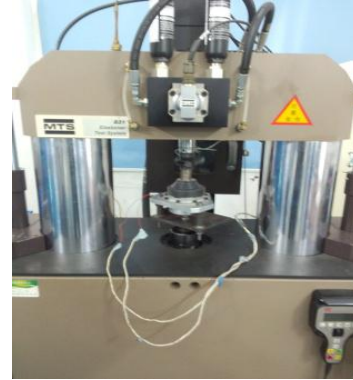


Figure 2: Experimental setup

The novel MR mount prototype was manufactured and measured by MTS testing system to obtain the experimental results as shown in Fig.2.

## 3. General regression neural network (GRNN)

General regression neural network (GRNN) has strong nonlinear mapping and self-learning advantages for non-parameterized model of MR mount. Moreover, GRNN needs to adjust only one parameter, known as smoothing factor. The training of the GRNN depends on data samples, so this network can reduce the influence to its approximation capability caused by man-made subjective assumptions significantly. The structure of GRNN is shown in Fig.3, it consists of four layers: input layer, pattern layer, sum layer and output layer.  $X_1 \cdots X_n$  and  $Y_1 \cdots Y_m$  are the input and forecasting output values of the GRNN respectively,  $p_i$  is the transfer function of the pattern layer,  $S_D$  is sum for all of outputs of the pattern layer,  $S_{Nj}$  is weighted sum for all of outputs of the pattern layer.

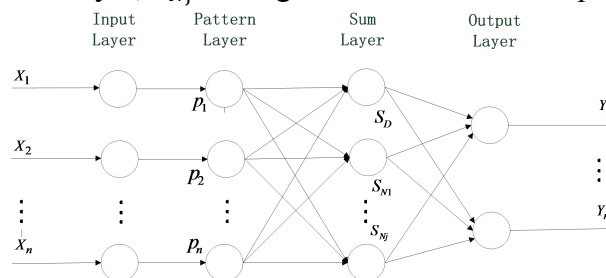


Figure 3: Structure of GRNN

Based on the GRNN model, the direct model was identified using the dynamic experimental data of MR mount. The principle diagram of the GRNN direct model is shown in Fig.4. The input variables were selected and consisted of the displacement  $x_{k-1}$ , velocity  $v_{k-1}$ , current  $I_{k-1}$ , restoring force  $F_{k-1}$  of the past and the displacement  $x_k$ , velocity  $v_k$ , current  $I_k$ , frequency  $f_k$  for the present. The output variable was the restoring force for the present, where N was normalized processing, D was unit sampling time of delay.

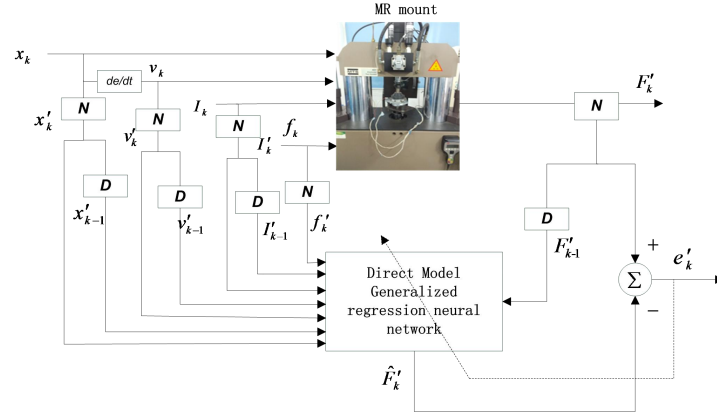


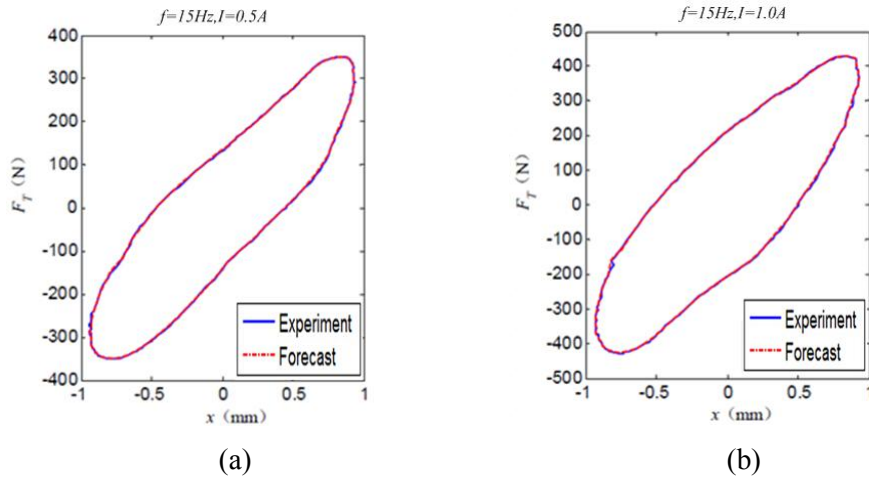
Figure 4: The principle diagram of the GRNN direct model

The index  $E_F$  was established for evaluating prediction accuracy of GRNN direct model quantificationally. The evaluation formula as follow [10]:

$$E_F = \left(1 - \frac{\sum_{k=1}^n |F_k - \hat{F}_k|}{n \cdot \max(|F_k|)}\right) \times 100\% \quad (1)$$

where,  $E_F$  was relative prediction accuracy of the output damping force,  $F_k$  was the experimental value of the output damping force,  $\hat{F}_k$  was predicted value of the output damping force through the GRNN direct model,  $n$  was sample size.

The experimental value and predictive value are shown in Fig.5. According to the expression (1), the GRNN neural network prediction accuracy can be calculated, as shown in Table 1.



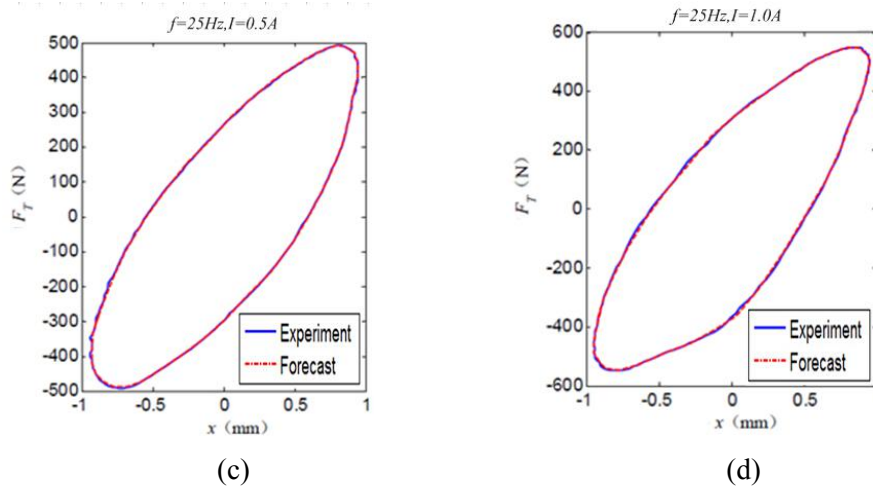


Figure 5: Experimental and predicted value of the output damping force

Table 1 Relative prediction accuracy of the output damping force

Excitation	15Hz,0.5A	25Hz,0.5A	15Hz,1.0A	25Hz,1.0A
Prediction accuracy	99.51%	99.67%	99.42%	99.44%

Table 1 shows that non-parametric direct model of MR mount has high prediction precision for the damping force based on GRNN neural network. The lowest value of prediction accuracy was 99.42%. It meets the requirements of the control system design.

## 4. Semi-active control of MR mount

### 4.1 Full-vehicle model based on MR mount

A 10-DOF full-vehicle dynamic model based on magneto-rheological mount is established in Fig.6. It has a sprung mass, four unsprung masses and independent suspension. The vehicle dynamics model is composed of 3 DOF vertical, pitch and roll motion of sprung mass, 4 DOF vertical motions of four unsprung mass, 3 DOF vertical, pitch and roll motion of the engine. The engine is supported by a four-point engine mount, where the 1st is MR mount and the others are passive rubber mount. The derived vehicle dynamics model is expressed by the following equations.

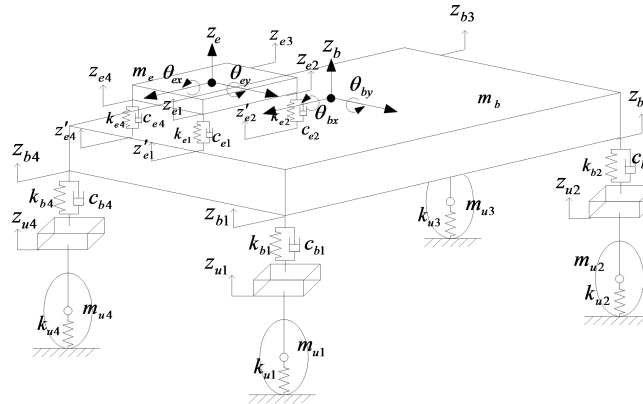


Figure 6: Full-vehicle dynamic model based on MR mount:

Vertical motion of engine:

$$m_e \ddot{z}_e = -f_{e1} - f_{e2} - f_{e3} - f_{e4} - u_1 + F_z \quad (2)$$

Roll motion of engine:

$$J_{ex}\ddot{\theta}_{ex} = l_{e1}f_{e1} - l_{e2}f_{e2} + l_{e3}f_{e3} - l_{e4}f_{e4} + l_{e1}u_1 + F_{\theta x} \quad (3)$$

Pitch motion of engine:

$$J_{ey}\ddot{\theta}_{ey} = t_{e1}f_{e1} - t_{e2}f_{e2} + t_{e3}f_{e3} - t_{e4}f_{e4} + t_{e1}u_1 + F_{\theta y} \quad (4)$$

$m_e$  is engine mass;  $J_{ex}$ ,  $J_{ey}$  are rotational inertia of the engine;  $\ddot{z}_e$ ,  $\ddot{\theta}_{ex}$ ,  $\ddot{\theta}_{ey}$  are vertical acceleration, roll angular acceleration and pitch angular acceleration of the engine, respectively.  $f_{ei}$  are passive mount forces;  $(t_{ei}, l_{ei})$  are absolute value of the coordinates  $(x_{ei}, y_{ei})$  of the mount upper endpoint relative to the engine coordinate, in which  $i=1\sim 4$ ;  $u_1$  is controllable damping force of MR mount;  $F_z$ ,  $F_{\theta x}$ ,  $F_{\theta y}$  are the vertical, roll and pitch exciting force (torque) of the engine respectively.

Vertical motion of sprung mass:

$$m_b\ddot{z}_b = -f_{b1} - f_{b2} - f_{b3} - f_{b4} + f_{e1} + f_{e2} + f_{e3} + f_{e4} + u_1 \quad (5)$$

Roll motion of sprung mass:

$$J_{bx}\ddot{\theta}_{bx} = -l_{b1}f_{b1} - l_{b2}f_{b2} + l_{b3}f_{b3} + l_{b4}f_{b4} + l_{eb1}f_{e1} - l_{eb2}f_{e2} + l_{eb3}f_{e3} - l_{eb4}f_{e4} + l_{eb1}u_1 \quad (6)$$

Pitch motion of sprung mass:

$$J_{by}\ddot{\theta}_{by} = t_{b1}f_{b1} - t_{b2}f_{b2} - t_{b3}f_{b3} + t_{b4}f_{b4} - t_{eb1}f_{e1} - t_{eb2}f_{e2} - t_{eb3}f_{e3} - t_{eb4}f_{e4} - t_{eb1}u_1 \quad (7)$$

$m_b$  is vehicle body mass;  $J_{bx}$ ,  $J_{by}$  are rotational inertia of the vehicle body;  $\ddot{z}_b$ ,  $\ddot{\theta}_{bx}$ ,  $\ddot{\theta}_{by}$  are vertical acceleration, roll angular acceleration and pitch angular acceleration of the vehicle body respectively;  $f_{bi}$  are suspension force;  $(t_{bi}, l_{bi})$  are absolute value of the coordinates  $(x_{bi}, y_{bi})$  of the suspension upper endpoint relative to the body coordinate system.  $(t_{ebi}, l_{ebi})$  are absolute value of the coordinates  $(x_{ebi}, y_{ebi})$  of the mount lower endpoint relative to the engine coordinate.

Four unsprung mass vertical motion:

$$m_{u1}\ddot{z}_{u1} = f_{b1} - k_{u1}z_{u1} \quad (8)$$

$$m_{u2}\ddot{z}_{u2} = f_{b2} - k_{u2}z_{u2} \quad (9)$$

$$m_{u3}\ddot{z}_{u3} = f_{b3} - k_{u3}z_{u3} \quad (10)$$

$$m_{u4}\ddot{z}_{u4} = f_{b4} - k_{u4}z_{u4} \quad (11)$$

$m_{ui}$  are unsprung mass,  $k_{ui}$  are tire stiffness,  $z_{ui}$  are vertical displacement of unsprung mass.

For the conventional passive suspension, its suspension forces are set by:

$$f_{bi} = k_{bi}(z_{bi} - z_{ui}) + c_{bi}(\dot{z}_{bi} - \dot{z}_{ui}), i = 1\sim 4; \quad (12)$$

Similarly, the mount force are normally imposed:

$$f_{ei} = k_{ei}(z_{ei} - z'_{ei}) + c_{ei}(\dot{z}_{ei} - \dot{z}'_{ei}), i = 1\sim 4; \quad (13)$$

Where:  $k_{bi}$  and  $c_{bi}$  are the suspension stiffness and suspension damping respectively.  $k_{ei}$  and  $c_{ei}$  are the mount stiffness and mount damping respectively.

When the pitch and roll angle of the body are small, the relationship between the vertical displacement of the upper end points of suspension and the body coordinate are:

$$\begin{cases} z_{b1} = z_b - l_{b1}\theta_{ex} - t_{b1}\theta_{ey} \\ z_{b2} = z_b + l_{b2}\theta_{ex} + t_{b2}\theta_{ey} \\ z_{b3} = z_b - l_{b3}\theta_{ex} - t_{b3}\theta_{ey} \\ z_{b4} = z_b + l_{b4}\theta_{ex} + t_{b4}\theta_{ey} \end{cases} \quad (14)$$

When the pitch and roll angle of the engine are small, the relationship between the vertical displacement of the mount upper end points and the engine coordinate origin are:

$$\begin{cases} z_{e1} = z_e - l_{e1}\theta_{ex} - t_{e1}\theta_{ey} \\ z_{e2} = z_e + l_{e2}\theta_{ex} + t_{e2}\theta_{ey} \\ z_{e3} = z_e - l_{e3}\theta_{ex} - t_{e3}\theta_{ey} \\ z_{e4} = z_e + l_{e4}\theta_{ex} + t_{e4}\theta_{ey} \end{cases} \quad (15)$$

The relationship between the vertical displacement of the mount lower end points and the body coordinate are:

$$\begin{cases} z'_{e1} = z_b - l_{eb1}\theta_{bx} - t_{eb1}\theta_{by} \\ z'_{e2} = z_b + l_{eb2}\theta_{bx} - t_{eb2}\theta_{by} \\ z'_{e3} = z_b - l_{eb3}\theta_{bx} - t_{eb3}\theta_{by} \\ z'_{e4} = z_b + l_{eb4}\theta_{bx} - t_{eb4}\theta_{by} \end{cases} \quad (16)$$

Define  $q = (z_e \ \theta_{ex} \ \theta_{ey} \ z_b \ \theta_{bx} \ \theta_{by} \ z_{u1} \ z_{u2} \ z_{u3} \ z_{u4})^T$ ,  $U = (u_1 \ 0 \ 0 \ 0)^T$ ,  $F_e = (F_z \ F_{\theta x} \ F_{\theta y})^T$ . The standard form of the differential equations is obtained:

$$M_w \ddot{q} + C_w \dot{q} + K_w q = B_w U + D_w F_e \quad (17)$$

$M_w$ ,  $C_w$  and  $K_w$  are the mass matrix, damping matrix and stiffness matrix respectively,  $B_w$  is the input matrix of controllable damping force,  $D_w$  is the engine excitation matrix.

Define  $X = (q \ \dot{q})^T$ ,  $Y = [z_{e1} \ z_{e2} \ z_{e3} \ z_{e4}]^T$ , The state equation and output equation can be written as follows:

$$\dot{X} = AX + BU + DF \quad (18)$$

$$Y = CX \quad (19)$$

Where:  $A = \begin{bmatrix} \text{zeros}(10,10) & \text{eye}(10,10) \\ -M_w^{-1}K_w & -M_w^{-1}C_w \end{bmatrix}$ ,  $B = \begin{bmatrix} \text{zeros}(10,4) \\ M_w^{-1}B_w \end{bmatrix}$ ,  $D = \begin{bmatrix} \text{zeros}(10,3) \\ M_w^{-1}D_w \end{bmatrix}$ .

## 4.2 Fuzzy controller design

In recent years, fuzzy control strategy were more and more applied to complex nonlinear systems due to its flexibility, strong robustness and mathematical model independence. A schematic diagram of the fuzzy control of MR mount is set up as shown in Fig.7. In this paper, a fuzzy controller with dual-input and single-output is designed, the second-order frequency of engine and acceleration of the mount upper endpoint are choose as input. The controllable damping force of MR mount is set as output. The actual control damping force is calculated by the GRNN direct model according to the current of the GRNN inverse model.

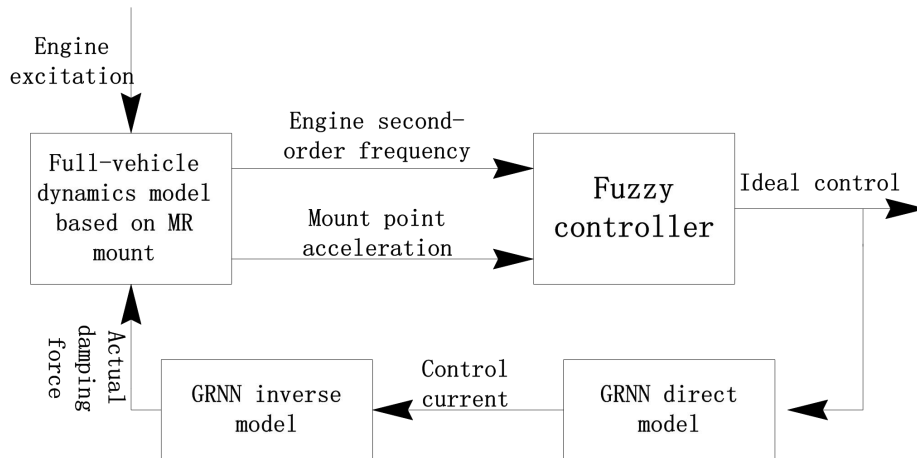


Figure 7: Schematic for MR mount fuzzy control



The input second-order frequency range is [0 100] Hz, acceleration range is [-20 20] m/s<sup>2</sup>, output damping force range is [-300 300]N. The fuzzy control rules can be obtained based on experience and theoretical derivation. As shown in Table 2. The universal set of input acceleration is divided into eight partitions: positive big(PB), positive middle (PM), positive small (PS), positive zero (PZ), negative big(NB), negative middle (NM), negative small (NS), negative zero (NZ). The universal set of input frequency is divided into five partitions: positive zero (PZ), positive small (PS), positive middle (PM), positive big (PB), positive large (PL). The universal set of output damping force is also divided into eight partitions.

Table 2: Fuzzy control rule

$f$	$a$	NB	NM	NS	NZ	PZ	PS	PM	PB
PZ		NB	NM	NS	NZ	PZ	PS	PM	PB
PS		NB	NM	NS	NZ	PZ	PS	PM	PB
PM		NM	NS	NS	NZ	PZ	PS	PS	PM
PB		NZ	NM	NM	NB	PB	PM	PM	PZ
PL		NZ	NM	NB	NB	PB	PB	PM	PZ

### 4.3 Simulation experiments

A full-vehicle simulation model was established based on MR mount and passive mount by using Matlab software. The vibration acceleration signal of NO.1 mount upper endpoint are measured when the rotational speed are 400r/min (Startup condition), 750r/min (Idle condition) and 1500r/min (Steady-state conditions) respectively. Comparing between the passive mount and controllable mount is shown in Fig.8 to Fig.9.

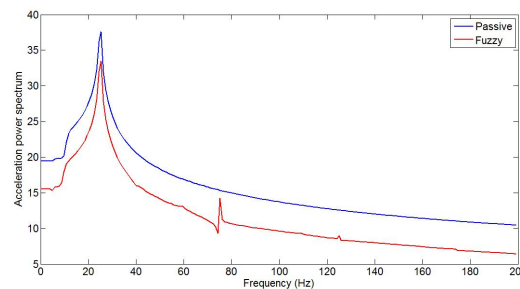
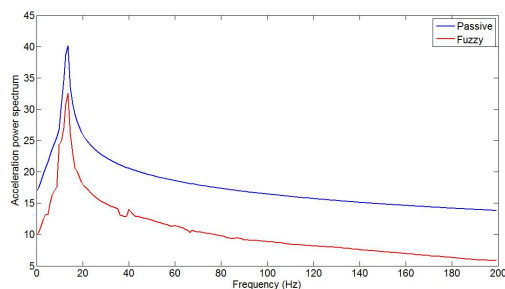


Figure 8: Acceleration response at 400r/min      Figure 9: Acceleration response at 750r/min

Above results show that the vibration acceleration and acceleration power spectral peaks can be reduced significantly when it is compared to passive engine mount in a speed of 400r/min, 750r/min and 1500r/min respectively.

Table 3 shows the acceleration RMS value of MR mount. The acceleration RMS value of MR mount decreased 58.51%, 37.63% and 22.75% in a speed of 400r/min, 750r/min and 1500r/min respectively. It implies that the proposed fuzzy controller is effective in vibration isolation.

Table 3: Acceleration RMS value at MR mount

Rotational speed	400r/min	750r/min	1500r/min
Passive	5.3869m/s <sup>2</sup>	3.9489 m/s <sup>2</sup>	5.5168 m/s <sup>2</sup>
Fuzzy	2.2348 m/s <sup>2</sup>	2.4629 m/s <sup>2</sup>	4.2615 m/s <sup>2</sup>
Decay	58.51%	37.63%	22.75%

## 5. Conclusions

- (1) The direct and inverse model of MR mount were established by generalized regression neural network (GRNN) identification method using MR mount dynamic experimental data.

The accuracy of the proposed identification method was further analysed and evaluated. The result shows that the GRNN has very high identification accuracy to meet the requirements of control system design.

- (2) The full vehicle 10 degrees of freedom dynamic model with MR mounts was derived considering the GRNN direct and GRNN inverse model. A fuzzy controller is designed to isolate vibration of engine to base. The simulation results show that the fuzzy controller has good broadband vibration isolation characteristics. The acceleration amplitude in the second order frequency of engine decreased significantly.

## ACKNOWLEDGEMENTS

This work is supported by Chongqing foundation and advanced research project (cstc2015jcyjBX0097) and Chongqing key industry common key technology innovation project (cstc2015zdcy-ztxx30001). These financial supports are acknowledged greatly.

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