BIFURCATION AND CHAOS OF A LINEAR STRUCTURE WITH A NONLINEAR ENERGY SINK

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A nonlinear energy sink (NES) is an effective device to reduce structural vibration while keeping the system frequency unchanged. However, a nonlinear energy sink may leads to complex dynamics such bifurcation and chaos. The investigation treats bifurcation and chaos in forced vibration of a harmonically excited linear structure coupled with a nonlinear energy sink. The bifurcations with the varying NES mass and NES nonlinear stiffness are numerically examined via the Poincaré map. Dynamical behaviours are identified by phase trajectories amplitude spectrums and Poincaré maps. The bifurcation diagrams reveal that the responses of the structure and the energy sink are periodic except a few bursts of chaotic motions. In addition, the dynamic behaviours of the structure may be different from those of the nonlinear energy sink for appropriate parameters.

Keywords: nonlinear energy sink, bifurcation, chaos, numerical simulations

1. Introduction

A nonlinear energy sink is an effective device to reduce mechanical and structural vibration passively [1,2]. Recently, much attention has been paid to suppress forced vibrations of structures subjected to external excitations [3-18]. The structures were modeled as single-degree-of-freedom oscillators [3-8,11], two-degree-of-freedom linear oscillators [9,10,17,18], linear strings [12,13], linear beams [14,15], and single-degree-of-freedom nonlinear oscillators [16]. A simplest model of a nonlinear energy sink is an essential nonlinear oscillator consisting of a small mass, a cubic stiffness and a linear damper [3-6, 8,9,11-18].

Most available investigations focused on periodic steady-state responses [3,5,7-9,11-18]. In addition to experimental works [3,4,11,18] and numerical simulations[3,7,10,14,16-18], approximate analytical methods are a powerful approach to predict the steady-state responses by yielding amplitude-frequency response curves and examining their stabilities[3,7-9,11-16]. Most used approach is the complexification averaging method [3,5,7,9-11,14-16]. Besides, a mixed multiple scale/harmonic balance algorithm was proposed and applied [8,12,13]. The method of harmonic balance is also used to analyze the periodic steady-state response [7]. In addition to periodic motions, there were some numerical simulations on quasi-periodic motions [4-6,10,13] and the complexification averaging method was applied to investigate quasi-periodic motions [4,5,10].

The cubic stiffness in a nonlinear energy sink creates new nonlinearity in the coupled system of a structure and the energy sink. The nonlinearity may change dynamics of the system qualitatively as well as quantitatively. Specifically, the nonlinearity may lead to new complex dynamics such as chaos. The possibility of chaotic motion was initially revealed in [5]. It is well known that the route to chaos is essential and significant to understand nonlinear behaviors of a system. However, the route to chaos has not researched for a system composed by a structure and a nonlinear energy sink. To address the lack of researches in this aspect, the present work explores the route to chaos by examining bifurcations in the Poincaré maps regarding to two key design parameters of a nonlinear energy sink, namely the mass and the nonlinear stiffness.

The manuscript is organized as follows. Section 2 presents a basic model of a structure with a nonlinear sink. In Section 3, bifurcation diagrams are numerically calculated. In Sections 4, chaos is numerically identified. Section 5 ends the manuscript with concluding remarks.

2. Formalizations

Consider a harmonically excited structure with a nonlinear energy sink. The structure is model as a linear single-degree-of-freedom system with stiffness k_1, linear damping coefficient c_0, the mass m_0 and excited by the periodic force F(t)=Acos(ωt). The nonlinear energy sink consists of m, cubic stiffness k, linear damping c. Figure 1 shows the model.

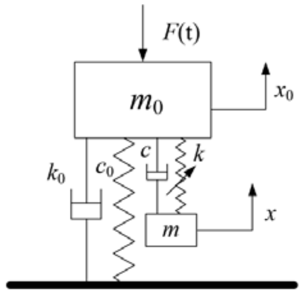


Figure 1: A linear oscillator with a NES.

Measured from their static equilibriums, the displacements of masses m_0 and m are denoted as x_0 and x, respectively. Newton's second law yields the dynamic equations of the system

$$\begin{aligned} &m_0\ddot{x}_0 + k_0x_0 + c_0\dot{x}_0 + c\left(\dot{x}_0 - \dot{x}\right) + k\left(x_0 - x\right)^3 + A\cos\left(\omega t\right) = 0 \\ &m\ddot{x} + k\left(x - x_0\right)^3 + c\left(\dot{x} - \dot{x}_0\right) = 0 \end{aligned} . \tag{1}$$

3. Bifurcation diagrams

This section examines nonlinear behaviours of the system based on the numerical integrations calculated via the Runge-Kutta scheme. Choose the parameter values as m_0=24 kg, k_0=20 kN/m, c_0= 1.2 N·s /m, c= 1.2 N·s /m, A=10N, and ω=28.8675 rad/s. Two key design parameters of the nonlinear energy sink, namely, mass m and cubic stiffness k are considered as a varying parameter respectively. Bifurcations in the Poincaré maps are employed to demonstrate the effect of the two parameters on dynamical behaviours. The displacement components in the Poincaré maps are focused. The first 4800 points in the Poincaré maps are calculated for fixed parameters, and only the last 200 points are plotted in bifurcation diagrams to eliminate transient responses.

The change of mass m is focused with fixed k=10000 kN/m^3. Figure 2 depicts the displacements components in the Poincaré maps of the structure response and the nonlinear energy sink response. The numerical results show that the responses of the structure and the energy sink are periodic except a few bursts of chaotic motions. Such chaotic motions are dynamic complexity induced by the nonlinear energy sink, because linear structures behave periodically only. Figure 3 presents bifurcation diagrams of the displacement components in the Poincaré maps of the structure and the energy sink for the change of cubic stiffness k for m=0.5kg. The structure and the energy sink vibrate periodically except for the bursts of chaos for the small and the large stiffness k. The amplitude of the periodic motion of the structure increases with the stiffness, while that of the energy sink remains almost unchanged.

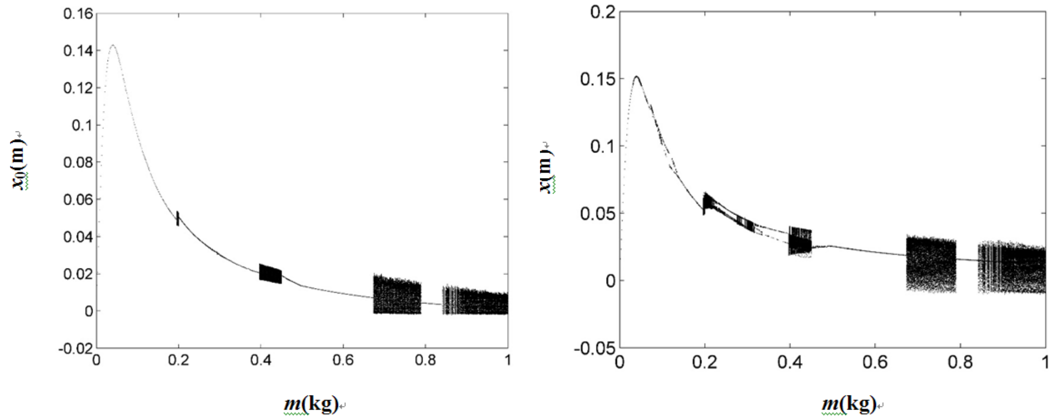


Figure 2: The bifurcation diagrams of the structure and the NES responses for varying *m*.

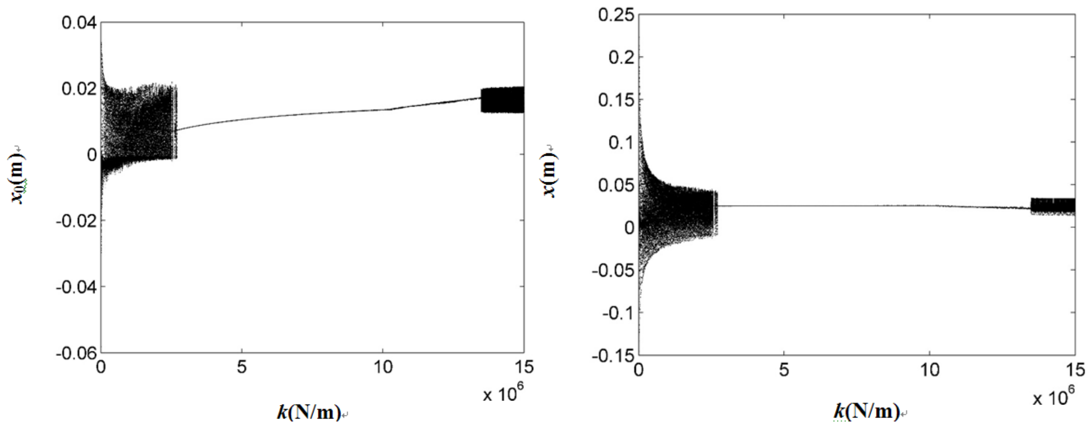
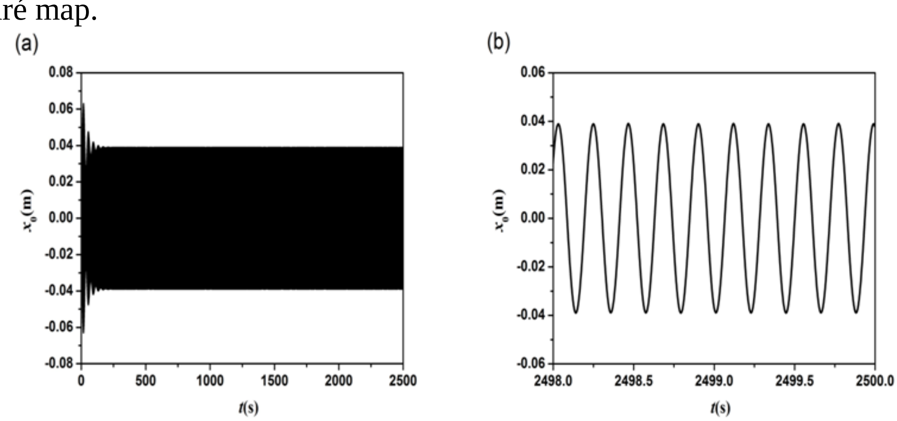


Figure 3: The bifurcation diagrams of the structure and the NES responses for varying *k*.

4. Periodic and chaotic motions

For the periodic responses, the vibrations of the structure and the energy decreases with the increasing energy sink mass, except for the very small energy sink mass. It should be remarked, the response of the energy sink seems more complex than that of the structure, as shown in Figs. 2. Fix k=10000 kN/m^3. Figures 4 and 5 show that the structure vibrates periodically but the energy sink chaotically for m=0.2496kg. Figures 6 and 7 show that the structure is with period-1 motion while the energy sink period-2 motion, for m=0.3384kg, and the fact implies the occurrence the period-doubling bifurcation for the energy sink, also shown in Fig. 2 Figures 8 and 9 show vibrations of both the structure and the energy sink are chaotic. In above-mentioned figures, chaos is identified by the time history, the amplitude spectrum, the phase portrait, and the Poincaré map, while periodic motion is demonstrated by the time history with its local enlargement, the phase portrait, and the Poincaré map.



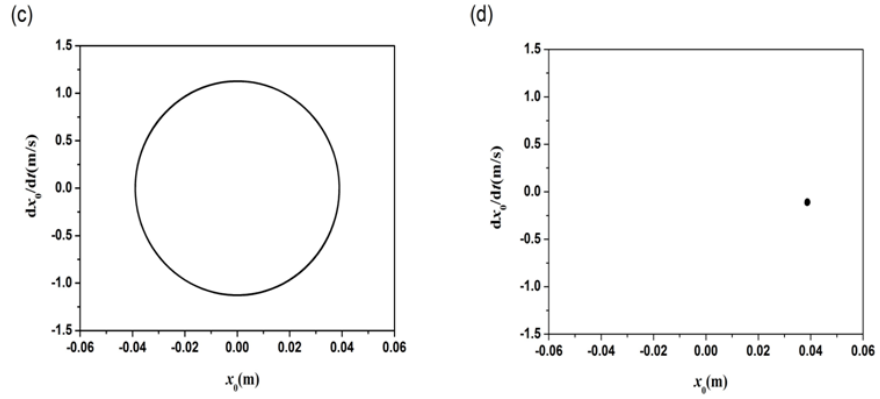


Figure 4: Periodic vibration of the structure for *m*=0.2496kg (a) the time history, (b) the enlargement of the time history, (c) the phase portrait, (d) the Poincaré map.

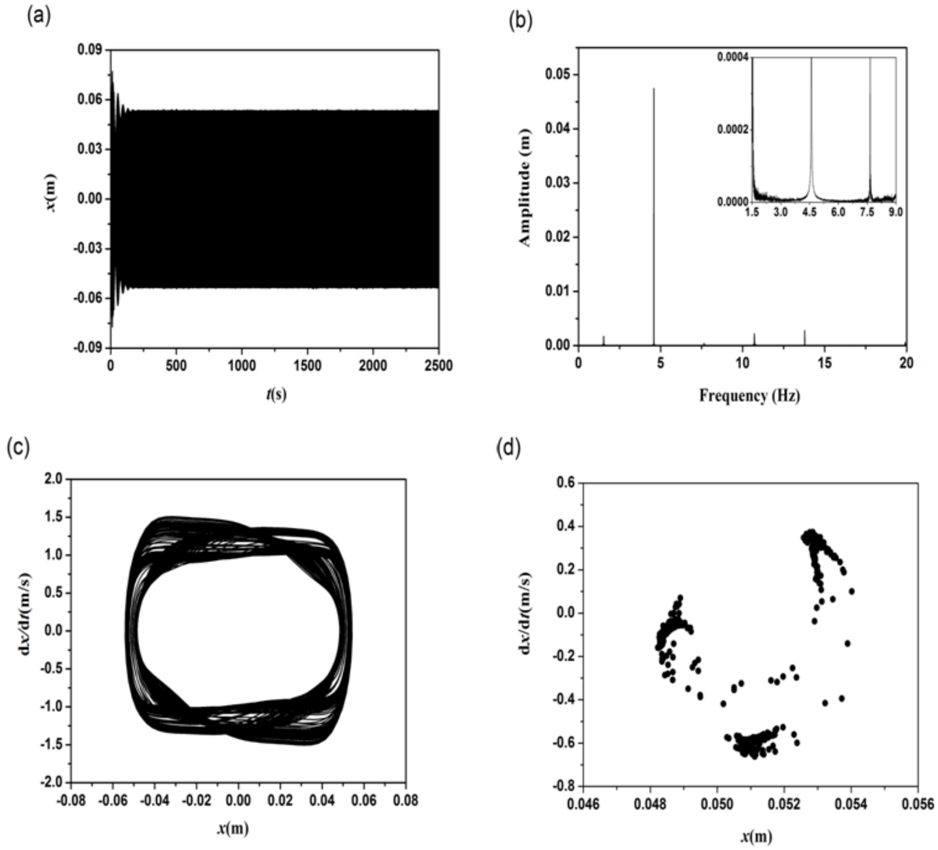
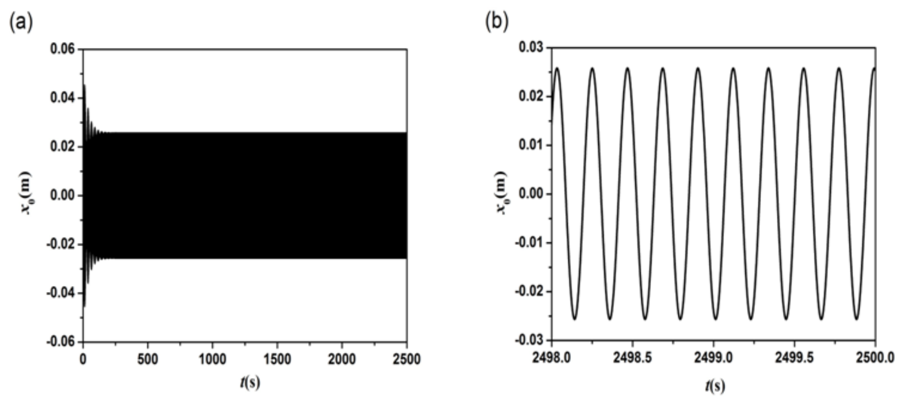


Figure 5: Chaotic motion of the energy sink for *m*=0.2496kg (a) the time history, (b) the amplitude spectrum, (c) the phase portrait, (d) the Poincaré map.



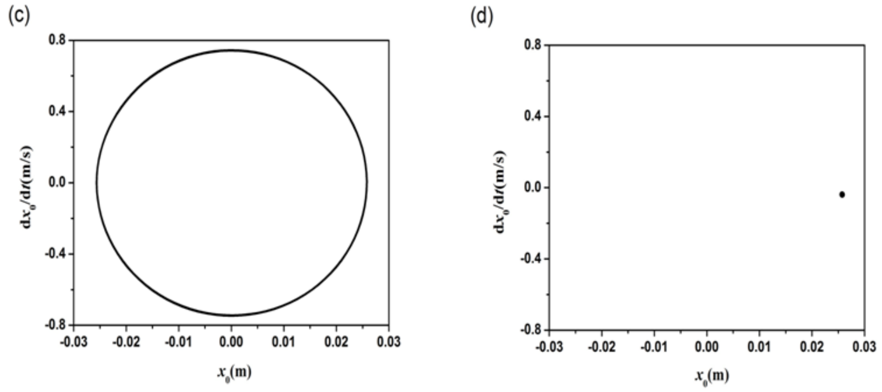


Figure 6: Period-1 motion of the structure for *m*=0.3384kg (a) the time history, (b) the enlargement of the time history, (c) the phase portrait, (d) the Poincaré map.

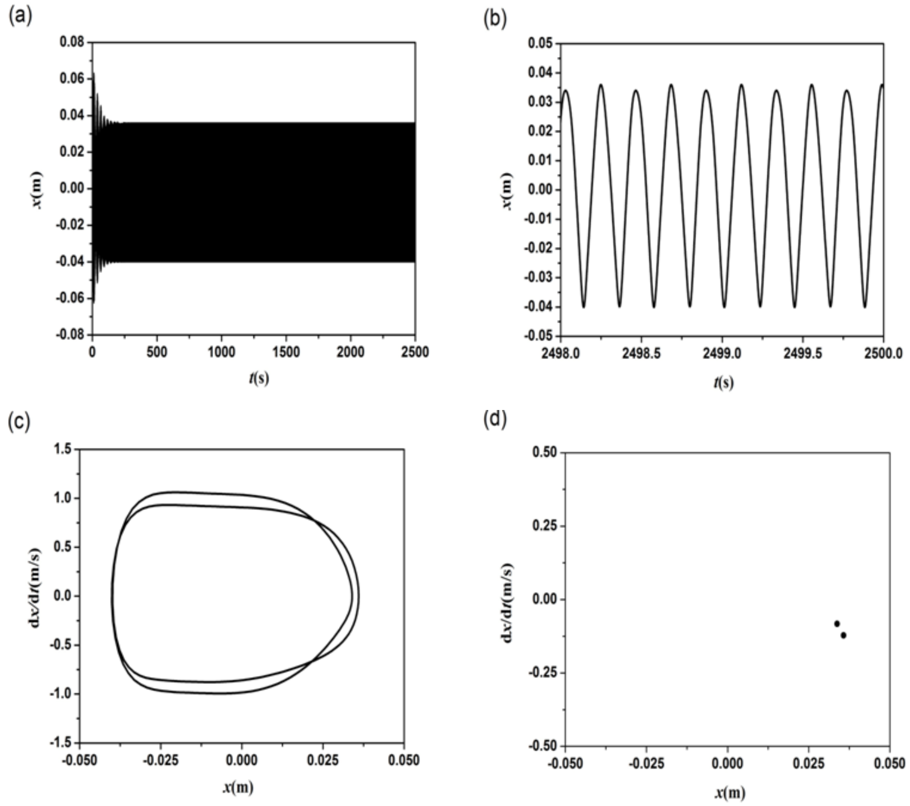
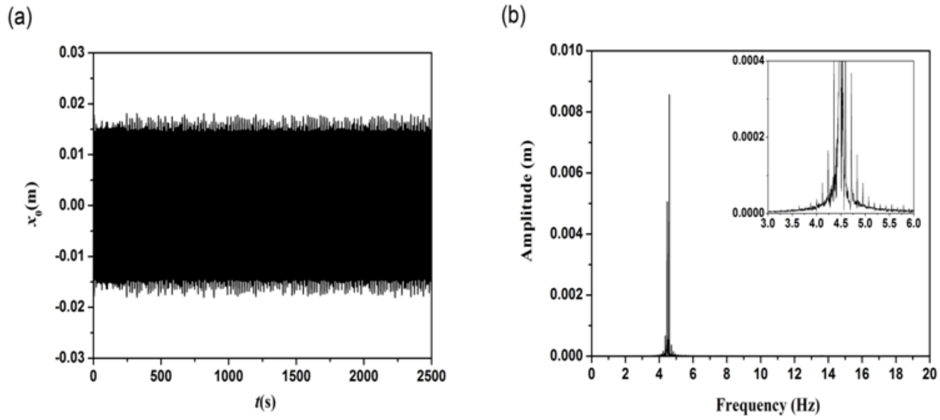


Figure 7: Period-2 motion of the energy sink for *m*=0.3384kg (a) the time history, (b) the enlargement of the time history, (c) the phase portrait, (d) the Poincaré map.



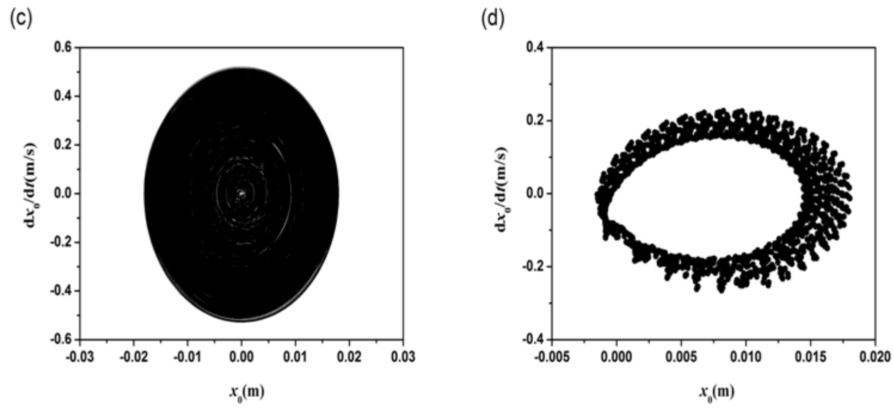


Figure 8: Chaotic motion of the structure for *m*=0.72kg (a) the time history, (b) the amplitude spectrum, (c) the phase portrait, (d) the Poincaré map.

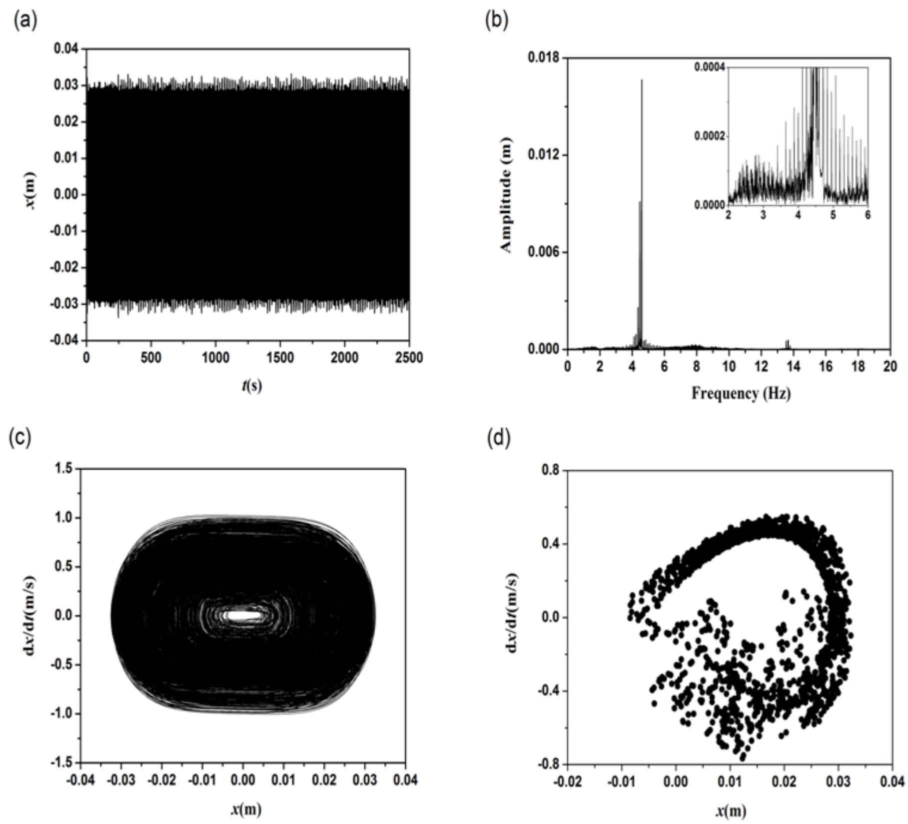
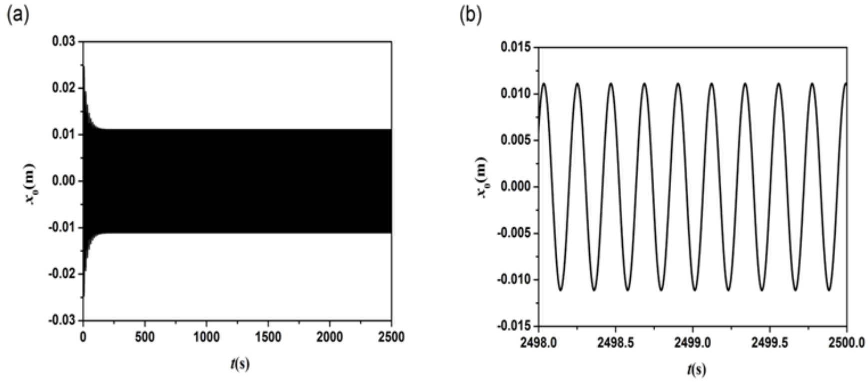


Figure 9: Chaotic motion of the energy sink for *m*=0.72kg (a) the time history, (b) the amplitude spectrum, (c) the phase portrait, (d) the Poincaré map.

For different cubic stiffness *k* with fixed *m*=0.5kg, periodic motions of the structure and the energy sink are respectively shown in Figs. 10, and chaotic motions can be found in Figs. 11.



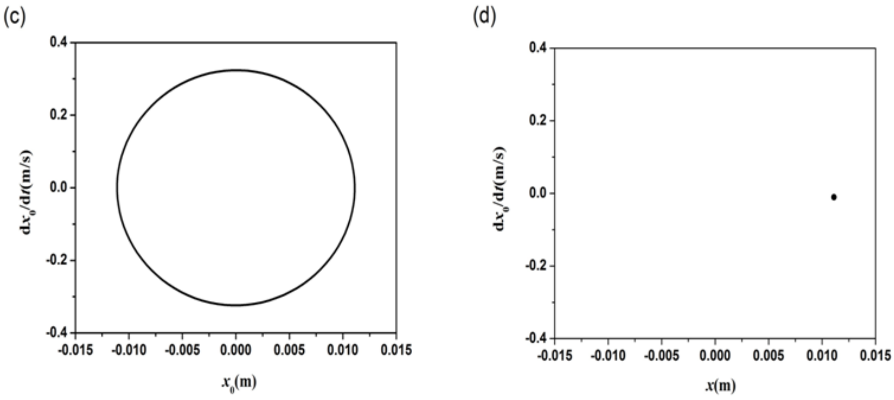


Figure 10: Periodic vibration of the structure for k=5600 kN/m^3 (a) the time history, (b) the enlargement of the time history, (c) the phase portrait, (d) the Poincaré map.

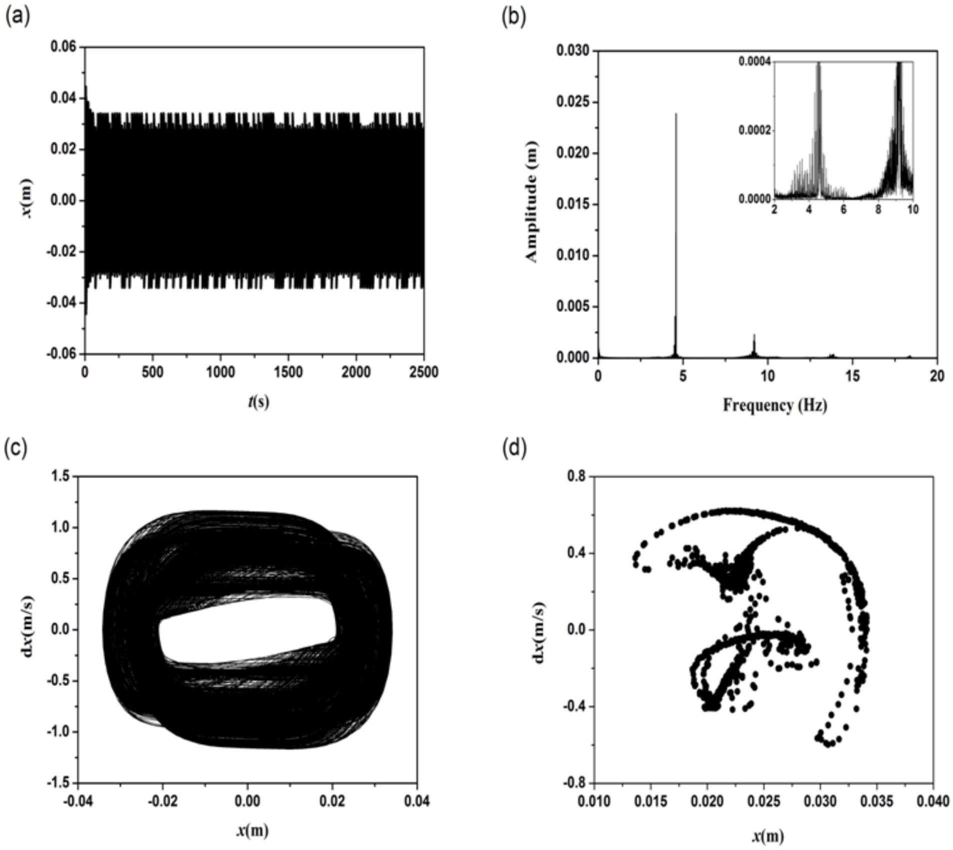


Figure 11: Chaotic motion of the energy sink for k=14400kN/m^3 (a) the time history, (b) the amplitude spectrum, (c) the phase portrait, (d) the Poincaré map.

5. Conclusions

The investigation treats steady-state response in forced vibration of a periodically excited linear structure coupled with a nonlinear energy sink. The bifurcations are numerically examined via the Poincaré map. Phase trajectories, amplitude spectrums and Poincaré maps are used to identify dynamical behaviours. The investigation demonstrates that a nonlinear energy sink may result in chaotic motion of the structure and the nonlinear energy sink and the dynamical behaviours bifurcate with the varying mass the cubic stiffness of the nonlinear energy sink.

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