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A TWO-WAVE TYPE MODEL FOR ESTIMATING THE DYNAMICAL RESPONSE OF FRAME STRUCTURES

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As part of a scheme aimed at the reduction of flanking vibration in buildings by active damping techniques one derivative method of predicting the modal resonances of framed structures utilised a method of wave analysis. Earlier work by Budrin, Nikiforov, Kihlman, and others had established the need to include at least both fluxural and longitudinal wave types at intersecting nodes either of plates or beams. Such analyses were restricted to single junctions, and the present work follows the study by Bhattacharya, Mulholland, and Crocker (1) who calculated energy flow via a "two-joint" model where a pair of infinite plates were connected by a tie bar. The simplification arising from the absence of reflected travelling components or near fields at infinity is acceptable where the concern is with averaged statistical flow. However in the present report, related to damping methods, some prediction of resonances in a finite structure is important, and reflected wave components must be generally included.

Model of frame

Figure (1) shows the rectangular space frame driven by a harmonic forcer of amplitude $\bar{F}$ at the node formed by elements 1, 3, and 4. Transverse flexural waves are present in the two principal planes of the assumed square section, whilst longitudinal wave components are generated in axial directions.

If v_i and w_i are the transverse displacements in y and z directions for the i^{th} element member, then the standard wave equation solutions for forward and reflected waves are given, on dropping the term $e^{-j\omega t}$

$$v_i = A_i e^{jk_i x_i} + B_i e^{-jk_i x_i} + C_i e^{-k_i x_i} + F_i e^{-(k_i l_i - k_i x_i)} \quad \text{in eight equations}$$

$$w_i = I_i e^{jk_i x_i} + J_i e^{-jk_i x_i} + L_i e^{-k_i x_i} + M_i e^{-(k_i l_i - k_i x_i)} \quad \text{in eight equations}$$

For longitudinal displacements in the i^{th} member

$$u_i = G_i e^{jp_i x_i} + H_i e^{-jp_i x_i} \quad \text{in eight equations}$$

where wave numbers $k_i = \omega/C_F$; $P_i = \omega/C_L$ and x is generally an axial coordinate.

The eighty complex wave coefficients A_i etc are obtained by solving the sufficiently available equations given by the element-end conditions of
(a) Continuity of linear displacement (b) Angular displacement
(c) Equilibrium of axial and shear forces in each axial direction.
(d) Equilibrium of bending moments about each axis. Conditions (c) and (d) discount inertias at the nodes, as does the displacement solution v_i etc discount beam rotational inertia.

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In the case of the joint (1, 4, 3), Figure 2, from which a corner cube has been removed (Figure 2) to show the sign convention adopted, the above conditions give:

(a) Displacements: $u_1 = -w_4$; $u_1 = -w_3$; $u_3 = v_4$; $u_3 = w_1$; $u_4 = -v_1$; $u_4 = -v_3$.

(b) Angles: $\frac{dw_1}{dx_1} = \frac{dw_3}{dx_3}$ plus two similar equations for remaining axes.

(c) $E_1I_1\frac{d^3w_1}{dx_1^3} + E_4I_4\frac{d^3v_4}{dx_4^3} + E_3A_3\frac{du_3}{dx_3} = -\bar{F}$, plus two further equations with zero external force.

(d) $E_3I_3\frac{d^3v_3}{dx_3^3} + E_4I_4\frac{d^2v_4}{dx_4^2} - T_1\frac{dv_1/dx_1}{dw_1/dx_1} = 0$, plus two further equations.

T_1 is the torsional constant for member 1, and in the last term v_1 and w_1 are only relatable via a mutual dependence on the coordinate x_1.

A two dimensional model (first used as an experimental confirmatory excercise) can be used to illustrate the sign convention used.

Of this set of fifteen relationships per mode, typical relationships are:

Type (a) $G_1 + H_1 + I_4 + J_4 + L_4 + M_4e^{-k_4l_4} = 0$

Type (b) $jI_1 - jJ_1 - L_1 + M_1e^{-k_3l_3} - jI_3 + jJ_3 + L_3 - M_3e^{-k_3l_3} = 0$

Type (c) $-jI_1 + jJ_1 - L_1 + M_1e^{-k_1l_1} - jA_4 + jB_4 - C_4 + Fe^{-k_4l_4}$

$+ j\,(Ap_1/Ik_1^3)(G_3-H_3) = -\bar{F}\,/(EIk_1^3)$

with similar relationships of type (d).

A further six relations based on zero slope and deflection at one grounded end, of which five only are necessary, donate twenty equations per node, to provide eighty in all for a solution of the full set of wave coefficients A_i etc. A further 16 wave coefficients due to torsional wave presence have been excluded since the upper limit to the computing viability was already near. The matrix size was condensable to 66 square by eliminating 12 chosen coefficients in terms of others, whose solution allowed back substitution for the eliminated terms.

Comments: In this approach, the solution for unknown wave coefficients provides individual displacements v_i, w_i, u_i for any location x via the wave equation. This contrasts with the more usual discretised stiffness/mass matrix solution where eigenvalue, eigenvector derivation will give the forced amplitudes via a normal mode solution only at the nodes. In the presented method the response $v_1(x)$, $w_1(x)$, is computed over a sweep of frequencies which also defines the natural frequencies via the resonance infinity values.

In contrast with the stiffness/mass matrix approach which must include transformations from local element to global system coordinates, the presented method retains only local element descriptions since the effective correlation of element axes is maintained in the continuity of displacement and angle of the end conditions (a) and (b).

One calculated result is given below for the drawing point (1, 4, 3) in the case of steel rig of hollow inch-square section with dimensions l_1 1.5m, l_4 2m, l_3 1.5m.

(1) M.C. BHATTACHARYA, K. MULHOLLAND and M.J. CROCKER 1971 Propagation of sound energy by vibration transmission via structural junctions. Jl Sound and Vibration 18(2), 221.

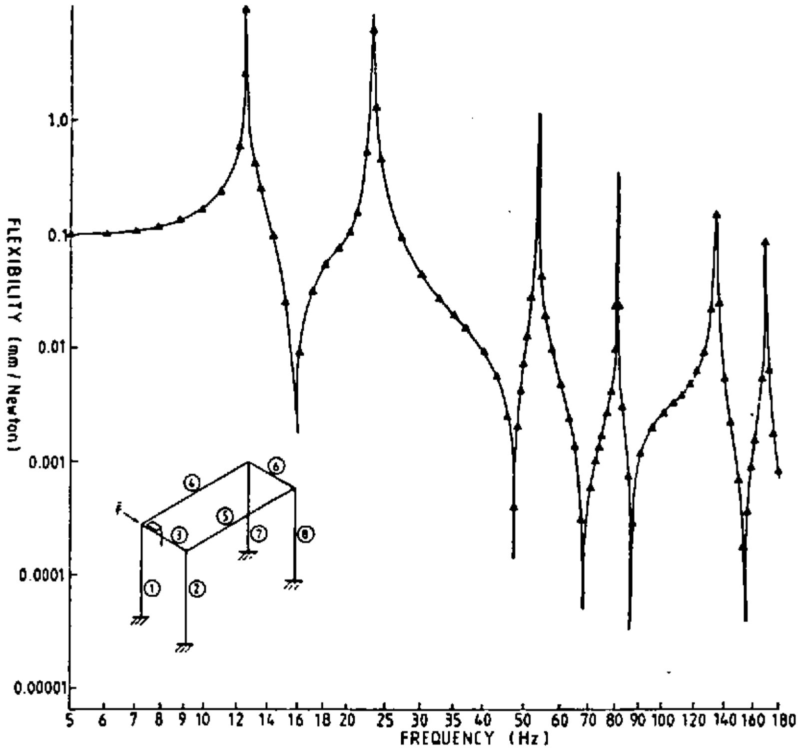

FIGURE 3

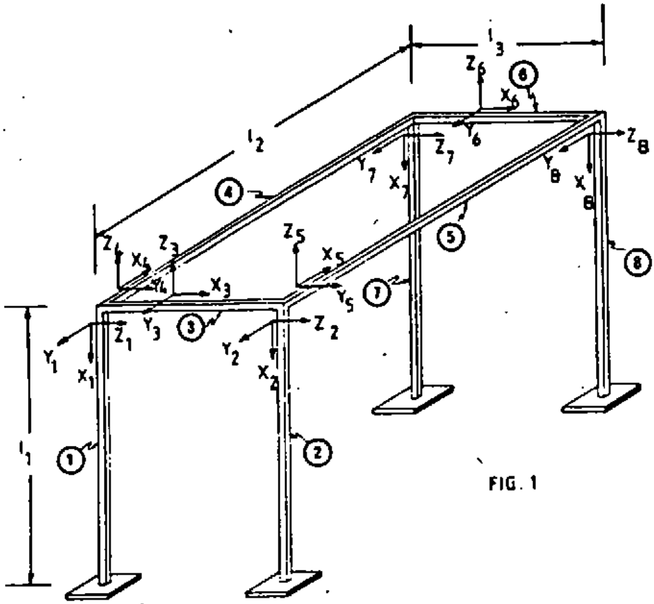

FIG. 1

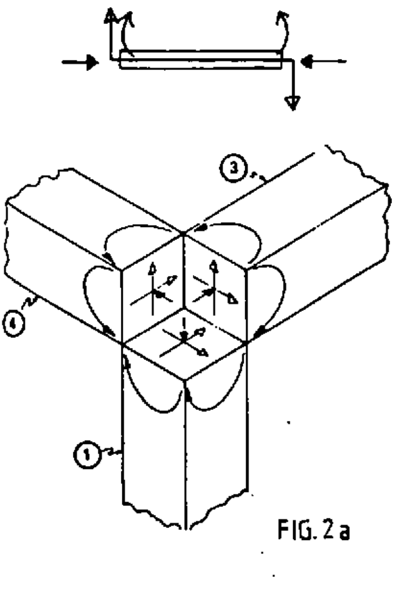

FIG. 2a

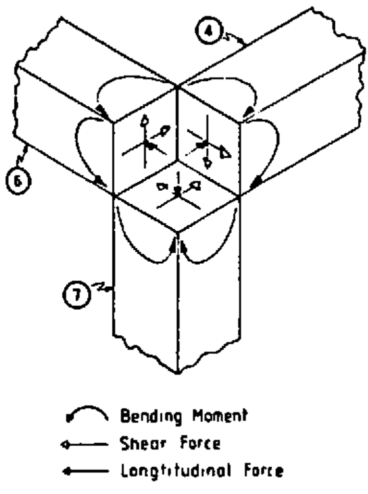

FIG. 2b