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THEORETICAL ASPECTS OF THE SEA-FLOOR REVERBERATION

BY

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ABSTRACT

This paper presents a new theory which extends the wave-expansion formalism to irregular surfaces of arbitrary roughness without the complexity of additional terms which are required in the theories based on the same principle.

The theory is viewed against the back-ground of currently employed models and the experimental data.

NOTATION

λ = $K \cos \theta$

$\underline{u}$ = $K \sin \theta$

K = $2\pi/\lambda_a$, λ_a = acoustic wave-length

θ = angle of incidence

$\langle \ldots \rangle$ = ensamble average

$R = \frac{N \cos \theta - fM}{N \cos \theta + fM}$; Rayleigh reflection coefficient

f = $\left[1 - M^{-2} \sin^2 \theta\right]^{\frac{1}{2}}$

N = ρ^1/ρ ; $M = c/c^1$

ρ^1; c^1 = density and speed of sound in the lower medium

ρ ; c = density and speed of sound in the upper medium

$\underline{A} \cdot \underline{B}$ = scalar product

σ = r.m.s. of surface elevations

$z = \zeta(\underline{x}) = \zeta(x; y)$, 2 - dimensional rough surface

$C(\underline{x}) = C(x; y)$ = correlation function of $\zeta(\underline{x})$

Ko = incident wave-number

K = scattered wave-number

$\underline{x}$ = $(x; y)$ = horizontal coordinates

R_c = local radius of curvature of $\zeta(\underline{x})$

L = horizontal roughness scale of $\zeta(\underline{x})$

a = particle size

$\mathcal{R}$ = $R + \frac{2N(N-1)}{(N \cos \theta + fM)^2} \sin^2 \theta$; Rayleigh modified reflection coefficient

H.O.T. = Higher Order Terms

$f(\underline{u})$ = amplitude coefficients of the scattered field

α = roughness parameter; denotes the degree of $\zeta(\underline{x})$ modulations

$P(\underline{u})$ = $\frac{\sigma^2}{4\pi^2}\iint C(\underline{x})\, e^{-i2\underline{u}\cdot\underline{x}}\, dxdy$; elevation spectrum of $\zeta(\underline{x})$

$T[\underline{u};\alpha] = \frac{1}{4\pi^2}\iint \exp\left[-4\lambda^2\sigma^2\left\{1 - C(\underline{x})\right\}\right] \times e^{-i2\underline{u}\cdot\underline{x}}\, dxdy$;

T - spectrum

[·] = denotes reference

$Q(\underline{K};\underline{K}o)$ = general scattering strength function

$Q(\underline{K}=\underline{K}o)$ = monostatic (back-scatter) scattering strength function

1.0. Introduction

Reverberation is conveniently divided into three groups: 1) sea-surface, 2) volume and 3) sea-floor reverberation. It is the latter that will be discussed through the scattering strength function defined by (1.1) for a monostatic (back-scatter) configuration, in the plane of incidence

$$Q(\underline{K}=\underline{K}o) = \lambda^2 \left\langle \left| S(\underline{u};\, \underline{u}_o) \right|^2 \right\rangle \qquad (1.1)$$

where $\lambda = K \cos\Theta$ and $S(.)$ denotes a scattering matrix the elements of which correspond to various directions (modes) into which the incident field is fragmented.

Currently, there are three conceptually different approaches to evaluate (1.1); 1) H-K theory which is based on the Helmholtz integral within the Kirchoff's approximation [1], 2) W.E. Theory initialed by Rayleigh and subsequently extended and generalised by Kuo [2] and 3) phenomenological approach [3] based on the assumption that the scattering surface, $\zeta(\underline{x})$, may be replaced by the individual scatterers superimposed on the flat boundary.

In this paper only the first two models will be discussed to present the background for proposing a new theory which will be called a Phase Spectrum, (P.S.), theory. This new theory is based on the wave-expansion principle. The (P.S.) theory not only combines some of the features from both H-K and W.E. models but offers some interesting interpretations which are not readily inferred from these two theories.

2.0. H-K Thoery

In this formalism (1.1) is given by (2.1)

$$Q(K=Ko) = \frac{\langle |R|^2 \rangle}{4\pi^2} \frac{K^2}{\cos^2\Theta}\, T[\underline{u};\alpha] \qquad (2.1)$$

and is subject to the condition (2.2)

$$2 R_2 K \cos\Theta \gg 1 \qquad (2.2)$$

for local flatness [4] and to (2.3)

$$\frac{\sigma}{L} \ll \frac{L}{\lambda_a}^2 \cos\Theta \qquad (2.3)$$

to exclude shadowing effects [4]

Although (2.1) applies to very rough (highly modulated) $\zeta(\underline{x})$ conditions (2.2, 3) restrict (2.1) to a gently sloping surface without any sharp edges. It also requires that the (horizontal) roughness scale is many times the acoustic wave-length. Equation (2.1) applies to the far field conditions.

3.0. W.E. Theory

In this approach the same problem is handled quite differently. The scattered field, in the plane of incidence, is postulated and the relevant boundary conditions are satisfied exactly. This is in contrast to (2.1) which is obtained by the reverse process for a transmitting and statistically rough $\zeta(\underline{x})$, Kuo [2] obtained (3.1).

$$Q(\underline{K} = \underline{K}_o) = \lambda 4|\hat{R}|^2 \; P(\underline{u}) + \text{H.O.T.} \qquad (3.1)$$

In principal (3.1) applies to $\zeta(\underline{x})$ of <u>any</u> slope and modulation. For practical purposes however it is really useful when the H.O.T. terms are discarded. Although under this condition (3.1) applies to $\zeta(\underline{x})$ of small slopes and modulation it is <u>not</u> subjected to the restrictive conditions (2.2, 3) required by (2.1).

4.0. Status of the H-K, and W.E. Theories

At present (2.1) is more popular than (3.1). This primarily is due to the fact that the highly modulated $\zeta(\underline{x})$ are of more interest than the slightly modulated boundaries, although (2.1) is a subject to more fundamental shortcomings than (3.1).

To apply (2.1) correctly in any real situation (2.2, 3) must be satisfied. It is not difficult to see that (2.1) may fail at the low frequency limit on account of (2.2). At the high frequency limit (2.1) will probably be satisfied but on account of (2.3) the H-K model, (2.1), will be restricted to a gently sloping surface. The validity of (2.1) will become more precarious when applied to a multiply structured $\zeta(\underline{x})$.

A frequent criticism levied against (3.1) is that the assumption of the plane-wave superposition (4.1), for the scattered field,

$$\int f(\underline{u}) \exp\left[i\ (\underline{u}.\underline{x} + (u)\ z\right]\ du_x\ du_y \qquad (4.1)$$

on which (3.1) is based fails to be true within the irregularities of $\zeta(\underline{x})$. It may be shown [5,6] that in this region there is an additional set of waves known as the inhomogeneous waves which decay exponentially when the point of observation recedes from the boundary. It is not frequently realised that the same situation exists within the H-K formalism. This stems from the fact that (2.1) is based on a very simple Green function [7] which is required to develop the H-K integral. This function is obtained in such a way that it corresponds to the outgoing wave at a large distance from $\zeta(\underline{x})$; no account is taken of the inhomogeneous waves.

Secondly the H-K integral on which (2.1) is based hinges on the Green's theorem which is valid for continuous fields, [7]. The essence of Kirchoff's approximation is such that this continuity requirement is <u>not</u> preserved.

To remove these defects within the H-K formalism would require to invoke a more comprehensive Green function. At the same time however, a clear physical picture would be lost. This is in contrast to the W.E. approach.

The experimental data suggest the correctness of (3.1) rather than (2.1) throughout the whole range of angles of incidence. To improve the 'performance' of (2.1) a shadowing function is employed as a premultiplying factor to force (2.1) to follow the field data.

It is clear therefore that if the W.E. formalism could be extended to very rough $\zeta(\underline{x})$ without the computational complexity associated with (3.1), when applied to the highly modulated boundaries, it would be preferable to the integral formulation (2.1). This in fact is possible and is set forth in the next section, 5.0.

5.0. Phase spectrum theory

This theory is based on the Rayleigh's principle. The scattered and scattered-transmitted fields are postulated in accordance with (4.1). To evaluate (1.1) boundary conditions of continuous pressure and normal velocity across the $\zeta(\underline{x})$ are developed in terms of the incident, scattered and scattered-transmitted fields. Subsequently a pair of simultaneous equations is obtained in which the exponential function (5.1) is of fundamental importance.

$$\exp\left[-i\alpha\zeta(\underline{x})\right] \qquad (5.1)$$

Equation (3.1) is based on expanding (5.1) into power series in terms of σ and $\zeta(\underline{x})$, [2]. Although (5.1) converges for all the values of its argument the convergence is very slow for $\alpha \gg 1$ and many terms are required. It is for this reason that (3.1) is practically useful for $\alpha \ll 1$, gently sloping and slightly modulated $\zeta(\underline{x})$.

To circumvent this difficulty it is postulated, [8], that

$$\exp\left[-i\alpha\zeta(\underline{x})\right] = \iint B(\underline{u};\alpha)\, e^{\,i\underline{u}\cdot\underline{x}}\, du_x\, du_y \qquad (5.2a)$$

$$B(\underline{u};\alpha) = \frac{1}{4\pi^2}\iint e^{-i\alpha\zeta(\underline{x})}\, e^{-i\,\underline{u}\cdot\underline{x}}\, dx\, dy \qquad (5.2b)$$

The equations arising from the boundary conditions are expressed in terms of the 'spectral coefficients $B(\beta)$, (5.2), and a general solution is obtained for $Q(\underline{K};\underline{K}o)$ which in principle applies to $\zeta(\underline{x})$ of any slope modulation and statistics, [9].

To proceed it is necessary to introduce some restrictions on the general solution. It will therefore be assumed that $\zeta(\underline{x})$ will be restricted to be gently undulating (small slopes) but of <u>any</u> modulation within the Gaussian statistics. Also the observational configuration will be confined to the monostatic (back-scatter) geometry. Under these conditions it is found [9] that $Q(\underline{K};\underline{K}o)$ reduces to

$$Q(\underline{K}=\underline{K}o) = \lambda^4\left[|\mathcal{R}|^2 - |R|^2\right] P(\underline{u}) + $$
$$+\lambda^2 |R|^2 \; T\left[\underline{u};\alpha\right] \qquad (5.3)$$

This is a very interesting result when it is recalled that apart from the pre-multiplying factor the 1st and 2nd term in (5.3) depict the scattering characteristics of a slightly, (3.1), and highly (2.1), rough $\zeta(\underline{x})$. These two terms are combined linearly. It could therefore be argued that the small and large components of gently sloping $\zeta(\underline{x})$ independently scatter the incident field.

It follows from the development of the theory [9] that the assumption of small slopes implies a 2nd order statistics. This in turn suggests that to handle $\zeta(\underline{x})$ with large slopes a higher order statistics will ensue. It is readily seen that the P.S. theory (5.3) is not the same for both pressure release and rigid surface. This stems from the modified Rayleigh's reflection coefficient. This in contrast to the H-K model, (2.1).

Similarly, $Q(\underline{K};\underline{K}o)$ and $Q(\underline{K}=\underline{K}o)$ may be readily obtained for the scattered-transmitted field to ascertain the scattering characteristics from within the medium.

Since (5.3) is based on the Rayleigh's principle, (4.1), formally it is not subject to (2.2, 3). When the source or receiver or both are situated close to or within the irregularities of $\zeta(\underline{x})$ it may be prudent to verify the relevant boundary conditions to test the validity of the postulated fields.

6.0. Application

To apply (5.3) it is fundamental to determine a realistic statistical representation of $\zeta(\underline{x})$. To simplify matters it will be assumed that $\zeta(\underline{x})$ is singly structured. When the model of $\zeta(\underline{x})$ is based on the concept of discrete arrangement of particles described as a Markoff random process it is then found [9] that

$$P(\underline{u}) \simeq \frac{a}{3\pi} (\underline{u})^{-3} \qquad (6.1)$$

and

$$\sigma^2 \left[1 - C(\underline{x})\right] = \frac{2}{3} \; a \; \underline{x} \qquad (6.2)$$

Surface elevation spectrum, (6.1), applies only where inequality (6.3) is satisfied

$$a \ll \text{acoustic wave-length} \qquad (6.3)$$

otherwise one would have discrete scattering objects and not the assembly.

When (6.1, 2) are applied to (5.3) and $\zeta(\underline{x})$ is assumed to be isotropic, (5.3) reduces to (6.4).

$$Q(K = Ko) = \frac{1}{3} \frac{a}{\lambda_a} \cos^4 \theta \left\{ \left[|\mathfrak{R}|^2 - |R|^2 \right] (\sin^3 \theta)^{-1} + |R|^2 \left[70 \left(\frac{a}{\lambda_a}\right)^2 \cos^4 \theta + \sin^2 \theta \right]^{-3/2} \right. \qquad (6.4)$$

Equation (6.4) represents a closed form approximation to (5.3)

7.0. Theory vs. Experiment, [9]

Experimental verification is very encouraging indeed. At this stage it is more significant to note the similarity in shapes rather then the absolute levels.

At high frequencies both absolute values and shapes correlate quite satisfactorily. At low frequencies the shapes remain compatible. The levels are somewhat different. This is not really surprising since the (P.S.) theory, (5.3), primarily depicts the interfacial scattering. At low frequencies there will be some volume scattering from within the sea-bed, which is not included in the P.S. theory, (5.3).

8.0 Conclusions

A theory has been developed which extends the W.E. formalism to gently sloping $\zeta(\underline{x})$ of any modulation. It is also indicated that for this type of $\zeta(\underline{x})$ the small and large components of $\zeta(\underline{x})$ scatter the incident field independently. For $\zeta(\underline{x})$ with large slopes higher order statistics may be more appropriate.

The theory-experiment verification is quite encouraging subject to acoustic and statistical data on $\zeta(\underline{x})$ being available.

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