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ACOUSTICS OF FLUID-FILLED, SMALL CAVITIES : THEORETICAL MODELS AND APPLICATIONS.

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ABSTRACT

The purpose of this paper is to present the main developments in the theory of the acoustics of dissipative fluids at rest in bounded spaces (principally cavities of different sizes and shapes), from the early beginnings (in the 1850's) until now, including a brief indication of possible approaches for future developments.

Several applications of interest, and current research concerned with miniaturisation, are presented, with emphasis being given to the work done on the acoustic gyrometer and on its potential to be miniaturized, which includes the miniaturization of acoustical transducers (on silicon chips).

1. INTRODUCTION

The aim of this paper is to outline the main developments towards obtaining a description of the linear, dynamic behaviour of viscous heat-conducting compressible fluids at rest. It will deal with bounded spaces in general but focus particularly on small cavities and thin fluid layers. For more than 100 years, there has been a very strong motivation to get a good description of wave propagation in tubes, which takes into account the dissipative effects, as a lot of well known applications require good theoretical models for waveguides (including the widely used capillary tubes). In addition, since the early 1930's there has been increasing attention given to small cavities, particularly for the design and calibration of transducers [1,2,3,4,5,6], and since the 1970's, for measuring the properties of gases from precise measurements of the speed of sound [7,8]. More recently there have been devices like the acoustic gyro [9], and the application of trapped thin fluid layers for vibration damping and the optimisation of transducers performance.

Generally speaking, the need for accurate models of the acoustic fields in smaller and smaller fluid-filled cavities, fluid films or guides is becoming very important as the growing demand for miniaturized tools (transducers, etc.), and the widespread use of silicon technology, has focuses our interest on designs on silicon chips.

The purpose of the paper is three fold: -first to show the developments in the theory related to acoustics in dissipative fluids in bounded spaces, with a brief historical outline then a more in-depth discussion of the advances of the last ten years (section 2), -secondly to give examples of applications which need models derived from the theoretical results, attention being focused on the acoustic gyro (section 3), -and thirdly to describe the work that is required by the current levels of miniaturization and from new, non conventional demands on the behaviour of the devices properties (concerning the response, frequency range, precision, etc.), which will motivate future research (section 4).

ACOUSTICS OF FLUID-FILLED SMALL CAVITIES

2. DEVELOPMENTS IN THE THEORETICAL ASPECTS;SHORT HISTORY.

2.1 The early studies on dissipative phenomena in fluids (1820-1900).

Energy loss from the sound wave in fluids can basically be separated into three distinct processes : -the transfer of momentum from the sound wave due to the effect of viscosity -the transfer of heat from the regions of "high" temperature to those of lower temperature, converting the energy of the sound wave into random thermal motion of fluid molecules -the transfer of energy from the translational energy of the fluid to internal energy modes and back which takes place during collisions between the fluid molecules, this process of transferring energy back and forth introducing a time lag between the time needed for the local pressure to decrease and the time needed for the internal energy modes to give back to the translational modes [11,12,13].

The theory has a relatively short history dating from the 1820's. The subjects of heat conduction and shear viscosity were the first to be treated (1822-1845) although a thorough understanding of the mechanisms of acoustic dissipation only come about somewhat later (1868-1899).

The first fundamental investigation was that concerning the diffusion of heat from regions of "high" temperature (linked, in acoustics, to compressions) to those of lower temperature (linked, in acoustics, to rarefactions). Jean B.J. Fourier (1768-1830) was involved with this problem at the very beginning of the 19th century, but he completed his work on heat flow only in 1822 in a book entitled "Analytic Theory of Heat" (note that at the same period he also discovered what is now called Fourier's theorem) [14].

The second investigation to be noted on dissipative phenomenon which is another remarkable one, is that by Stokes on the dissipation due to the shear viscosity effects (1845) [15] who gave us the form of the dissipative shear viscosity term in the fundamental equation of dynamics. Sir George G. Stokes (1819-1903) worked on the theory of viscous fluids between 1845 and 1850. He deduced what is now called Stokes's law that could be applied to the motion of a small sphere falling through a viscous medium to obtain its velocity under the influence of a known force such as gravity. (His work also included studies on fluorescence, sound, and on light; he was amongst the first to suggest in 1896 that the X-rays, newly discovered by Roentgen, were an electromagnetic radiation akin to light.)

But it is Kirchhoff's pioneering papers of 1868 [16], treating the viscous and thermal dissipation effects in acoustics, that can be considered to mark the beginning of modern theory of acoustic propagation in viscous and thermal conducting fluids. Gustav R. Kirchhoff was born on 12th March 1824 and died on 17 October 1887. Kirchhoff's major contribution to physics was his experimental discovery and accompanying theoretical analysis in 1859 of a fundamental law of electromagnetic radiation : for all material bodies, the ratio of absortive and emissive power for such radiation is a universal function of wavelength and temperature. He introduced later (1862) the concept of a black body. Outstanding among his other contributions was his early work on electrical currents (1845-1849) and on the propagation of electricity in conductors (1857). Kirchhoff was a master at formulating a logical concept of physical phenomenon thus leading to a coherent systems free from hypothetical elements, and in 1868 he produced the very fertile description of sound propagation in gases mentioned above. This theory was mainly based on the Navier-Stokes equation which includes the effects of shear and bulk viscosity, and the Fourier equation of heat conduction modified to account for linear acoustics approximations. He derived an algebraic equation (dispertion equation) and he found a solution to it,that is, the propagation constant, for plane waves and outgoing spherical waves in an unbounded medium and for waves propagating along the axis of a circular tube. In the latter case, he assumed boundary conditions of zero particle velocity and acoustic temperature at the tube wall. Then, for the case of wide tubes and long wavelengths compared with the thickness of the boundary layers, he calculated the attenuation factor and speed of propagation from the real and imaginary parts of the propagation constant respectively, which are solutions of the dispersion equation, making approximations to the lowest orders possible.

Lord Rayleigh has included a detailed account of this theory in his book "Theory of Sound"[17], which is a "vademecum" in every acoustical research laboratory. This paper would not be complete without reference to Rayleigh (but it is not superfluous as we will see below). Nobel prize winner in physics in 1904, Lord Rayleigh (baron J.W.Strutt), was born on 12th November 1842 and died on 30th june 1919. He published papers, reports on experimental and theoretical work on optical and acoustical radiation, electromagnetism,general mechanical theorems, vibrations of elastic media, capillarity and thermodynamics. An illustration of Rayleigh's uncanny ability to forecast developments in physics is provided by his paper (1899)"On the cooling of air by radiation and conduction and on the propagation of sound". In this he addressed the problem of the anomalously high sound attenuation observed in air (much greater than that predicted by the transport properties of viscosity and heat conduction). He predicted that the solution to the difficulty might be found in a relaxation mechanism involving reciprocal transfer of energy between translational and internal energy states of the molecules of the gas through which the sound passes. This suggestion was adopted by various subsequent investigators and led to the establishment of the vigorous field of molecular acoustics, which by the second half of the twentieth century had thrown new and important light not only on ultrasonic propagation but also on the structure and interaction of molecules [18].

Thus at the end of the last century, the basic equations and ideas were available to interprete the so-called classical loss mechanisms (that is those due to the viscous and thermal conduction effects), and understand the molecular relaxation mechanism. A complete calculation of sound absorption would necessarily include not only the contribution of each mechanism singly but also their interaction. Fortunately, for frequencies below 10 MHz, absorption due to classical losses and molecular relaxation are additive in gases. For a long time this result was assumed but it has been demonstrated recently (1972) [19] by making successive approximations to a solution to the Boltzmann equations. In addition, as we will see later, in waveguides or cavities such as we are concerned with here, most of the power loss occurs within the thin boundary layers, through viscosity and thermal conductivity. Then, under normal conditions, molecular relaxation can be neglected in the dissipation process [20] . Nevertheless, as we will see in the basic equations (section 2.3), the molecular relaxation effects may be taken into account by simply assuming that the heat coefficient at constant pressure is a complex number depending on the relaxation time, without changing anything in the formal solution of the problem. Therefore, as the main thrust of this paper deals with the acoustic propagation in small bounded spaces, it appears that we are concerned only with research devoted to viscous and heat conductive dissipation processes. The work of Kirchhoff on acoustic propagation appears to be a starting point for much of the subsequent research bearing on acoustic fields in bounded visco-thermal media.

2.2 Studies on acoustic propagation in visco-thermal fluid in the twentieth century, until the eighties.

Half of the twentieth century was to pass before any pertinent investigation provided us with a model sufficiently adequate for solving most of the problems of interest such as the propagation of all kind of modes, both propagating and evanescent, in waveguides. It is in fact only eighty years after the publication of the major paper by Kirchhoff, that Lothar Cremer (1905-1990) published an article entitled "On the acoustic boundary layer outside a rigid wall" [21] showing that, for the reflection of a harmonic plane-wave on an infinite rigid plane wall. the acoustic behaviour of the medium, in the neighbourhood of the boundary but outside the boundary layer, is adequately described by a simple propagational acoustic mode.and that most of power losses occur through viscosity and thermal conductivity within the boundary layer. As a consequence, assuming the temperature fluctuations and the particle velocity to be nearly equal to zero at the boundaries, the properties of plane-wave reflection on a plane surface are described by the ratio of the normal component of the acoustic particle velocity to the acoustic pressure at the boundary, called the "apparent specific admittance", determined by the shear viscosity and the coefficient of thermal conduction, and the ratio of the acoustic wavenumber to its component normal to the plane of the wall (that is, on the angle of incidence for a propagating mode). This is an innovative result as is demonstrated underneath (section 2.3.b, equation 25) which has significant consequences. We would like to mention briefly here that Cremer was also innovative in other fields in acoustics such as building acoustics,where his attention ranged from structure-borne sound, including impact noise insulation, sound attenuation in ducts, the transmission loss of simple and double walls and cylindrical shells, the theory of floating floors and the effects of sound bridges in building structures. In concert halls acoustics, he has been responsible for the design of a number of well-known halls, including the Berlin Philharmonic, the Opera House in Munich and the Liederhalle in Stuttgart ... As for musical acoustics, he has studied the behaviour of organ pipes and violins. Finally he has contributed to the theory of electromechanical transducers and to the field of psychological acoustics.

Coming back to the Cremer's equivalent boundary layer admittance, the first subsequent development was produced by Beatty two years later [22]. Making use of Cremer's result, he extended the classical Kirchhoff approximate result for the attenuation of harmonic plane waves in rigid walled tubes to the case of higher order propagating modes, but the theory is not valid near the adiabatic cutoff frequency and for evanescent modes. Developments of this work giving solutions, that is giving the approximate value of the propagation constant, at the cutoff frequency and for evanescent modes, have been made recently (1985-1988) [23 to 26]. As an example of these results, figure 1 shows the ratio of the actual attenuation coefficient α of the (m,n) modes to the attenuation coefficient of the plane wave mode (Kirchhoff) versus frequency parameter (ratio of the frequency to the adiabatic cutoff frequency), for a circular tube 0.2m in diameter.

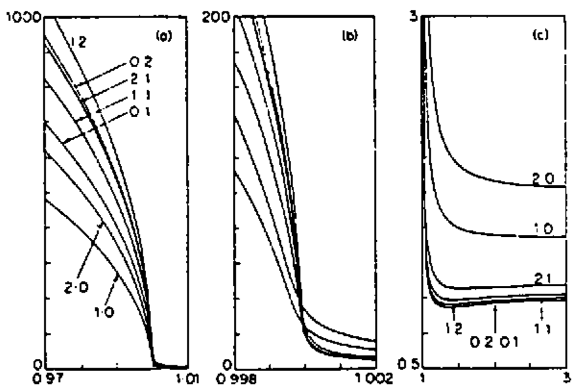

Fig. 1 : Attenuation ratio of (m,n) modes versus frequency parameter for circular tube.

Another extension of the Kirchhoff theory to spherical shells published by M R Moldover & al. in 1986 [7] leads to an equation which determines the approximate complex resonance frequencies of the gas in the shell, which has application for measuring the speed of sound in gases with high precision.

All of these results (and there are others), have been derived directly from the Kirchhoff or/and Cremer results mentioned above. In addition, the work carried out on the acoustic gyrometer [9] (that we will introduce under) largely makes use of the Kirchhoff and Cremer results.

However, recent works on the acoustics of very thin fluid layers or miniaturized fluid-filled cavities [27], and the need for accurate models of the acoustic field in ever smaller fluid-filled cavities has focused our attention towards solutions of the basic equations that were not available (or not enough accurate) until now. The next subsection is devoted to this subject.

2.3 Advances of the last ten years on the acoustical behaviour of dissipative bounded media.

a-The basic equations.

A viscothermal fluid oscillating around some steady state can be described by a set of thermostatic parameters and a set of thermodynamical variables. The thermostatic parameters are the ambiant values P of the pressure, T of the temperature, and ρ of the density. The thermodynamical variables are the pressure variation p, the particle velocity $\mathbf{v}$, the variable part of the density ρ', the entropy variation s per unit mass and the temperature variation τ . The nature of the fluid is then accounted for by phenomenological quantities : the shear viscosity μ, the bulk viscosity η, the coefficient of thermal conductivity λ, the heat coefficients at constant pressure and constant volume per unit of mass C_p and C_v , their ratio γ , the increase in pressure per unit increase in temperature at constant density β, and the fractional decrease in volume per unit increase in pressure at constant temperature χ_T . Three types of source can set the fluid into oscillation : external forces per unit of mass $\mathbf{F}$, mass sources described by a rate of creation of fluid q per unit of mass, and heat sources described by a rate of heat creation per unit of mass r .

The following constitute a complete set of linear equations [11,12,13,28] :

-The Navier-Stokes equation:

$$\rho\partial_t\mathbf{v} + \mathbf{grad}\, p - (\eta+4\mu/3)\, \mathbf{grad}(\mathrm{div}\, \mathbf{v}) + \mu\, \mathbf{curl}\, \mathbf{curl}\, \mathbf{v} = \rho\mathbf{F} \quad , \qquad (1)$$

-The conservation of mass equation:

$$\partial_t\rho' + \mathrm{div}(\rho\mathbf{v}) = \rho q \quad , \qquad (2)$$

-The conservation of energy, which reduces to:

$$\rho T\partial_t s - \mathrm{div}\, (\lambda\, \mathbf{grad}\, T) = \rho r \qquad (3)$$

In addition, the thermodynamical state laws of the fluid allow us to express all thermodynamical quantities with only two independent variables, leading, for example, to the following state laws :

$$s = (C_p/T)\, \tau - (P\beta\chi_T/\rho)\, p \quad , \qquad (4)$$

$$\rho' = \rho\chi_T\, (p - P\beta\tau) \quad . \qquad (5)$$

The molecular relaxation effect may be taken into account here by the simple assumption that the heat coefficient at constant pressure C_p in equation (4) is a complex number C_p^* depending on the relaxation time θ ; for usual bi-atomic gases C_p^* is given by :

$$C_p^* = C_p - C_v^{(v)}\,\theta\,\partial_t/(1+\theta\,\partial_t) \quad , \tag{6}$$

where $C_v^{(v)}$ is the contribution of the vibration of the molecules to the heat coefficient at constant volume, and where the operator $(1+\theta\,\partial_t)^{-1}$ formally means

$$\theta^{-1} e^{-t/\theta} \int dt'\, e^{t'/\theta} \quad . \tag{7}$$

But in waveguides or cavities such as those which are our concern here, most of the power loss occurs within the boundary layers, through viscosity and thermal conductivity, and thus the molecular relaxation effect can be neglected in the dissipation process ($C_p^*=C_p$) .
Under the usual gauge conditions, the particle velocity $\mathbf{v}$ of any disturbances governed by this system of linear equations can be considered as a superposition of a rotational velocity $\mathbf{v}_v$ (due to viscosity effects) and a solenoïdal velocity $\mathbf{v}_l$, due to acoustic ($\mathbf{v}_a$) and heat conduction ($\mathbf{v}_h$) effects :

$$\mathbf{v} = \mathbf{v}_l + \mathbf{v}_v \quad , \quad \mathbf{v}_l = \mathbf{v}_a + \mathbf{v}_h \quad . \tag{8}$$

Consequently, equation (1) can be split into two equations in such a way that combining equations (1) to (3) along with the relations (4) and (5) yields, outside the sources :

$$\partial_{ct}\,\tau - (\rho c/\gamma\beta)\,\mathrm{div}\,\mathbf{v}_l = \beta^{-1}\,\partial_{ct}\,p \quad , \tag{9}$$

$$(\partial_{ct} - l_h\,\Delta)\,\tau = [(\gamma-1)/(\gamma\beta)]\,\partial_{ct}\,p \quad , \tag{10}$$

$$(\partial_{ct} - l_v\,\Delta)\,\mathbf{v}_l = -(\rho c)^{-1}\,\mathbf{grad}\,p \quad , \tag{11}$$

$$(\partial_{ct} - l'_v\,\Delta)\,\mathbf{v}_v = \mathbf{0} \quad , \tag{12}$$

$$\mathrm{div}\,\mathbf{v}_v = 0 \quad , \tag{13}$$

$$\mathbf{curl}\,\mathbf{v}_l = \mathbf{0} \quad , \tag{14}$$

where the characteristic lengths l_v , l'_v and l_h are defined as follows, with c being the speed of sound :

$$l_v = (\eta+4\mu/3)/(\rho c) \quad , \quad l'_v = \mu/(\rho c) \quad , \quad l_h = \lambda/(\rho c C_p) \quad . \tag{15}$$

It is convenient, for calculating the acoustical propagation, to find the homogeneous wave equation for p, τ and $\mathbf{v}_l$ (which of course is essentially redundant given the equations 9 to 12). One can demonstrate [29,30] that these quantities satisfy the same propagation equation. For example, the temperature τ can be written as the sum of an acoustic temperature τ_a and an entropic temperature τ_h which are respective solutions of the homogeneous equations :

$$[(\partial_{ct})^2 - (\Gamma+R)\,\Delta]\,\tau_a = 0 \quad , \quad [(\partial_{ct})^2 - (\Gamma-R)\,\Delta]\,\tau_h = 0 \quad , \tag{16.a-b}$$

where $2\Gamma = 1 + (l_v + \gamma l_h)\,\partial_{ct}$,

$$2R = \{1+2[l_v-(2-\gamma)l_h]\,\partial_{ct} + (l_v-\gamma l_h)^2\,(\partial_{ct})^2\}^{1/2} \quad .$$

Note that equation (16.b) is a diffusion equation because the Taylor expansion of the function (Γ-R) shows that the operator ∂_{ct} can be factorised : τ_h is associated with the heat transfer due to thermal conduction.
In the frequency domain (∂_{ct} =ik), equations (16.a-b) give the following :

$$(\Delta + k_a^2)\,\tau_a = 0 \quad , \quad (\Delta + k_h^2)\,\tau_h = 0 \quad , \tag{17.a-b}$$

where $k_a^2 = k^2(1+ikl_{vh}-k^2 l'_{vh} l_h)^{-1}$, (17.c)

$$k_h^2 = -ik[l_h(1-ikl'_{vh})]^{-1} \quad , \tag{17.d}$$

with $\quad l_{vh} = l_v + (\gamma-1)l_h \quad , \quad l'_{vh} = (\gamma-1)(l_h-l_v) \quad .$

Note that equation (12) gives, in the frequency domain :

$$(\Delta + k_v^2) \mathbf{v}_v = 0 \quad , \tag{18}$$

where $\quad k_v^2 = -ik/l'_v \quad ,$

and that equations (17) and (18) provide us with the exact values of the three wavenumbers k_a, k_h and k_v , associated respectively with the acoustic movement, the entropic movement due to heat conduction and the vorticity movement due to viscosity effects (these results did not become available until 1989 [28]) .

From equations (9) to (12) it is easy to obtain expressions for the pressure variation p and the laminar velocity $\mathbf{v}_l$ as functions of the temperature $\tau = \tau_a + \tau_h$ (τ_a and τ_h being solutions of equations 17.a-b) ; in the frequency domain these results can be written as follows :

$$p = [\gamma\beta/(\gamma-1)] (\xi_{ha}\tau_a + \xi_{hh}\tau_h) \quad , \tag{19}$$

$$\mathbf{v}_l = (i/\rho\omega) [\gamma\beta/(\gamma-1)] (\zeta_a \, \mathbf{grad} \, \tau_a + \zeta_h \, \mathbf{grad} \, \tau_h) \quad , \tag{20}$$

with $\quad \xi_{\mu\nu} = 1-il_\mu k_\nu^2/k \quad$ and $\quad \zeta_\nu = \xi_{h\nu}/\xi_{v\nu} \quad ,$

where the subscript "μ" stands for "h" or "v" and where the subscript "ν" stands for "a" or "h", where the subscripts "a,h,v" correspond to the three kinds of mode (acoustic, entropic and vorticity modes) .

<u>b-Boundary conditions, applications [30].</u>

In most applications, the thermal conductivity and the specific heat per unit volume of the wall material greatly exceed the corresponding quantities for the gas. Then, neglecting heat-transfer parallel to the wall on account of the fact that the temperature varies slowly in that direction and neglecting the very slight temperature jump at the wall [31], the continuity of the normal heat flux at the interface is practically equivalent to the requirement that the total temperature τ be constant at the wall, i.e. :

$$\tau_a + \tau_h = 0 \quad \text{on the walls} . \tag{21}$$

In addition, for a perfectly rigid wall, neglecting the very slight velocity slip at the interface [31], we assume henceforth that the total particle velocity $\mathbf{v}$ is equal to zero at the boundaries :

$$v_{au} + v_{hu} + v_{vu} = 0 \quad \text{on the walls} , \tag{22}$$

$$\mathbf{v}_{aw} + \mathbf{v}_{hw} + \mathbf{v}_{vw} = 0 \quad \text{on the walls} , \tag{23}$$

where equation (22) represents the component of the velocity normal to the wall and equation (23) the components parallel to the wall (the sum of the acoustic velocity $\mathbf{v}_a$ and the entropic velocity $\mathbf{v}_h$ is the laminar velocity $\mathbf{v}_l$ introduced at equation (8)) .

Expressing each quantity as the product of a function of "u" (coordinate normal to the wall) and a function of $\mathbf{w}$ (coordinates in the plane tangent to the wall), substituting the result (20) into equation (22) and (23), and taking into account that the boundary conditions must be satisfied for arbitrary values of $\mathbf{w}$ all over the boundary surfaces, equations (21) to (23) yield a new general dispersion equation (see [30]) :

$$(1-\zeta_h/\zeta_a) \, v_{vu}/v_{vw} = v_0^{-1} [(\partial_u\tau_a)/\tau_a - (\partial_u\tau_h)/\tau_h] \quad , \tag{24}$$

where each function now represent the only function of "u" mentioned above (for the value of "u" on the wall), and where the subscript "w" stands for any component of the vector "w" (v_0 is the

modulus of the outward normal vector to the boundary surface).
Various applications of these results (especially equation 24) are presented below, for cartesian, cylindrical and spherical coordinates.

For the case of cartesian coordinates, let a semi-infinite medium bounded by an infinite, plane, rigid wall be set at x=0 (the x-axis being inward directly). As a consequence of equation (24), the effects of the thermal and shear modes on the boundary conditions can be treated by using the concept of the specific admittance $Y_a = \rho c\, v_{la}/p_a$ at the boundary :

$$Y_a = (ik)^{1/2} \{[1-(k_{ax}/k_a)^2](l'_v)^{1/2} + (\gamma-1)(l_h)^{1/2}\} \quad . \qquad (25)$$

This result was first given by L. Cremer in 1948 (see section 2.2 above), from a specific calculation in cartesian coordinates.

For the case of cylindrical coordinates, equation (24) provides the "exact" equation which gives the complex axial wavenumbers for all kinds of modes (propagating or evanescent) in infinite cylindrical waveguides (such a result was not available until we obtained it in 1988 [30]). This equation can be written as follows :

$$(1-\zeta_h/\zeta_a)(k_v^2/\chi_v^2)\{[m^2/B_m^{(h)}]-[(k_{az})^2/k_v^2]B_m^{(v)}\}=B_m^{(a)}-(\zeta_h/\zeta_a)B_m^{(h)} \quad ,(26)$$

with $\quad B_m^{(\mu)} = \chi_\mu R\, J'_m(\chi_\mu R) / J_m(\chi_\mu R) \quad ,$

where J_m is the mth order cylindrical Bessel function, R being the radius of the tube,

$\chi_\mu = [k_\mu^2-(k_{az})^2]^{1/2} \quad .$

This exact equation has been used in 1988 [25] to generalize the expression for the axial wavenumber obtained in 1949 [32].

For the case of spherical coordinates, we can treat the example of a perfectly rigid spherical shell (the origin of the coordinates being at the centre), and using the solutions which involve the nth spherical Bessel functions j_n, equation (24) can now be expressed as :

$$(1-\zeta_h/\zeta_a)\, n(n+1) / (1+b_m^{(v)}) = b_m^{(a)} - (\zeta_h/\zeta_a)\, b_m^{(h)} \quad , \qquad (27)$$

with $\quad b_m^{(\mu)} = [k_\mu R\, j'_n(k_\mu R)] / j_n(k_\mu R)$, R being the radius of the sphere .

This result was first given in the literature in 1988 [7], from a direct calculation in spherical coordinates. This "exact" equation can be used to determine the wavenumber k_a and, from it, the resonance frequencies of the fluid enclosed in the sphere.

3. RECENT APPLICATIONS : MODAL ACOUSTIC FIELD STRUCTURES , CORIOLIS COUPLING .

3.1 Introduction.

The set of equations that are presented in the preceeding section allows us to solve several practical problems associated with useful applications using guides or cavities. We intend here to focus on applications making use of fluid-filled cavities, which need a good understanding of the spatial structure of acoustic fields (mode coupling, effect of local and non local sources, and so on), and which need accurate results for resonant frequencies, quality factor, Coriolis effect, etc...

The first application that we would like to mention briefly here involves the thermophysical properties of gases, especially at low temperature and high pressure, wich are determinated merely by measuring the acoustic resonance frequencies of a spherical resonator 12cm in diameter (deviation from sphericity being less than 10μm) [7,8]. The temperature is controlled to an accuracy of the order of 0.001 K (although the precision with which the temperature can be expressed has an accuracy of the order of 0.01 K), and the resonator is operating in the frequency range 1 to 40 kHz. The high quality factors (typically 1,000 to 10,000) allow a precise measurement of the resonance frequencies, making use of a detailed model for the resonance curves. The ratio of the speed of sound in a gas of interest and the radius of the spherical resonator is then measured with an accuracy of the order of magnitude 10^{-5} (in the P,T range 1 to 200 bars, 200 to 320 K), which then enables us to obtain values for the thermophysical properties mentioned above.

This brief summary on the spherical resonator as a tool for obtaining precise values of the speed of sound is just given here as a very significant example of the use of the sound to measure physical properties of gases. Other examples can be found in the literature as well as other kinds of applications. We restrict ourselves here to applications which need a good modelling of the acoustic field in cavities. So, in this section (3), we intend to give a brief review of relevant studies, emphasizing mode coupling due to the Coriolis effect as an interesting example and a useful phenomenon which can be used to design new types of rate gyro. Other examples bearing on the use of thin fluid films for the damping of vibrations and the optimization of transducers are presented in section (4).

3.2 Mode coupling in cavities ; the acoustic rate gyro.

a-The problem.

The aim of this subsection is to present a systematic way for calculating mode couplings inside fluid-filled cavities, taking into account the necessarily imperfect wall of the cavities (loudspeakers and microphones at the surface of the walls, shape imperfections, etc.) and the inertial effects due to a rotation of the cavity. Only simple shaped cavities (rectangular, cylindrical or spherical) are considered, in order to obtain accurate solutions. Coriolis coupling and other inertial effects are considered as a volume source and we will show that this leads to coupling coefficients which are found to act very similarly to the "geometrical" terms (or other terms linked to the boundary layer effects).For this reason, the differences and similarities between the two kinds of coupling will be discussed together, so as to permit the transposition of any result from one coupling to the other.

As was emphasized in sub-section 2.3 , for cavities whose dimensions are greater than the viscous and thermal boundary layer thicknesses, that is which are greater than about 10 micrometers , the acoustic movement predominates almost everywhere (outside the boundary layers), and consequently the acoustic behaviour can be considered as a simple propagational acoustic mode in a dissipative gas (or perfect gas if very accurate results are not needed), which therefore must satisfy the Helmholtz equation (17-a), where the acoustic wavenumber k_a is given by equation (17-c). This leads, for the acoustic pressure p_a , when the field is generated by an external force **F**, (to a first approximation) to :

$$(\Delta + k_a^2)\, p_a = \rho\, \mathrm{div}\mathbf{F} \quad , \text{ in the whole volume (D)} . \qquad (28)$$

Because of the conditions at the interface, the three modes (acoustic mode, entropic mode and vorticity mode) interact strongly inside the boundary layers : entropic and vorticity modes are generated by the reaction of the wall in the presence of the acoustic field, giving rise in turn to a small velocity component for the acoustic field on the boundaries. The acoustic part of this reaction may be described by the admittance-like boundary condition Y_a for the acoustic movement given in section 2.3-b (equation 25). In addition, any real cavity will not provide only this kind of boundary condition : the necessary sources and microphones together with the shape imperfections (compared with the corresponding perfect shaped cavity which allows us to solve the problem with a separable coordinate system),and so on, act as perturbations of the acoustic field. All these effects are accounted for by the boundary conditions for the acoustic movement as follows:

$$(\partial_n + ik\varepsilon)\, p_a = 0 \quad \text{, on the perfect shaped walls } (\partial D) \;, \tag{29}$$

where ∂_n is the normal outwardly directed component of the spatial derivative on the walls.

Note that for a perfect cavity, the boundary (∂D) is "nothing other than" the real wall and "ε" is the admittance-like boundary condition Y_a. In this "non realistic" case, the boundary condition would be as follows :

$$(\partial_n + ikY_a)\, p_a = 0 \quad \text{, on the walls .} \tag{30}$$

Because Y_a depends on the direction of the acoustic velocity on the walls, this admittance-like expression must be used carefully.

The solution for the problem can be expressed using the classical integral equation

$$p_a(\mathbf{r}) = \iiint_D G(\mathbf{r},\mathbf{r}')\,\rho\,\mathrm{div}\mathbf{F}\,d\mathbf{r}' + \iint_{\partial D} [G(\mathbf{r},\mathbf{r}')\,\partial_{n'} p_a(\mathbf{r}') + p_a(\mathbf{r}')\partial_{n'} G(\mathbf{r},\mathbf{r}')]\,d\mathbf{r}' \;,$$

where the Green function $G(\mathbf{r},\mathbf{r}')$ is assumed to obey the same boundary condition (30) as the perfect walled tube. The Green's function and the acoustic pressure inside the cavity may then be calculated using an eigenfunction expansion. The orthonormal eigenfunctions ψ_N chosen are the solutions of the homogeneous problem :

$$(\Delta + k_N^2)\,\psi_N = 0 \quad \text{, in the volume (D) ,} \tag{31}$$

$$(\partial_n + ikY_a)\,\psi_N = 0 \quad \text{, on the perfect shaped walls } (\partial D) \;.$$

As the admittance Y_a, the same as in equation (30), is very small, the solutions for ψ_N are calculated using a first-order expansion with respect to Y_a. The resulting functions are roughly equivalent to those satisfying the Neumann boundary conditions (i.e. the eigenvalue problem is nearly self-adjoint). But the eigenfunctions and the eigenvalues are both complex, avoiding by that means any pole in the coefficients of the expansions [33].Substituting the expressions of the boundary conditions for the acoustic pressure (equation 29) and for the Green's function $G(\mathbf{r},\mathbf{r}')$ (i.e. equation 30), along with the eigenfunction expansions for the Green's function, and the eigenfunction expansion for the pressure variation,i.e.:

$$p_a(\mathbf{r}) = \Sigma\, A_N\, \psi_N(\mathbf{r}) \tag{32}$$

into the integral equation, leads to the set of algebraic equations [34,35]:

$$[\,(D) + (\alpha)\,]\,(A) + (S) = 0 \tag{33}$$

where (A) is the unknown matrix column of A_N,

(S) is the matrix column which represents the energy transfer between sources and eigenmodes, the Nth term being given by $S_N = \iiint_D \psi_N\,\rho\,\mathrm{div}\mathbf{F}\,d\mathbf{r}$, (34)

(D) is the diagonal matrix of the terms $(k_N^2 - k_a^2)$ which especially emphasize the resonances,

(α) is the non diagonal matrix defined by its elements $\alpha_{MN} = \iint_{\partial D} \psi_N\, ik(\varepsilon - Y_a)\,\psi_M\, d\mathbf{r}$ which introduces the mode coupling due to the perturbations mentioned above.

In this formulation, a slight displacement δx of an element of the wall from its "perfect shape" can be expressed as an equivalent admittance shift for the boundary conditions at the initial location. This admittance shift is locally expressed as:

$$(\varepsilon - Y_a) \;\#\; i\tan(k\delta x) \;\#\; ik\delta x \;.$$

Moreover, this admittance approximation can be used for negative perturbations as well as for positive ones, meaning that slight shape extentions beyond an ideal shape may be calculated using the initial or an "average" perfect geometry, and this then permits optimizing the convergence of the perturbation method. The (α) matrix is then symmetric (it should be emphasized that this is the case only for small perturbations).

b-Inertial mode coupling [34,35].

As far as the linear approximation is valid, the Coriolis effect is the only inertial effect to be considered, because the other inertial terms have no acoustic effect. The Coriolis term

$$\rho\, \mathrm{div}\mathbf{F}_c$$

in the right hand side of equation (28-a) can be successively written as follows:

$$\rho\, \mathrm{div}(2\Omega\Lambda\mathbf{v}) = 2\rho[\mathbf{v}.\mathrm{curl}\Omega - \Omega.\mathrm{curl}\mathbf{v}] = -2\rho\Omega.\mathrm{curl}\mathbf{v}\ ,$$

where Ω is the vector rate of rotation of the cavity with respect to an inertial frame, and where $\mathbf{v}$ is the total particle velocity of the fluid perturbation inside the cavity. As $\mathbf{v}$ is the sum of a laminar velocity $\mathbf{v}_l$ (corresponding to the acoustic and the entropic movement) and of a vorticity velocity $\mathbf{v}_v$ due to the viscous effect (see equation 8), the last result reduces to :

$$\rho\, \mathrm{div}\mathbf{F}_c = -2\rho\Omega.\mathrm{curl}\mathbf{v}_v\ , \qquad (35)$$

which involves only the vorticity particle velocity $\mathbf{v}_v$.

Therefore, the only inertial effects acting upon the acoustic field inside a cavity are the effects of the Coriolis acceleration upon the vorticity component of the field, which is important only inside the boundary layers near the walls (see sub-sections 2.3 and 3.2). In addition, the outgoing (from the wall) solution for the vorticity velocity $\mathbf{v}_v$ of the diffusion equations (12) and condition (14) in cartesian coordinates (which can be used even if the walls are not plane, so long as the curvature is sufficiently small) permits us to obtain an explicit expression for the Coriolis source term. Finally, the Coriolis effect can be accounted for in the non-diagonal matrix (α) in the following manner :

$$\alpha_{MN} = 2(i\omega)^{-1} \iint_{\partial D} \psi_M\, \Omega.(\mathbf{u}\Lambda\mathbf{grad}_T\psi_N)\, d\mathbf{r}\ , \qquad (36)$$

where $\mathbf{grad}_T$ denotes the components of the operator **grad** in the tangent plane to the wall and **u** a coordinate along the normal to the wall, outwardly directed, its origin being on the wall.

This results shows that the Coriolis effects on the acoustic field leads to coupling between modes (as do the other perturbing effects mentioned above in section 3.2-a). If we consider that the changes in the field structure perturb the source term itself (the loudspeaker) only in a negligible way, this coupling can be interpreted as energy transfer between modes. This is supported by figure 2, which shows theoretical (full line) and experimentally measured variations of the amplitude of the mode generated by a loudspeaker when increasing the rotation rate of the cavity, illustrating the energy transfer from that mode to the others. The agreement is quite good, showing futhermore that the experimental setup used,a modal antenna, was able to measure accurately variations less than 1 dBSPL (the acoustic level in the cavity was about 110 dBSPL).

Figure 3 shows a schematic representation of the matrix (α) when the eight first modes are taken into account : the non nul elements are represented by a letter which qualify the origin of the coupling effects involved (given in the figure caption).

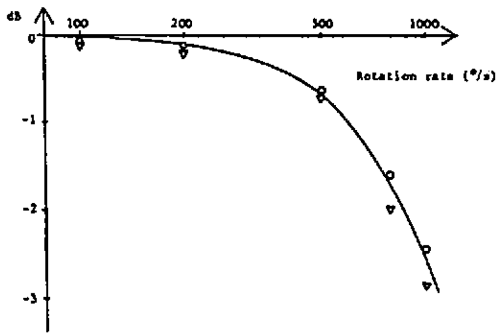

Fig. 2 : Variations of the amplitude of the excited mode when the rotation coupling increases (reference : level without rotation).
Solid curve: theoretical values (using actual quality factor);
O : measured values for $\Omega > 0$; measured values for $\Omega < 0$.

IDM	M		DM		M	M	
M	IDM		M		SCMG	M	
		ID				SC	
DM	M		IDM		M	M	
							D
M	SCMG		M		IDM	M	
M	M	SC	M		M	IDM	
				D			ID

Fig. 3 : Elements non equal to zero in the coupling matrix (α) :
- I Ideal cavity (without any perturbation) ,
- D Dissipative terms (viscous and heat conducting effects) ,
- S Source effect when the loudspeaker is not well "centred" ,
- C Coriolis effects ,
- M Microphone position -i.e. when it is at a wrong place .
- G Geometrical faults .

Most of these coupling always exist . All of them are generally symmetric, except the Coriolis effect which is always antisymetric (this last property can be very useful in practice) [36].

Figure 4 shows the ratio of the amplitude of the main mode created by the coriolis effect to the amplitude of the main primary mode created by the loudspeaker (which is an accurate picture of the sensitivity of the acoustic rate gyro), in a cavity designed to strengthen these two modes : the solid line is the theoretical result, and the small circles are experimental values. The coupling remains linear even for large magnitudes (coupling ratio above 0.4).

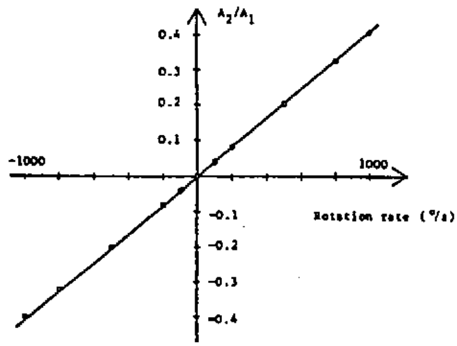

Fig. 4 : Coupling created by the rotation of the cavity around the z-axis .
Solid curve: theoretical values (using actual quality factor);
O : measured values.

This illustrates the fact that the α_{NM} terms may be used for large perturbations, as they do not correspond to any small-amplitude approximation. In addition, the theoretical and experimental results clearly reveal that rate gyros based on acoustic mode coupling in cavities can reach a sensitivity better than 0.01 °/s (the angular velocity of the minute hand of a watch is ten times faster).

The advantages of an acoustic rate gyro over the rapidly spinning wheel gyro (for example) would be a lower power consumption, a higher reliability and long lifetime, as well as a lower manufacturing cost and a lower temperature sensitivity. And as far as the microphones and the loudspeakers can be miniaturized, this kind of gyro is capable of being made very small. The present silicon technology permits us to reach an "ultra-miniaturization", opening new fields : on the one hand inviting the design of reciprocal miniaturized transducers (loudspeakers and microphones), and on the other hand attracting us towards modelling the acoustic fields in cavities whose dimensions have the same order of magnitude as boundary layer thicknesses, excited by non localized sources. This work is now in progress and already is motivating research for the future. The main purpose of the next section is to present the theoretical aspects of these problems, showing first the approximations currently in use for non miniaturized devices, which do not remain valid for miniaturized transducers (when good results are expected), and then showing the way to how we may obtain an improved analytical solution and accurate models for ultraminiaturized transducers.

4. WORK CURRENTLY IN PROGRESS ON MINIATURE CAVITIES.

4.1 Background.

In recent years, the growing demand for miniaturized transducers and the widespread use of silicon technology, has attracted attention to the design of condenser and electret microphones on silicon chips [see for example ref. 37] . As the primary purpose generally seems to be for mass-produced, low-cost microphones, where high precision is not needed, the theoretical models used are those existing for conventional microphones [4,5,38], and the major concern has been simply to solve the technological problems associated with etching the silicon. However, similar models are also used to describe the dissipative properties of fluid layers in a variety of applications, such as the damping in porous acoustic materials [39,40,41], the damping of vibrations in rotor-bearing systems and attenuation of flexural vibrations of panels in machinery enclosures [see for example 42,43], and overall, the need for accurate models of the acoustic fields in very small cavities or guides is becoming very important.

The significant parameter of these systems, when considering the geometry, the shape, and so on, is the thickness of the fluid layer between the vibrating membrane and the backing electrode (in the case of microphones or loudspeakers) or between the two vibrating plates (in the other applications), compared with the thickness of the viscous and thermal boundary layers. The situations we are concerned with involve a thin layer of fluid between the two surfaces where "thin" may have several meanings : - the thickness of the fluid layer is very much less than its others dimensions, - it is much smaller than the wavelength of sound in the fluid at the frequencies of interest, - it is smaller than, or has the same order of magnitude as, the viscous and thermal boundary layers (which under normal conditions are between 10 and 100 micrometers for the frequencies of interest). However, the thickness remains much greater than the mean free path (which in air at atmospheric pressure is about 0.1 micrometer), so we are able to define a fluid "particle" which is small compared with the thickness of the fluid layer but large compared with the mean free path (and thus we ensure that the continuum hypothesis remains valid).

There will be a need in the near future for improved theoretical models for calculating the behaviour of the membranes or plates in such devices, as the models now available are not valid for fluid layer thicknesses less than about 10 micrometers (if good results are required). Specifically, we will need accurate models for miniaturized, simple-shaped condenser or electret microphones and loudspeakers on silicon chips, having smooth electrodes (no holes or grooves whose influences would overshadow many other effects), where the fluid layer is surrounded by a reservoir (with a capillary tube to equalize the static pressure). These transducers may have non conventional properties (for example limited to a narrow frequency range at high frequencies), as they may be used in very small cavities (which are also etched on a silicon chip for example).

As it seems that most of the assumptions and hypotheses assumed in the models in use until now can be avoided without being left with a problem which can only be solved numerically, we can have improved simple models which will be helpful for the study and design of miniaturized transducers. In the following sub-section (4.2), we will discuss briefly the conventional models and the approximations they involve, and in sub-section (4.3) we will give an indication of future developments. In both cases we will be interested in the response of a membrane clamped at its periphery, taking into account the effects of the fluid-film trapped between the membrane and a smooth rigid backing electrode at rest, and surrouded by a reservoir at its periphery, as it is an important example.

4.2 The conventional models (see, for example, [1] to[5],[38] and [42] to [46]).

The foundation of the conventional derivation is the assumption that the flow is laminar, tangential to the walls (which are assumed to lie in the w-plane, that is for example the xy-plane), with a z-component (i.e. normal to the membrane and the backing electrode) of the flow assumed to be zero. Moreover, it assumes that the pressure variation is uniform through the thickness of the fluid film. It includes viscous losses in the shear, oscillating, laminar flow produced as fluid is pumped forth and back by the vibrations of the membrane, but since the velocity profile is not greatly dependent on the inplane coordinates **w** and since it has a strong shear across the layer for any value of the coordinates (which is valid only for films of infinite extent), terms containing spatial variations of velocities in the tangential directions are neglected compared with terms containing spatial variations in directions normal to the walls, that is $|\mathbf{grad}_w| \ll |\partial/\partial z|$ when applied to the total particle velocity $\mathbf{v} = \mathbf{v}_w$; this assumption implies that the bulk vicosity is neglected and that the vorticity is not involved so far (even if the shape contains sharp edges). Sometimes the theory is extended to cover thermal effects due to heat conduction within the fluid and between the fluid and adjacent surfaces [27,42]; in this case, the temperature, and hence the density, may vary across the fluid gap but, as the associated laminar entropic velocity is not introduced, the thermal effects lead only to a polytropic law, instead of an isothermal (or an adiabatic) one, to describe the compressibility of the fluid. Moreover, all quantities which depends on the coordinate z, that is the particle velocity $\mathbf{v}=\mathbf{v}_w$, the temperature variation τ and the density variation ρ', are replaced by their mean value (denoted < >) across the thickness of the fluid. Therefore, assuming a harmonic motion (∂_{ct}=ik), equations (1) to (5) may be written as equations (37) to (39) :

$$[\partial_{ct} - \ell'_v(\partial_z)^2]\,\mathbf{v} = -(\rho c)^{-1}\mathbf{grad}_w p \quad , \qquad (37)$$

$$h_0\, \mathrm{div}_w\langle \mathbf{v}\rangle + i\omega\xi = -i\omega h_0\langle\rho'\rangle/\rho \quad , \qquad (38.a)$$

$$\langle\rho'\rangle = (\gamma/c^2)\,(p-\beta\langle\tau\rangle) \quad , \qquad (38.b)$$

$$[\partial_{ct} - \ell_h(\partial_z)^2]\,\tau = [(\gamma-1)/(\beta\gamma)]\,\partial_{ct}p \quad , \qquad (39)$$

where "h_0" is the thickness of the fluid gap between the "walls".

The left hand side of equation (38-a), $h_0\, \mathrm{div}_w\langle \mathbf{v}\rangle + i\omega\xi$, is the volume flux amplitude, ξ being the amplitude of the relative displacement of the membrane, which is the non-local driving term for the fluid motion. Note that the assumptions lead us to stipulate that the z-dependence for τ gives a z-dependence for ρ' but not for p which is assumed z-independent (see equation 38-b): this cannot be justified from the kinetic theory of fluids.

The associated boundary condition would be the same as those given in section 2.3-b (equations 21 to 23), except that several no longer apply as a consequence of the assumptions made. We do not need to write anymore that the z-component v_z of the particle velocity is zero on the backing plate; in addition, the continuity equation $v_z=i\omega\xi$ at the interface between the fluid and the membrane cannot be used here, as v_z is assumed to be equal to zero throughout the fluid film. Moreover, at any point located on the periphery of the fluid layer, either Dirichlet's condition or infinite medium (that is Sommerfeld) conditions are assumed, depending on the system considered (this last hypothesis greatly simplifies the solution of the problem).

In addition, the equations governing the forced vibrations of the membrane, clamped at the periphery, driven by an incident harmonic acoustic wave p_i assumed to be uniform over the surface of the membrane can be written as follows :

$$T\,[\,\Delta + K^2\,]\,\xi(\mathbf{w}) = p_i - p(\mathbf{w},h_0) \quad , \quad \text{for any value of } \mathbf{w} \, , \qquad (40)$$

$$\xi(\mathbf{w}_s) = 0 \quad \text{at the periphery} \quad (\mathbf{w}=\mathbf{w}_s) \; .$$

with $K^2 = k^2c^2\mu_s/T$, where T is the tension of the membrane, μ_s the surface density and kc (=ω) the angular frequency.

It is easy to demonstrate (see, for example, [47] to [49]) that the whole acoustic problem reduces to a complex Helmholtz non homogeneous equation, depending only on the two tangential coordinates **w** , as follows:

$$(\Delta_w + \chi^2)\, p(\mathbf{w}) = \zeta\, \xi(\mathbf{w}) \quad , \qquad (41)$$

where $$\chi^2 = k^2 [\, \gamma - (\gamma - 1) B_h \,] / (1 - B_v) \quad , \qquad (42)$$

$$\zeta = - \rho k^2 c^2 / h\, B_v \quad , \qquad (43)$$

with $$B_{h,v} = [\tan (k_{h,v}\, h/2)] / (k_{h,v}\, h/2) \quad .$$

Note that this expression for the complex wavenumber χ is similar to that obtained independently by Zwicker and Kosten [32] and Daniels [50] for circular tubes (where the rôle of w and z are interchanged), over a very wide range of frequencies and radii, which has been used and studied extensively over the last forty years . Asymptotic expansions (with respect to the ratio of the radius and the boundary layer thickness) have been presented [51] and conditions of validity obtained both numerically [52] and analytically [53] (providing one of the basis of the theory of sound absorbing materials [32,39,40,41]). More general solutions, including both boundary layer and volume dissipation, have been obtained analytically [20] and the corresponding solution in the time domain (i.e. pulses in tubes) is given in reference [54].

Coming back to the problem concerning the response of a membrane clamped at its periphery, taking into account the effects of the fluid-film trapped between the membrane and either a smooth or perforated rigid backing electrode and a backing cavity, we must mention that attempts, principally during the last three decades, to improve the classical model mentioned above have been successful in describing the frequency response of electrostatic transducers accurately, especially the 1" condenser microphone. It can be considered that the earliest work in this direction is that of D.H. Robey [44], which shows how to treat fully the viscous effects taking into account the rotational component of fluid velocity, when no grooves or holes are included on the backing electrode, assuming that the input impedance of a surrounding reservoir is exactly equal to zero and that the process is isothermal, writing the displacement of the membrane as an eigenfunction expansion but considering only the average of this displacement amplitude over the surface of the membrane... . One decade after, I.G. Petritskaya [46], making use of the results given by D.H. Robey, introduces the effect of the input impedance of a number of holes in the stationary electrode, to study the behaviour of electrostatic microphones, trying in particular to optimize the viscous damping. Therefore, J.E. Warren et al. [45], whilst making the hypothesis that the pressure field and the density are uniform in the direction normal to the membrane across the fluid layer, and assuming a laminar velocity and a Poiseuille flow between the electrodes, and an isothermal process, make use of a finite difference method to obtain numerical solutions, and show a number of useful results for optimising and characterizing properties of microphones. Then, making use of the work of D.H. Robey and I.G. Petritskaya, and at the same time reducing the theoretical complexity of the problem by means of asumptions compatible with the characteristics of 1" B&K microphones, A.J. Zuckerwar [38] expresses the sensitivity and equivalent lumped elements, which gives results which are in excellent agreement with experimental data taken on B&K 1" pressure microphones.

As most of the assumptions and hypotheses mentioned in this section do not remain valid for enclosed fluid layer thicknesses approching ten micrometers, and because these approximations can be avoided whilst still allowing an analytical solution, a new approach to solving the complete "exact" set of equations (9) to (23) can be made as presented in the following sub-section.

ACOUSTICS OF FLUID-FILLED SMALL CAVITIES

4.3 Conclusion : further developments.

First, in order to avoid the "main" difficulty that has been identified with the conventional models, the z-component of the fluid velocity is no longer assumed to be zero, and then the continuity equation for the normal velocity at the interface between the membrane and the fluid can be written as usual, this normal velocity being the sum of the z-componant of the solenoïdal particle velocity and the vorticity particle velocity. This continuity equation must be used to describe the strong coupling between the membrane and the fluid. In addition, it would be convenient to start with a solution of equation (40) governing the vibrations of the membrane, clamped at its periphery, in such a way that we will take advantage of the orthogonality properties of the solutions of the associated Dirichlet eigenvalue problem.

Therefore, the general solution for the displacement of the membrane is a simple eigenfunction expansion, and then, since the continuity equation for the normal velocity at the interface between the membrane and the fluid must be verified for arbitrary values of the coordinates w over all the boundary surface, the normal fluid velocity at this interface must be written as an expansion using the same eigenfunctions as used for the membrane. Hence, this normal fluid velocity, outside the interface, for any value of z in the separation between the backing plate and the membrane, can be written as the sum of this eigenfunction expansion and a function of w and z expressed as another eigenfunction expansion where it would seem logical to assume that the eigenfunctions will be solutions of the one dimensional Neumann problem in the z-direction.

Then, the propagation equations (17) must be used to obtain the general solution for the coefficients of these expansions. Therefore, solving the complete set of basic equations (9) to (14), tacking into account the boundary conditions (21) to (23) and making use of the results mentioned here above, will allow us to calculate the behaviour of the membrane (or plate), avoiding the approximations currently in use which do not remain valid for miniaturized transducers, in the frequency range of interest (up to 100 kHz). In the lower frequency range, lumped element models can be obtained for a first approximation as the dimensions of the system are much smaller than the acoustic wavelength, and in the higher frequency range more complicated models would be provided as the vorticity and entropic phenomenon will play an important role (which increases with the frequency). This work is now in progress [10]... We would like to mention here that an extension of some part of the work mentioned in this paper, in the time domain, using a heat source described by a rate of heat creation, is given in reference [55].

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