VIBRATION TRANSMISSION BETWEEN STACKED REINFORCED CONCRETE BEAMS USING MONTE CARLO SIMULATIONS WITH FINITE ELEMENT METHODS

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The prediction of vibration transmission in collapsed and fragmented reinforced-concrete buildings has the potential to inform decisions about the possibility to detect human survivors trapped in buildings after earthquakes by using structure-borne sound propagation. In collapsed buildings there are many uncertainties such as the collapse pattern of the building and the contact conditions between the debris. In this research, a statistical rather than a deterministic model is being developed to predict vibration transmission through the debris of a collapsed building. This paper assess the potential to use Statistical Energy Analysis (SEA) to model vibration transmission between two reinforced concrete beams when they are resting on top of each other. An experimentally validated finite element model of two beams was used to carry out a Monte Carlo simulation with 30 beam junctions in random formations. Coupling loss factors were determined with FEM data using Experimental SEA and these were compared against theoretical models based on a lump spring connector.

Keywords: experimental modal analysis, finite element methods, Monte Carlo simulations, statistical energy analysis

1. Introduction

Earthquakes have the highest rate of mortality among all the natural disasters. From 1970 to 2009, 36% of fatalities that have occurred due to natural disasters are due to earthquakes [1]. When victims are trapped inside a collapsed building, the challenge is to detect and locate survivors within a period of time that will allow them to be rescued. The majority of documented live rescues are accomplished within the first six days [2]. However, important variables affect the survivability including the structure type and void space formation, the cause of the structural collapse, the survival location in the building and the speed and sophistication of available search and rescue capabilities [3]. The prediction of vibration transmission in collapsed and fragmented reinforced-concrete buildings has the potential to inform decisions about the possibility to detect trapped human survivors by using structure-borne sound propagation. This research forms part of a funded project concerning an approach to search for human survivors using structure-borne sound propagation in collapsed and fragmented structures through the development, validation and use of theoretical models.

The aim of this paper is to assess the potential to use Statistical Energy Analysis (SEA) to model vibration transmission between two reinforced concrete beams when they are stacked on top of each other (i.e. no bonded connection). Finite Element Methods (FEM) are used to create a model of an X-shape beam junction which is experimentally validated against the results of experimental modal analysis. The FEM model is used to create an ensemble of 30 beam junctions for a Monte Carlo simulation. Experimental SEA is used to determine coupling loss factors between the two beams from the FEM data for comparison against theoretical models based on lump spring connectors.

2. Methods

2.1 Experimental work

2.1.1 Test specimens and setup

The experimental samples consist of two reinforced concrete beams (C25/30, S500) with the same dimensions (2.4 m length, 0.2 m width and 0.3 m depth). Beams 1 and 2 are reinforced with four and eight longitudinal steel bars of 16 mm diameter, respectively. The transverse reinforcement of both beams consists of 8 mm diameter stirrups placed at 200 mm centres along the beams.

An X-shape beam junction was formed after placing beam 2 on top of beam 1, which is supported by two aluminium square bars (see Fig. 1). The angle between the two beams was 41°.

Figure 1 - Test setup showing the test equipment and the X-shape junction formed by beams 1 and 2.

2.1.2 Experimental modal analysis

Experimental modal analysis was carried out to identify the eigenfrequencies and mode shapes of the setup. The beams were excited using an impact hammer (Brüel & Kjær Type 8200) and the out-of-plane response was measured using three accelerometers (Brüel & Kjær Type 4371). Brüel & Kjær Pulse Reflex software was used for signal processing and modal analysis. During the modal testing, the accelerometers remained at fixed positions whilst the impact hammer was moved along the excitation points.

2.2 Finite element models

Two FEM models of the beam junction were developed in Abaqus v6.14 [4] and eigenfrequency analysis was carried out to identify their dynamic characteristics (eigenfrequencies and modeshapes). The solid element C3D20R (20 nodes) and the beam element B32 (3 nodes) were selected from the element library of Abaqus to model the concrete and the steel bars respectively. The mesh density fulfils the requirement for at least six elements per wavelength in structural and vibroacoustic problems [5].

The linear spring element, SPRING1 was selected from the Abaqus element library to approximate the elastic support provided by the square-section aluminium bars to the lower beam, beam 1. The contact between the two beams was modelled using the general contact algorithm of Abaqus/Standard and was defined to have either only elastic normal behaviour (FEM model 1) or both elastic normal and rough tangential behaviour (FEM model 2). The latter was implemented assuming an infinite friction coefficient such that the common nodes of the contact area moved together in the horizontal plane. After model updating, the stiffness of the springs was estimated to be 4.1E05 N/m and the normal stiffness of the contact was estimated to be 7.54E08 N/m and 2.66E08 N/m for FEM models 1 and 2 respectively. This gave a factor of 2.8 between the normal contact stiffness of FEM models 1 and 2.

Table 1 shows the physical and mechanical properties of the materials used in the model. More information regarding the estimation of the material properties can be found in [6].

Table 1: Material properties.

Material	Density, ρ [kg/m^3]		Young's modulus, E [N/m^2]	Poisson's ratio, ν [-]
Concrete	Beam 1	2328.7	36875E06	0.2
	Beam 2	2245.2	32475E06	
Steel	7800		200E09	0.3

2.3 Monte Carlo simulation

The experimentally validated FEM model 2 (with normal contact stiffness equal to 3.33E08 N/m based on previous experiments [6]) was used as a basis for creating a sample of 30 beam junctions using Monte Carlo simulation [7]. In each junction, the relative position of the two beams was sampled from a uniform distribution whereas the angle between the two beams remained constant at 41°. Both beams had the same material properties (average values of beams 1 and 2) and were assumed to have simply supported ends as indicated by the orange symbols in Fig. 2. In order to increase the modal density beyond that of the beams in the physical experiments, beams 1 and 2 were assumed to have lengths equal to 6.0 and 5.0 m respectively.

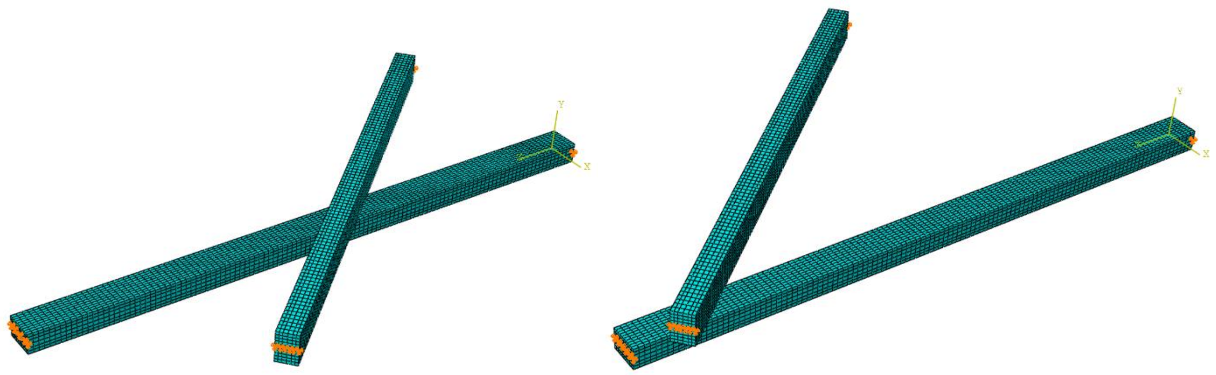

Figure 2 - Example of two beam junctions used in the Monte Carlo simulation.

2.4 Experimental Statistical Energy Analysis (ESEA)

FEM models used mode-based steady-state dynamic analysis to calculate the dynamic response of the 30 beam junctions up to 3200 Hz considering only the out-of-plane (i.e. y-direction) bending modes. The critical damping, ζ, was set to be equal to 0.05. The beams were excited using rain-on-the-roof excitation (i.e. forces with unity magnitude and random phase); all the nodes of the lower surface were excited on beam 1, and all nodes of the upper surface were excited on beam 2.

Each beam represents one subsystem and the output from the FEM models was used to calculate the subsystem energy and power input that apply to an SEA model of each beam junction. These FEM data were then used in ESEA to determine coupling loss factors.

The general ESEA matrix solution for two subsystems is given by [8]

$$\begin{bmatrix} \sum_{n=1}^{2} \eta_{1n} & -\eta_{21} \\ -\eta_{12} & \sum_{n=1}^{2} \eta_{2n} \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \begin{bmatrix} \frac{W_{\mathrm{in(1)}}}{\omega} & 0 \\ 0 & \frac{W_{\mathrm{in(2)}}}{\omega} \end{bmatrix} \tag{1}$$

where η_{ij} is the coupling loss factor from subsystem i to j, η_{ii} is the internal loss factor for subsystem i and E_{ij} is the energy of subsystem i when the power is input into subsystem j.

The energy associated with each subsystem is given by [8]

$$E = m\langle v^2 \rangle_{t,s} \tag{2}$$

where m is the mass and $\langle v^2 \rangle_{t,s}$ is the temporal and spatial average of the mean-square velocity of all the unconstrained nodes of the beam subsystem.

For rain-on-the-roof excitation at P nodes the power input, W_{in} is given by [8]

$$W_{\mathrm{in}} = \frac{\omega}{2} \sum_{p=1}^{P} \left(\mathrm{Im}\{\hat{F}\} Re\{\hat{w}\} - \mathrm{Re}\{\hat{F}\} Im\{\hat{w}\} \right)_p \tag{3}$$

where F is the force and $\hat{w}$ is the peak out-of-plane displacement associated with each node.

2.5 Theoretical model based on lump spring connector

For N identical point connections between two beams, the coupling loss factor from beam i to beam j can be calculated using [8]

$$\eta_{ij} = \frac{N}{\omega m_i} \frac{Re\{Y_j\}}{|Y_i + Y_j + Y_{\mathrm{c}}|^2} \tag{4}$$

where m_i is the mass of beam i.

The driving-point mobility of a thin beam of infinite extent, for excitation of bending waves in the central part of the beam is given by Eq. 5 [8]

$$Y = \left((1+i) 2.67 \rho S \sqrt{c_{\mathrm{L,b}} h f} \right)^{-1} \tag{5}$$

where S is the cross-sectional area of the beam, h is the depth of the beam, f is frequency, and $c_{\mathrm{L,b}}$ is the phase velocity of the beam for quasi-longitudinal waves.

The driving-point mobility of the point connection, Y_{c}, can be calculated using Eq. 6 [8]

$$Y_{\mathrm{c}} = \frac{i 2\pi f}{k} \tag{6}$$

where k is the dynamic stiffness of the point connection acting as a spring (N/m).
For rigid point connections the stiffness can be assumed to infinite; hence $Y_{\mathrm{c}} = 0$.

3. Results

3.1 Experimental validation of the FEM model

Close agreement was achieved between FEM and experimental eigenfrequencies for both models of the X-shape junction, as shown in Fig. 3. For all the mode pairs in the frequency range from 700 to 3200 Hz, the percentage difference in eigenfrequencies was less than 5%.

Figure 4 compares FEM and experimental results in terms of mode shapes using the Modal Assurance Criterion (MAC) [9]. Note that only bending and torsional modes were included in the validation procedure of the FEM models. For FEM model 1, close agreement was achieved for the majority of mode pairs between 1000 and 3200 Hz; MAC values were greater than 0.8 for 17 of the mode pairs. Below 1000 Hz, the agreement is weak with the exception of the first two mode pairs which have MAC values greater than 0.8. For FEM model 2, close agreement was achieved for the vast majority of the mode pairs in the frequency range from 700 to 3200 Hz with MAC values greater than 0.8 for 23 mode pairs.

The results of this validation indicate that both FEM models are sufficiently accurate to describe the dynamic behaviour of the X-shape junction formed by two reinforced concrete beams stacked on each other in the frequency range from 1000 to 3200 Hz. However, the FEM model with the rough tangential behaviour is more appropriate for modelling the mode pairs below 1000 Hz.

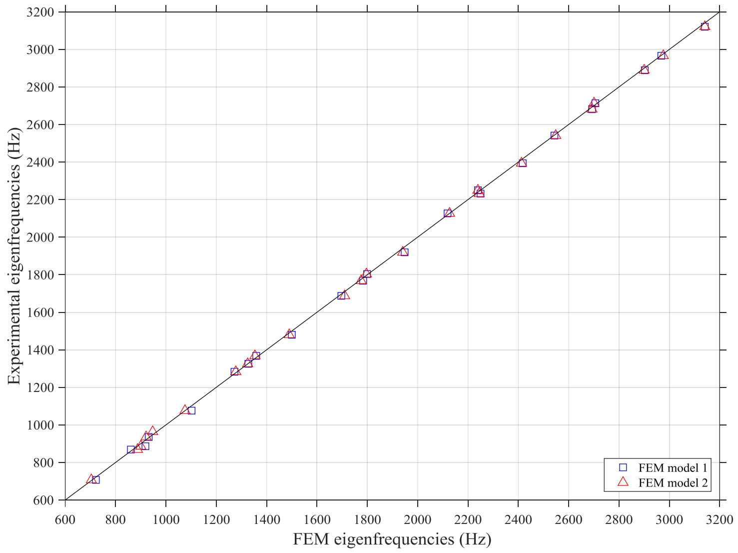

Figure 3 - Comparison between FEM and experimental eigenfrequencies for the X-shape beam junction.

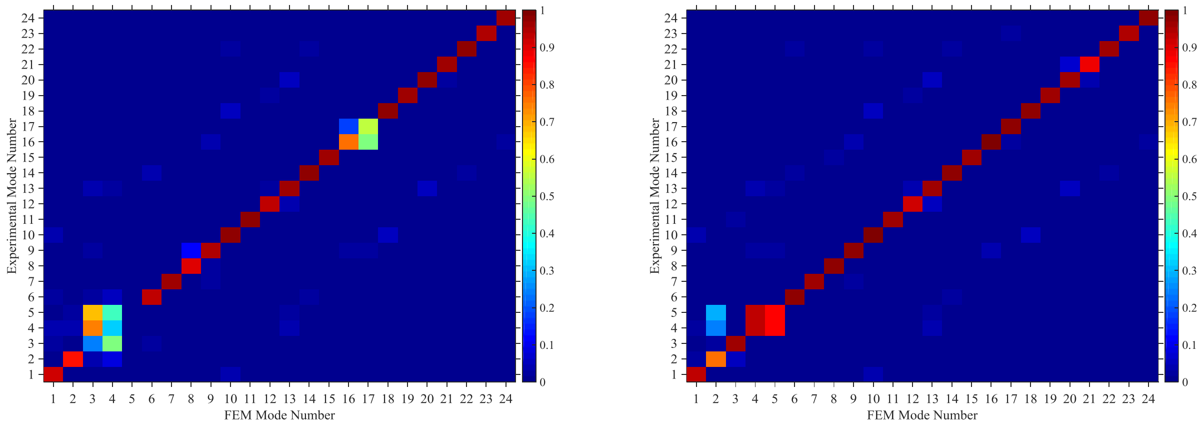

Figure 4 - (left) MAC values for the FEM model 1, (right) MAC values for the FEM model 2

3.2 Comparison of coupling loss factors from lump spring theory and FEM ESEA

Figure 5 compares the coupling loss factor, η_{12} for the 30 beam junctions from FEM ESEA with three prediction models based on a lump spring connection, one assuming an infinitely stiff spring and the other two using stiffness values from model updating of FEM models 1 and 2. Results are shown for six frequency bands with a bandwidth of 460 Hz in the frequency range from 441 to 3200 Hz. ESEA gave negative coupling loss factors in bands below 441 Hz; hence these were not included on Fig. 5.

The 30 results from FEM ESEA show a spread between 9 and 24dB. Between the 1130 and 2970 Hz bands the majority of these curves tend to lie between the prediction models assuming a rigid connection (upper limit) and using the spring stiffness from FEM model 2 (lower limit).

There is reasonable agreement between the ensemble average FEM ESEA result and the predicted value using the stiffness value from FEM model 1. The differences were below 5 dB for the 670, 1130 and 1590 Hz bands and between 5 and 8 dB for the 2050, 2510 and 2970 Hz bands. Considering the relatively low mode counts this is typical of the agreement between SEA and measurements observed for heavyweight building elements [8].

There is poor agreement between the ensemble average FEM ESEA result and the predicted value using the stiffness value from FEM model 2. This is attributed to the fact that the rough tangential behaviour is not considered in the lump spring model.

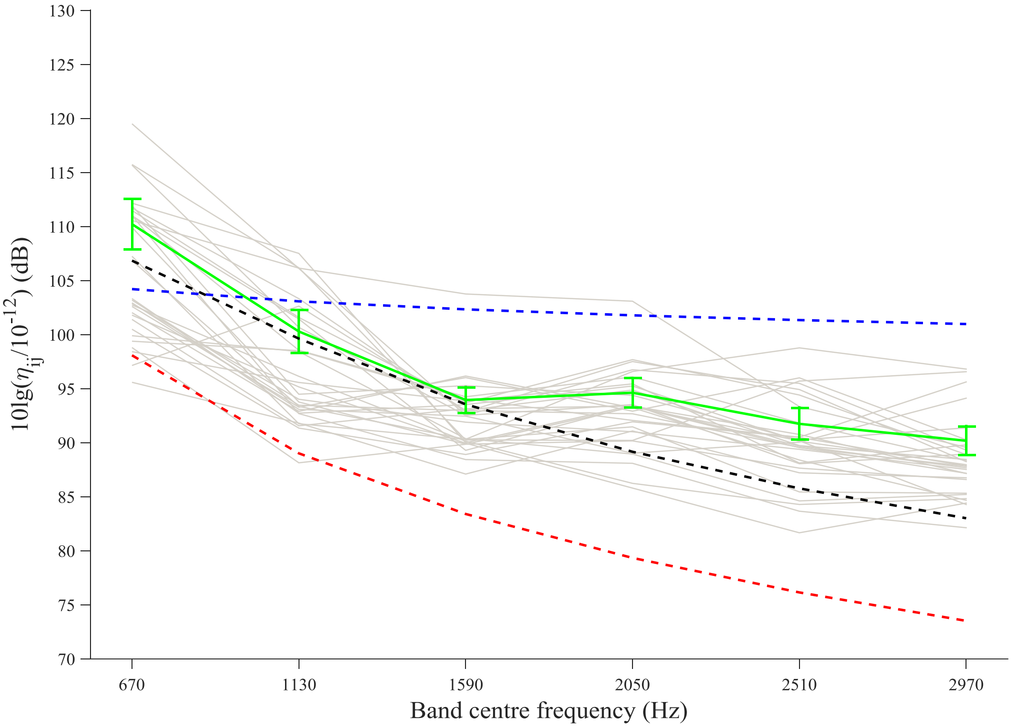

Figure 5 - Comparison of FEM and theoretical coupling loss factor η_{12}. Light grey lines correspond to η_{12} calculated with FEM ESEA for the 30 junctions, the green line shows the ensemble average of the 30 η_{12} values from FEM ESEA with 95% confidence intervals, the blue dashed line shows the predicted value of η_{12} when Y_c=0 and the black and red dashed lines show the predicted η_{12} when Y_c is calculated using Eq. 6 with the stiffness determined from model updating of FEM models 1 and 2 respectively.

4. Conclusions

For an X-shape junction of two beams stacked on top of each other, the eigenfrequencies and mode shapes from FEM modelling showed close agreement with experimental modal analysis. Two FEM models were used with elastic normal behaviour (FEM model 1) or elastic normal and rough tangential behaviour (FEM model 2). Both FEM models achieved close agreement in the frequency range from 1000 to 3200 Hz but only FEM model 2 was in close agreement below 1000 Hz.

Using FEM, an ensemble of 30 random beam junctions was generated using Monte Carlo simulations and ESEA was used to determine coupling loss factors between the two beams. These were compared with predicted coupling loss factors based on a lump spring connector. The predicted value using the stiffness from the model updating of FEM model 1 was in close agreement with the ensemble average coupling loss factor. Predicted values assuming a rigid connector and the stiffness from the model updating of FEM model 2 tended to provide upper and lower bounds for the ensemble results.

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