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"The computational efficiency of a new minimization algorithm for eigenvalue analysis"

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1. INTRODUCTION

The application of displacement methods to structural dynamic analysis leads to the general eigenvalue problem

$$Kx = \omega^2 Mx \qquad (1.1)$$

for which a classical solution consists in building up the dynamic flexibility matrix

$$D = K^{-1} M \quad , \qquad (1.2)$$

and computing its eigen-values and -vectors

$$\left.\begin{array}{l} \omega_1^2 \leq \omega_2^2 \leq \ldots\ldots \leq \omega_n^2 \\ x_{(1)}, x_{(2)}, \ldots\ldots , x_{(n)} \end{array}\right] \qquad (1.3)$$

by direct or iterative methods. For a finite element analysis, this classical solution requires that K and M be assembled and K be inverted. The order of these matrices is frequently so high that it is impractical or prohibitive to compute the eigensolutions.

Two approaches have been used to solve practically the large eigenvalue problems. The first one aims at an approximation preserving the low frequency spectrum while reducing the order of the eigenvalue problem (6, 7, 8, 9).
The second consists in seeking the stationary points of the Rayleigh quotient

$$\lambda(x) = \frac{x'Kx}{x'Mx} \qquad (1.4)$$

by minimization techniques (1, 4).

More precisely, if we consider the restricted class of vectors x which consists only of vectors that are orthogonal to the first r-1 eigenvectors :

$$x' M x_{(s)} = 0 \qquad s = 1, \ldots r-1 \qquad (1.5)$$

$$\min_x \left\{ \lambda(x) \right\} = \omega_r^2$$

and this minimum is reached for $x = x_{(r)}$

When minimizing the Rayleigh quotient, it is no more necessary to build up physically the structural matrices K and M. Indeed such an algorithm requires only the computations of the Rayleigh quotient and its gradient vector at any point. This can be performed by re-reading the elementary matrices separately from tape or disk units (1).

Such an algorithm will be presented in this paper, and in order to show its computational efficiency, it will be applied to a large scale problem : a delta wing analysed numerically and experimentally by Taborek (5).

2. MINIMIZATION OF THE RAYLEIGH QUOTIENT

2.1. Research of the fundamental mode

The first techniques for minimizing the Rayleigh quotient have be confined to the "Steepest descent" method , and were unsatisfactory because of the show convergence provided by this algorithm.

By using the well known Fletcher-Reeves method (2), W.W. Bradbury and R. Fletcher (3) developped a more powerful algorithm which was applied successfully by K.L. Fox and M.P. Kapur to structural eigenvalue problems (4). However, they gave no satisfactory solution to the difficulty arising from the homogeneous form of the Rayleigh quotient; the resulting deflation process for obtaining higher modes could not be easily developped.

In this paper, a new minimization algorithm will be presented which relies upon the generation of a set of H conjugate gradients, H representing the Hessian matrix of the local second order derivatives computed at each iteration :

$$H = \left\{ \frac{\partial^2 \lambda}{\partial x_i \, \partial x_j} \right\} \tag{2.1}$$

Quadratic convergence is guaranteed in the neighbourhood of the eigensolution, by analogy with the conjugate gradient method applicable to quadratic functions.

It is important to note, at a practical point of view, that the H conjugate directions can be computed without building up physically the H matrix : indeed the orthogonalization process involves only bilinear forms.

2.2. Convergence to higher modes

The usual matrix deflation process cannot be used if we want to perserve the sparse nature of the structural matrices : therefore a gradient projection scheme will be used that constrains at each iteration the minimization search to lie in the subspace orthogonal to the previously determined eigenvectors. By refering to Fox and Kapur (4), it can be noted that the gradient projection scheme is considerably simplified by the use of a minimization algorithm that does not need the definition of a metric.

2.3. Scaling transformation

The ill-conditioning of the minimization problem (1.4) being directly related to that of inverting the stiffness matrix K, it follows that the accumulation of round-off errors and the eventual instability of the conjugate gradient algorithm depends on the ellipticity of the potential energy $\frac{1}{2}x'Kx$ just as in statics (10, 11, 12).

An artificial source of ill-conditioning is the use of such elements that a bad distribution of the stiffness occurs, or the definition of such a typical length that the different types of displacements (deflections, rotations, curvatures) do not present the same order of magnitude.

The numerical experiments have shown that the convergence of the algorithm is very sensitive to the ellipticity of the stiffness matrix (1) : therefore a scaling transformation is discussed that removes the artificial ill-conditioning due to the choice of physical units.

3. NUMERICAL APPLICATION

In order to prove its computational efficiency, the algorithm has been applied to a large scale problem : the delta wing analysed numerically and experimentally by R.J.J. Taborek (5). The structure is represented by the figure 1 of the abstract corresponding to (6). The whole structure is treated as a Kirchhoff plate : a specially adapted conforming plate element has been generated which takes into account the variable distance between the upper and lower skins, and the anisotropy introduced by the skin stiffness.

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