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A REAL-TIME, STOCHASTIC MELODY GENERATING SYSTEM

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Introduction

Purely analogue sound and music synthesis systems tend to offer rather restricted timbral possibilities and control facilities. On the other hand the degree of generality of some purely digital systems (eg (1)) can tend to make them slow and unintuitive to use. At present hybrid systems, consisting of a computer controlling an analogue synthesiser, offer an attractive compromise. An 'orchestra' of sounds can be made up by interconnecting modules on the analogue synthesiser. The computer then 'conducts' this orchestra by supplying it with musical data and directing its execution. The required rate of data flow to the synthesiser is low - perhaps a few tens of bytes per second - and even a microcomputer can cope with ease. The system described here uses the surplus processing time to actually generate pitch data, in real time, using a stochastic technique.

The attraction of using an analogue device on the lower, sound-generating level of the system is that the musical attributes of the sound (timbre, dynamic level etc.) can be controlled directly by turning knobs and listening. No explicit knowledge of the sound's physical properties is needed. As long as a desired sound is recognised by the operator when it occurs then he may 'home-in' on it in an entirely intuitive manner. In the system described here an attempt is made to implement this principle on a higher, melodic-structure level, allowing the operator to use his passive, recognising ability rather than demanding of him an explicit description of the output required.

There is a limit to the number of parameters that an operator can properly control at one time. It was therefore decided to reject the more general methods of data generation in which the next event has a complex dependence on many previous events. In the method chosen the output is controlled via probability distributions governing only the pitch, change of pitch and a change of change of pitch. It was felt that this was not an arbitrary simplification but one which might take advantage of the kind of information redundancy which is present to a great extent in conventional music and to some extent in all music. Some details of the generation method used and interaction between levels is given in (2).

Structure of the System

The p-level - control of pitch values

On this level we specify the *a priori* probabilities of occurrence of each of the 12 pitches of the equally tempered scale. (However by adjusting a tuning potentiometer almost any micro- or macro-tonal scale with a linear pitch distribution can be made available and a straightforward program modification would allow completely arbitrary pitch systems and modes to be used.)

Some examples of the pitch-distribution data used are given in Fig.1. These could be derived from analyses of existent music (though it should be obvious that output based on such distributions alone is unlikely to have more than the most superficial resemblance to the source.) In these experiments a guess was made at the form of a distribution likely to be suitable for a particular purpose and then successive refinements were made.

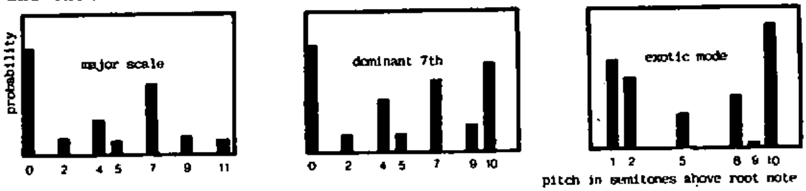

Figure 1. Pitch-Distribution Data

As would be expected, just a few pitches generated from such distributions are enough to give a very clear, say, major, minor or dominant 7th feel to the sequence. Control on the p-level alone however allows very unconventional melodic leaps to occur. For example, if the probability of C is high then all C's in the voice range allowed have an equal and high probability of occurrence. Thus transitions from, say, C_2 to C_6 will be common. This might be ideal in some contexts but often a user will want to impose restrictions on the melodic movement, which needs a higher level of control.

The Δp-level - control of melodic interval (change of pitch)

The Δp-level distributions control movement amongst those pitches allowed by the current p-level distribution. Two examples are given in Fig.2. Here the height of the Δp=1 column governs the probability of a movement from the previously generated pitch to the next highest pitch which has a nonzero probability. Similarly, Δp=-2 is associated with movement to the next-but-one allowed lower pitch. Thus the Δp-level controls movement by numbers of whole steps in whatever mode is implied by the p-level. For example, if we currently have a C-major scale prevailing on the p-level, then the Δp=1 column controls the transitions C → D, E → F, etc., although these intervals are of different sizes.

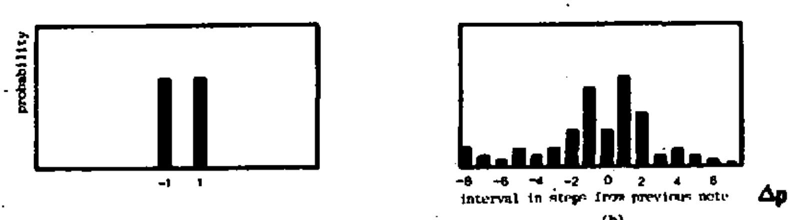

Figure 2. Interval Distribution Data

Roughly speaking, in traditional melodies the frequency of occurrence of intervals decreases as the interval size increases. The distribution shown in Fig.2(b), then, gives output with a plausible, if rather unimaginative, shape. Fig.3 shows how fairly tightly controlled arpeggio motion can be contrived by allowing only the constituent notes of a particular chord on the p-level, and forbidding intervals of greater than 2 steps on the Δp-level.

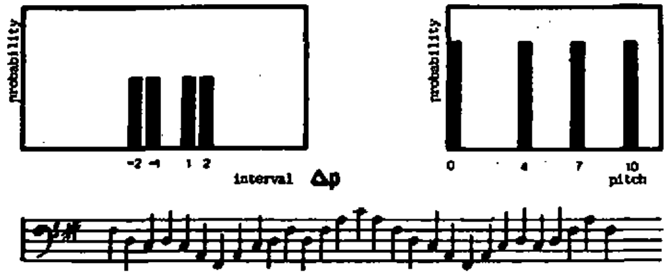

Figure 3. Controlled Arpeggio Motion

The $\Delta^2 p$-level - control of change of interval

In effect we have been considering a melodic line as a plot of pitch as a function of time. It can be seen that the p-level and the Δp-level are related, respectively, to the permitted values of the function itself and of its first time derivative (ie. its gradient). The next and highest level of control ($\Delta^2 p$) is associated with the second time derivative and so determines the permitted curvatures of the melody. Thus nonzero heights for columns corresponding to $\Delta^2 p=n$, where n is of largish magnitude, allow angular melodic movement. For negative values of n this may allow pitch maxima to occur and for positive values, pitch minima, depending on the present and previous values of Δp which have been chosen by the system.

It is by use of this $\Delta^2 p$-level, in conjunction with the lower levels, that some longish-term control of the general melodic shape may be exerted. Consider for example that we require to make the output tend towards that shown in Fig.4(a). It is clear that on the p-level we must allow all the notes of the scale of C-major. The nonzeroness of the particular probability is all that counts here and the actual probabilities have been arbitrarily set equal. On the higher, Δp, level we must allow $\Delta p=-1,+2$ only. However, such two-level control alone is too permissive. There is nothing here to prevent the occurrence of long runs of adjacent downward steps or upward double steps. We therefore look to the $\Delta^2 p$-level to apply the necessary restrictions.

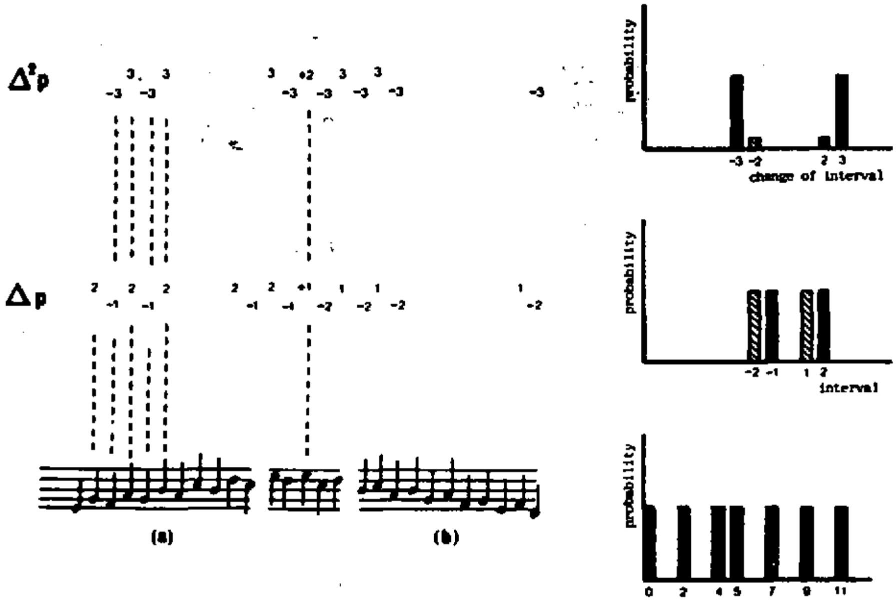

Figure 4. Longer-term Control

Clearly we need to allow $\Delta^2p=\pm3$ to produce this motion. Rather conveniently, this condition actually forces the melody to move upward with exactly the required shape. Since all the voices of the system have a limited range we must allow the melody to descend eventually. Fig.4(b) shows an appropriate descent pattern, which could be forced simply by changing to $\Delta p=-2,+1$ only, on the Δp-level (the hatched columns) and leaving the p- and Δ^2p-levels as before.

However it is sufficient to allow these new values of Δp in addition to the old ones. That is, allowing $\Delta p=\pm1,\pm2$ will cause either continuous upward or continuous downward movement depending on which interval happens to occur first. Allowing only $\Delta^2p=\pm3$ ensures complete stability in the up or the down mode.

To prevent an impasse when the melody hits the top of the range we can set a very small probability ∂ for $\Delta^2p=2$. This is normally negligible in comparison with that of $\angle^2p=\pm3$ but provides an escape route by forcing a flipping from the stable upward to the stable downward motion. Similarly, setting a probability of ∂ for

A REAL-TIME, STOCHASTIC MELODY GENERATING SYSTEM

$\Delta^2p=-2$ forces a flipping at the bottom of the range. The pitch of the output will thus sweep up and down across the chosen range of the voice, automatically reversing at the extremes. If we increase the probabilities of $\Delta^2p=\pm2$ from the infinitessimal then the melody starts to turn around spontaneously without encountering the range limits. We can then control the average lengths of the upward and downward segments by adjusting the probabilities $\Delta^2p=\pm2$ in relation to those of $\Delta^2p=\pm3$.

Other simple melodic shapes can be forced by careful choice of which probabilities are nonzero on the Δp- and Δ^2p-levels.

For example, allowing $\Delta p=\pm1,\pm2$ only
and $\Delta^2p=-3,2,4$ only,

causes the familiar motion (which we may call 'M-shape' - see Fig.6(a)) of which Fig.5 is made up.

Figure 5. A Melody built up of "M-shape" motion

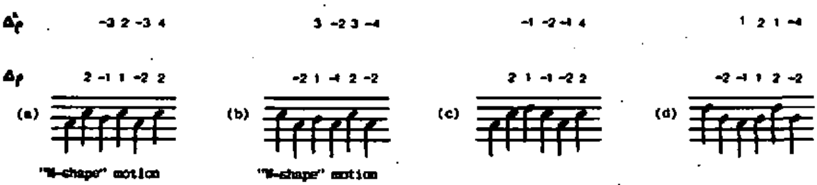

Figure 6.

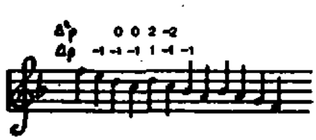

Figure 7.

By allowing instead $\Delta^2p=-4,-2,3$ we force the inverse, W-shape motion of Fig.6(b). The patterns shown in Fig.6(c) and (d) are also easily produced. Thus, merely by setting various values on the Δp- and Δ^2p-level to be either zero or nonzero, static melodic patterns can be contrived. The system allows the user to make continuous transitions between these states, in real time, by moving slider potentiometers each of which can be software assigned to control any probability value.

It is easy to show that there are simple patterns which cannot be forced by fixed distributions of p, Δp and $\Delta^2 p$. Fig.7 has repeated downward steps so we need to allow $\Delta^2 p=0$. It also has isolated upward steps so we need $\Delta p=1$. With this combination there is nothing to stop undesired repeated upward steps from occurring. We can, of course, always program the system to change the distributions as often as required and thereby achieve the pattern of Fig.7 or indeed any arbitrary melody. However, generally speaking, it is contrary to the economic and simple philosophy of the system to contrive specific output at the cost of the large amount of control data which would be required.

In spite of these restrictions the system shows some interesting generality. For example, the melodic shapes of Fig.6 are not confined to intervals of $\Delta p=\pm 1,\pm 2$.

Allowing $\Delta p=\pm x,\pm y$ only
and $\Delta^2 p=-(x+y),+2x,+2y$ only,

gives 'M-shape' motion for many values of x and y. The pattern is preserved but the interval structure is different in each case.

Perhaps more usefully, any of these patterns may be imposed on different scales or parts of scales merely by changing the distribution on the p-level. Thus Fig.5 uses static distributions of Δp and $\Delta^2 p$ (and indeed, here, p) with a momentary, forced $\Delta p=-4$ causing a downward shift of five steps when required.

Technical Specifications of the System

The system software is written in the assembly language of the Texas 990/4 microcomputer. This is a 16-bit machine with a ∿20 μs multiplication time. Although program development is very laborious the resulting code is efficient and fast. About 20 notes per second can be generated on each of four independent voices in real time. The voices are provided by a Synthi 100 voltage-controlled, modular synthesiser which gives considerable timbral flexibility. The computer-synthesiser interface consists of four 4-bit and sixteen 8-bit digital-to-analogue converters and a similar configuration of multiplexed analogue-to-digital converters. The former supply control voltages to the synthesiser and the latter afford the user real-time control of chosen probabilities.

A user-definable library of probability distributions for the three levels of control is stored in an area of random-access memory. Control data for a particular output sequence is also stored in memory. This indicates which distributions are to be used on each level for each voice at any time, and for how long they will prevail. The precise choice of pitch, interval and change of interval for each event is made from the appropriate distribution by sequentially accessing a permanent random-number table. Thus storing the entry point to this table is sufficient to allow an exact repetition of any output sequence if desired.

A REAL-TIME, STOCHASTIC MELODY GENERATING SYSTEM

In contrast to the pitch, the rhythm of sequences is explicitly determined by the user. It may be of almost any degree of complexity but this of course involves the entering of a proportionately large amount of control data. A facility is also provided for programmed timbral and other changes.

Conclusions

Experimental output obtained so far has been generated by relatively simple and unrefined probability distributions. Nevertheless it proves easy to control the tonality and general shape of the pitch sequences and to make plausible approximations to certain traditional musical styles. Whilst this latter is not the direct object of the investigations it provides a useful test of the system and suggests that the severe restrictions imposed on the output are to some extent appropriate ones.

It is planned to produce an augmented, refined and more user-friendly version of the system which will be routinely accessible to composers. It is hoped that they will be attracted by the facility to produce parallel sequences of musical events governed by statistical rules rather than explicitly defined. Further, the real-time manipulation of these rules via potentiometers should provide a new and useful domain of control.

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Acoustic of the Harp

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Historical Outline of the Harp

Egyptian	B.C.	Triangular in shape
Anglo-Saxon	C12th	solid wood sound box
Celtic Harps (16 are extant)	C14th---	ditto with string length scaling rules
Italian	C16-17th	ditto,ditto.
French	C18th	beginning of spruce soundboards
French pedal harp	1720-40	hook action
French, Erad	1792	single action forked action
	1810	double action forked action
Development of superior silk strings for the harp	1798	
Sulphur treatment of gut	1850	

This outline should be compared with the development of other instruments with large numbers of strings. Surviving instruments of the following are later than the surviving members of the harp family

Harpsichord	1525	
Claichord	1543	
Piano	1709	but used mainly after Mozart in the 1770's

The sixteen extant Celtic harps therefore form an excellent set of the earlyiest instruments and should be considered as containing evidence of the sophistication of the makers of that time. They show that harps by the C14th had developed from the triangular shape of the Anglo-Saxon type to one with a definite curve in the strin arm. Definite scaling rules for the strining were being used by makers of the earliest Celtic harps so that it may not be unwarranted to suggest that as makers on the western seaboard of Europe, a region not generally considered as famous for instrumant making at this time, were using rules for the definition of strings, they may also have been using similar rules for the placement of acoustical resonances of the body in advance of the makes of the harpsichord.

In the Celtic harps there are common features(which are often found in other harps as well):

1. Solid soundboard of hard wood. The boxes are made of sallow or hornbeam and it has been suggested that more delicate glues structures could not have withstood the damp climate of the wesrten isles. This is certainly the case i reverse, that modern small harps, the clarsach of Scotland, made in the Western Isles cannot be brought to the eastern cities of Scotland. Later Italian harps also used solid boxes but the dry climate there encouraged developements and differences in style and construction after the instrument had been introduced from Scotland.
2. Strings rising at about 30^0 from the soundboard. This feature leads to a decrease in the breaking tension of the strings, but not so much as occurs with a more acute angle which is found in some examples of early Italian harps. For instance, with gut the breaking tension at 30^0 is reduced to about 33% of the braking tension of a straight pull.
3. Scaling rules are used for string length which ensures that lengths follow a power law relationship over much of the string range: this will be discussed later.

Use of air holes in the soundbox. The immediate reaction to air holes in an instrument is that it utilises a Helmholtz air resonance and that this must form an essential part of the mechanism for sound output. As the loudness index of hardwood is about 150 and that for spruce about 600 the advantage of having this mechanism to support all the other resonances of the body is an important aspect of the Celtic harp. The sound holes mat also be operative at higher frequencies because it is known that in modern soundboxes there are other resonances of the standing wave type in nearly sealed soundboxes and hence the four holes, which are common on the Celtic harp, may also assist in radiating sound at these higher air modes.

Scaling Rules of the Harp: String Length

The author has made measurements of string length on most of the extant Celtic Harps and the string scaling rules have been established. These are listed in the table in terms of the multiplier per octave towards the bass which the maker would have used in setting out the instrument. All measurements used in this paper will be described in this way, and the octave is taken to be an interval of seven strings along the soundboard even in instruments of the C14th.

Brian Boru Trinity College	29 strings	C14th	1.8 middle 2.42treble
Queen Mary	29	c.1500	1.8
Lamont	30	C15th	1.67 middle
Fitzgerald-Kildare	36	C16-17th	1.78
Downhill	30	1702	1.62 or (there are two 1.71 possible stringings)
Bunworth	36	1734	1.7 middle
Otway	34	C17th	1.85 middle
Carolan	35	C17-18th	1.63 middle 1.97 treble
Mullagh Mast	33 early	C18th	1.55

and these should be contrasted with the stringing rules on the modern harp

Concert harp	48	C20th	1.71 or, depending on maker 1.74

The development from thC12th to the C14th was one of decresing the string lengths in the central part of the range of the instrument moving from the triangular shape to one with a curved string arm. The reason for this change may not be so much that the sound of the instrument was improved but that this simple change prvented strings in mid-range from breaking because it is here that strings are too long when graded strings are used.

From the C14th power scaling rules are adhered to in the central part of the compas of the instrument. Treble strings are often shorted that the scaling would indicated but this is so that the string arm may be attached strongly to the soundbox to withstand the pull of strings which are on one side of the string arm. In the bass strings are again less long than is predicted by the string scaling rule and this occurs because to height of the harp is truncated for reasons of portability. In the bass the deficit og string length can be overcome by using overwrapped strings, a practice that is also used in the harpsichord and in the piano.

Scaling Rules for the Harp: String Diameter

In modern instruments a knowledge of string diameter, linear density,Young's modulus and the breaking tensions of strings can be added to the measurements of string length, and hence it is possible to compare the overall stringing rules for the harpsichord and the piano with the harp. It is also possible to make other estimates of the demands on the soundboard of the harp which will be used to support the strings.

Before doing so it is necessary to define a new parameter that is not generally required in consideration of the piano or the harpsichord. FEEL is critical to the harpist in a way that is not so important to the player of the other two instruments. There are two aspects: to the player it is important that the transverse displacement of the string is reasonable for the force applied in order to produce good tone: secondly the displacement which is in the plane of the strings must not be so great that the strings collide once plucked. The first of these is important in a second order way to the pianist, more so to the harpsichordist because of the more fragile nature of the jack, and very importsant to the harpist. The second is only of importance to the construction of the harp for in the piano and in the harpsichord the strings are hammmered and plucked at right angles to the plane of the strings, whereas in the harp the pluck is in the plane of the strings. I have defined FEEL as the displacement transversely to the string during a pluck per unit plucking force

$$FEEL = \ /F = 1/4T$$

where T is the tension in the string of length l.

Table of string parameters   in terms of multipliers per octave to the bass

			PIANO	HARPSICHORD	HARP
String length	l		1.95	1.94	1.71 or 1.74
String Diameter	d		1.05	1.15	1.37 or 1.44
String Tension	T	$\propto (fld)^2$	1.04	1.24	1.37 or 1.56
String Tensile Force		$\propto T/d^2 \propto (fl)^2$	0.94	0.93	0.69 or 0.76
FEEL		$\propto 1/T \propto 1/l(fd)^2$	1.88	1.56	1.25 or 1.11
String Characteristic Impedance	Z	$\propto fld^2$	1.07	1.47	1.6 or 1.77
Frequency	f		0.5	0.5	0.5

In considering this table the main difference are seen to occur in string tension, string tensile force and in FEEL. The tension of harp strings increases dramatically towards the bass but the tensile force nevertheless decreases. The outcome of this is that strings are pitched progessively less than their breaking pitch as the bass is approached in the harp. On the other hand, FEEL changes less than on the piano and on the harpsichord, in keeping with the requirements that have been explained before: to the harpist the FEEL changes at a modest 11% to 25 % per octave and this gives a comfortable working feel to the instrument, whereas the feel for the piano,rising at 88%, would make the instrument unplayable with the fingers.

It may also be remarked at this stage that the increase in string tension, of 1.37 to 1.56 per octave, and the change in string characteristic impedance, of 1.6 to 1.77 per octave, both imply that the thickness of the soundboard should increase toward the bass by an amount which will, in this case carry the increased static tension of the strings, and in the second, maintain a correct balance between the string and soundboard impedances for the extraction of energy from the string.

Before leaving this section in which the historical developement of the harp has been considered, it is not unrealistic to enquire if the same set of scaling rules may have been used in older harps. The answer to this depends on having data on the strings which were used in early harps. For the Celtic harps there is , of course, no data available but it should be remembered that brass and iron were available in the form of wire from the C14-15th and a gauging for the strings may have been in use in the Celtic harps which are known to have been strung with wire. For harps which were strung with gut there is no doubt that it would have been posssible to have used a gauging that would have been close to that used on the modern concert harp. It is not unwarranted to suggest, therefore, that as the string of the harp from the earliest of times follows string length rules that are still being used today string diameters would have been scaled also.

There is strong evidence that by the late C18th strings were being made to an advanced technology, which would imply that similar strings were being made much earlier still. At the time that the harp was being perfected by th French piano makers, Erad, an inventor named Baud from Versailles who was seeking patent approval for a new method of constructing musical strings from twisted silk submitted samples of the strings to the scientific acadamy in Paris in 1798. The practice of submitting sealed packets of a potential invention was common practice at that time and it has only been recently that the academy has been examining such sealed deposits, and it has been found that Baud's contains a full set of strings for the harp made from twisted silk. The numbering on the strings defines a harp with 38 strings of which the lowest ten are constructed of wire wound over silk. The strings are built up of individual bundles of threads that are in turn grouped into sets of bundles. In Baud's strings the twist is all in the same direction (unlike the fabrication of core which follows a similar technique) in order that the surface may be as smooth as possible. An analysis of the diameters of this cache of silk strings from 1798 shows that the strings are scaled according to a power law realtionship, and that the diameter increases by 1.44 peroctave to the bass. This is to be compared with the scaling for modern gut that can be 1.37 or 1.44 per octave depending on the maker. There is strong evidence. therefore, that the manufacture of strings for harps (silk or gut) has remained the same for about a hundred years, in the same way that the scaling of string lengths on the harp has been static for longer. Hence, it may not be unrealistic to suggest that strings have been scaled in an appropriate way since the C14th when string length scaling is known to have been in use.

Scaling Rules for the Harp: String Inharmonicity and String Pitch Distortion

Real string suffer from two defects: inharmonicity of the upper partials and change in pitch due to string distortion. The first defect is due to the stiffness of the string through Young's modulus of the string material and is defined to be

$$f_n = n f_0 (1+Bn^2)^{\frac{1}{2}}$$

$$B = \pi^2 E S K^2 / T l^2 \simeq \pi^2 E d^2 / 64 \rho l^4 f_0^2$$

$$f_0 = (1/2l)(T/\sigma)^{\frac{1}{2}}$$

$$K = d/4 \;:\; S = \pi d^2/4 \;:\; \sigma = \pi \rho d^2/4$$

where E = Young's modulus, d = diameter, ρ = density of the string material Hence, in terms of octave scaling the parameters listed before can be used to evaluate the inharmonicity of strings through the factor B.

	PIANO	HARPSICHORD	HARP
Inharmonicity B	0.305	0.373	0.878 or 0.904

Here the scaling of the diameter and length of the harp conspire to make the inharmonicity larger than on the other instruments as the bass is approached (or rather, less as the treble is approached). On the other hand, it must be remembered that as Young's modulus of gut is $2.83 \pm 0.28 \; 10^9 \; Nm^{-2}$ compared with about 2 10^{11} Nm^{-2} for piano wire and 9 10^{10} Nm^{-2} for brass wire the base from which the inharmonicity of gut commences is much smaller than that for wire. A calculation for B for the three instruments confirms this

	PIANO	HARPSICHORD	HARP
B at middle C	3 10^{-4}	4.5 10^{-5}	8.1 10^{-5}

The change of pitch due to string distortion is caused because the length of the string is extended during the pluck. Consequentialy the tension of the string is increased and the pitch of the string is not stable when it sounds. The relative pitch change is given by

$$\Delta f/f = (\pi E d^2/16T - 1)(\Delta L/L)$$

where the first term arises because of the change of tension , and the second because of the increase in length of the string, and where $\Delta L/L$ is the fractional change in length of the string. It is difficult to quantify the string distortion for the value of $\Delta L/L$ is not defined. Preliminary experiments on the harp show that when the harp is played professionally $\Delta L/L$ is about 4 10^{-4}. If this value is used throughout, it is possible to evaluate the string distortion for the harp and to compare the distortion with the other instruments.

Firstly, calculations on the harp show that inharmonicity of gutstrings is less than the string distortion so that the correctness of tone quality of the harp may depend more on the feel of the strings than on their tension. In comparing string distortion between instruments the term in the above expression that arises from the change in length is smaller than that from the change in tension so that the change in frequency is given by

$$\Delta f/f \simeq \pi E d^2/16T \quad (\Delta L/L)$$

	PIANO	HARPSICHORD	HARP
Pitch Distortion peroctave for $\Delta L/L$= const.	1.06	1.06	1.37 or 1.33

Hence in this example where $\Delta L/L$ is a constant the change in tone quality of gut on the harp is worse than on the harpsichord and on the piano. In both cases, inharmonicity and pitch distortion, the cause is the requirement for a small change in the feel of the strings over the range of the harp.

When the complete range of the harp is considered the pitch distortion can be calculated for the overwound strings (usually of copper over steel) in the bass as well as for the gut strings. The pitch distortion of the wire strings increases in quite an irregular way (partly because they are not completely scaled on this instrument) from about 1.5 - 6% while the distortion for gut increases with a smoother trend from 0.1 - 0.25%. There is always a significant difference between the sound of the wire and the gut strings on the harp, and apart from the increased inharmonicity that is always present which gives the wire strings their bell-like quality, pitch distortion of the wire strings must play a significant role in their tonal quality. The pitch distortion of gut, on the other hand, is less throughout the range than the 2% level which has been suggested as the limit which is tolerable to the ear. The distortion of the wire strings can be improved by reducing the radius of the steel core, using s spun core , or better using a core of a lowere Young's modulus (eg nylon).

<u>Scaling Rules of the Harp: Decay Constants and Impedance Matching</u>

String vibration is reduced by internal friction, air viscosity, sound radiation and energy transfer to the soundboard. The most important of these for thin strings is known to be air damping, although in the case of gut internal friction may be important. If the major mechanism is air damping then only a small fraction of the vibrational energy is passed to the soundboard and thence to the air as sound.

For air damping the decay factor can be written as

$$T_1(f) = (\rho_s/2\pi\rho_a f)(\sqrt{2}/M + 1/2M^2)^{-1}$$

ρ_s and ρ_a are densities of string and of air

$$M = d/4\sqrt{2\pi f/\mu_a}$$

μ_a = kinematic viscosity of air

which can be reduced at low frequencies to

$$T_1 \propto d^2$$

and at high frequencies to

$$T_1 \propto d f^{-\frac{1}{2}}$$

which allow the scaling rules of the decay porcess to be compared with those of the strings

When the main process of damping of the strings is through the transfer of energy to the soundboard the decay depends on the relative impedances of the string and the soundboard and an equivalent circuit analysis suggests that the energy stored in the string will decay at a rate

$$T_2 = [2\pi d^2 \rho_s \, l \, f^2 \; G(n,f)]^{-1}$$

where G (n,f) is the real part of the soundboard admittance. Again then can be reduced to show the scaling rules if the assumption is made that the soundboard admittance is reasonably constant, which is not, in fact, as will be discussed later. The reduced form is

$$T_2 \propto d^{-2} l^{-1} f^{-2}$$

The resultant combined time constant for any string will be

$$T^{-1} = T_1^{-1} + T_2^{-1}$$

Considering now the scaling rules that are implied for the damping of string vibrations because of the construction of the instrument a comparison can be made between the piano, harpsichord and the harp.

	PIANO	HARPSICHORD	HARP
Air Damping T_1			
Low frequency	1.10	1.32	1.87
High frequency	1.48	1.63	1.94
Soundboard Damping T_2 per octave to the bass	1.86	1.56	1.25

There is some evidence that the decay rates in the harp follow the expectations of the above table. Measurements on the initial decay of sound of the pluck of note shows T_1=2.3 per octave, which is more a rapid change then calculated above, and T_2=1.9 per octave to the bass which is in agreement with the above. In the decay process in the harp there are always complicated decays and there are often long lasting exponential parts to the process. Generally a fast decay is followed bya slower decay in all but the highest and the lowest notes.

The final aspect of the scaling of the harp that should be considered is that of the thickness of the soundboard. Of all the three instruments that have been considered the harp has to have a static strenght to withstand the outward component of the tension of the strings on the soundboard. In the construction of the harp this feature is often given priority in design. The tension of the strings can be calculated fairly easily but the overall strength of the soundboard fixed to the soundbox with the distributed load of the strings is not such an easy problem to deal with. Nevertheless, an approximation can be made by considering the strength of the soundboard to be largely due to the spruce under the string across the width of the soundboard in which case the ultimate strength of the soundboard is $Y \propto H_b^2$, making the strength of the soundboard alter at about 1.58 per octave to the bass. This is to be compared with the scaling of the string tension which is between 1.37 and 1.56 per octave. Hence, soundboards appear to be matched in strength to the tension of the strings, at least in scaling.

Another comparison can be made between the soundboard and the strings. The characteristic impedance of the strings should match the impedance of the thickness of the wood into which they are working. Neglecting for the moment, that resonances exist in the soundboard, it is instructive to compare the characteristic impedance of the strings $Z_s \propto fld^2$ to the impedance of the body wood under the strings $Z_b \propto H_b^2$. For soundboards of the modern concert harp using gut strings measurements show that $Z_s/Z_b = 1$. In these sectins on the scaling rules used in the three similar instruments which have large numbers of strings, the scaling rules give valuable information about the acoustical sophistication that has been developed by makers over the centuries. Although more detailed measurements are required to confirm the absolute values of the parameters that have been mentioned it is clear that in the harp there is a correct balance between the expected acoustical behaviour and the actual from a point of view of scaling rules.

Design of the Stringing of the Harp

Measurements of the breaking tension of hut and its Young's modulus have been made and on the basis of these the design of the stringing of a particular instrument can be understood in terms of the pararmeters so far discussed: tension, feel, inharmonicity, string distortion etc.

From measurements of the tensile strenght of gut what becomes patently clear is that the working tensile stress is far below that at breaking. This finding is at varience with the usual practice with brass strings where the working tension is only a few semitones below the breaking tension. With gut there are two breaking tensions that are important: maximum breaking tension is achieved when the string is pulled straight, but the breaking tension on the harp is always less than this by about 620 cents because the gut string turns through about 30° once it is brought through the soundboard. The actual tension of the working string is less that this again by about 320 cents at the treble and 660 cents at the bass of the gut range. This means that overall the working tension is smaller by 950 cents at the treble and 1280 cents, more than an octave, at the bass. This seems to be the common arrangement of tension in many modern harps. Working under this gross reduction from breaking and still producing a good tone is only possible in the gut range of the harp because of the low value of Young's modulus.

When the whole string range of the harp is considered the break between gut and wire has to be considered. Plots of string tension show that the break is not traversed smoothly on most harps but is accompanied by a jump in tension from gut to wire which may be as large as a 40% increase between adjacent strings. There is comparable drop in feel at the break and the tone quality of the sound between adjacent string at the break has already been shown to be significantly different both in inharmonicity and string distortion effects. In the piano and in the harpsichord there are also noticable effects at the break between plain and overwound strings, but because the covers of the strings are made from the same material this can lessend somewhat by careful adjustment of linear density, length and by adjustments to the bridge to alter the input impedance to the soundboard. These adjustments are not fully available on the harp and the best remedy would seem to be to use filamentary nylon as the core of the over-wrapped strings in the bass of the harp.

Action of the Soundboard

The harp, like the piano and the harpsichord, have many strings entering the soundboard among its whole length so that the input admittance will not be the same for each string. The admittance seen by a string will depend upon where the string enters the standing wave pattern of the soundboard. It has been established, in a variety of harps, that the soundboard sustains resonances and these have been detected by Chladni powder methods and its optical interference methods. It is possible to detect a soundboard resonance attributible to the Helmholtz resonance of the air cavity, and also to trace a families related resonances on the soundboard. It has been shown that the sound output of a string depends on whether it enters the soundboard at a node or at an antimode. Strings tuned to a particular resonant frequency entering the soundboard at a nodal position, (high impedance), have difficulty in speaking whereas those entering at an antinode produce a booming sound.

The sound quality of the harp can be adjusted in two ways: if the overall stiffness of the board is increased than the sound is less but more uniform. This is probably due to the clustering of resonances and a lessening of the peak to valley ratio of the input admittance along the soundboard. If the soundboard is thinner, and hence more flexible, the sound output is enhanced,and in many ways the instruments has a more noticible final character. The other way in which the sound output may be adjusted is to shift the positions of the nodes and antinodes of the resonant modes of the board so that no string is tuned close to a nodal position.

Action of the Air inside the Soundbox

The air inside the cavity of the soundbox communicates with the atmosphere through five holes in the back of the box on the modern harp. In the Celtic harp the sound holes were often placed on the front of the instrument on the soundboard. Experiments have been conducted on the modern harp which show that the air sustains a Helmholtz resonance and resonances of the standing wave type. In the case of the Helmholtz resonance it is found, as is usual, below the first main resonance of the soundboard, and the two couple in the usual way to produce comporite resonance so that the air resonance is lowered in frequency and the true soundboard plate resonance is raised in pitch, and at each the soundboard has large displacements. As is other instruments, the sound output in the bass of the instrument is dictated by these two lowest resonances. In the modern concert harp the air resonance is found at about 178. Hz whereas the lowest wire string on the harp is at 36.7 Hz, that is more than two octaves below (2720 cents). The importance of the air resonance and the first plate resonance is seen immediately when an average spectrum of played music is taken; sound input rises very steeply to the air resonance with practically no sound emission from the instrument below its frequency. The bass wire string are therefore heard through their harmonics which are above the air resonance!

At higher frequencies than the air resonance standing waves are found in the air column of the soundbox. Experiments with the sound box holes closed, except for one, indicate that the box acts as a conical pipe and substains harmonic modes. With sound holes open these modes are still present, displaces in frequency and diminished im amplitude. Sound output from the rear holes of the soundbox is large at air resonances and experiments are planned to investigate the directivity of sound and to investigate the positioning of the sound holes in order to maximise the output.

When the shape of a Helmholtz resonator is simple there is only one true Helmholtz resonance, although there are other higher modes within the cavity. The harp soundbox is not simple and contains sections which are divided internally with braces so that it may be considered as a composite air body with each air hole working into its own section of air volume of the box. Such composite Helmholtz resonators are employed as absorbers in noise control problems, and as amplifiers in some musical instruments. A model of the air cavity of the harp has been made to invertigate the possible behaviour of the harp acting in this way. The box was considered in the model to act as a composite element composed of five intercommunicating Helmholtz resonators. Elementary circuit ideas were applied to this to construct the analogous circuit and the circuit analysed for sound output.

The analysis shows that for the dimensions of the real harp there are two air resonances of the Helmholtz type in the frequency range considered. When the areas of the internal windows between the braces is reduced in the computation, the resonant peaks tend to seperate one from another until there are five individual peaks associted with the individual seperate cavities in the box when the area linking the individual parts together is finally reduced to zero. In the real harp the two peaks are seperated in frequency, but in the case of having reduced windows between the sections, the peaks cover a wider frequency range but, being more numerous, give a more uniform response for sound output from the holes at the rear of the instrument.

<u>Transient Action of the Harp at Plucking</u>

The impluse of plucking any stringed instrument shock excites all the resonances of the instruments. These decay exponentially with their individual decay times and the remaining vibration of the instrument is that due to the string which generally has a decay longer than any of the body modes. This has been invertigated for the guitar in which it has been shown that at onset of the pluck the sound output comprises vibrations at all the body modes which decay within a few tens of milliseconds to leave the 'sustaining' tone of the string. The same effect is observed in the harp. Hence within the first few tens of milliseconds of each pluck of the instrument there is information about its whole tone quality and this is followed by the more pure harmonic tone of the strings. The resonances of the air and body of the harp are therefore not only of importance in determining the amount of sound that the instsument can produce at any frequency but are also of importance in determining the quality of the onset of the plucked notes.

On corollary of this effect is that it is possible to obtain information about some of the important resonances of an instrument without performing any experiments on the instrument! By analysing the sound at onset of the notes it is possible, in many cases, to find the main resonances of the air or body and this is especially easy, and certain, in the bass of the range of an instrument where the resonances are few and well spaced in frequency. Hence, recordings of old instruments, which cannot be played any longer, can be used to find the main low frequency resonances, with a certain degree of confidence.

Overall Sound Output fron the Harp

Making live averages of played scales or played music of the harp gives a good indication of the overall sound output. The main observations on the results so obtained are; Sound output is very small below the air resonance but rises very sharply to this resonance at about 178Hz; sound output is then fairly constant to about 1K Hz and then begins to fall exponentially with rising frequency to a cutoff at about 3 kHz; there are minor features within this overall pattern but these occur at low frequencies below Hz.

Intergrating the sound output of played music indicates that of sound output is produced by the harp below 1K Hz. This finding confirms the usual critiscism of the harp that it is very weak in the treble, and the maker's hope that it can be developed to be stronger in the upper range.

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