ACTIVE CONTROL OF THE NOISE OF A RIJKE TUBE

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A Rijke tube is a straight tube with two open ends and a hot wire gauze stretched across inside it. It can be used as an elementary model for combustion instabilities involving the propagation of longitudinal pressure waves in ducts.

There is an unsteady heat transfer Q from the hot gauze to the air. Q at a time t depends on the flow velocity u at a somewhat earlier time. For linear considerations this dependence is

$$Q'(t) = \beta\, u'(t-\tau) \tag{1}$$

with τ being the time lag and the factor β depending on the gauze temperature, mean velocity and diameter and length of the gauze wire. Q' and u' are the unsteady components of the heat transfer and velocity respectively.

Under certain conditions the unsteady heat transfer feeds energy to the waves in the tube. Instability occurrs if this energy gain is larger than the loss of acoustic energy at the ends of the tube. This produces a loud booming sound of discrete frequencies with the fundamental frequency dominant. An instability can be cancelled by the feedback system shown in Fig. 1.

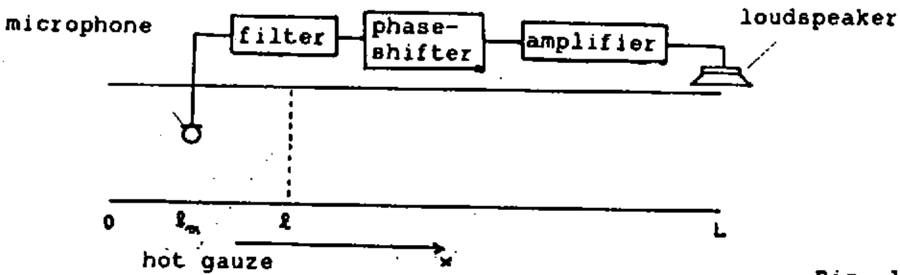

Fig. 1

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A microphone at $x=\ell_m$ in the tube picks up the sound and passes the signal to a filter which lets through only the fundamental frequency. This is then phase-shifted, amplified and fed back into a loudspeaker at one end of the tube. The acoustic field in the tube produced by the loudspeaker superimposes on the original field caused by the hot gauze. The pressure and velocity field caused by the hot gauze can be expressed as

$$p_1(x)=\begin{cases}\dfrac{q_1}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(1+R_Le^{\frac{2i\omega}{c}(L-\ell)}\right)\left(e^{\frac{i\omega}{c}(\ell-x)}+R_0e^{\frac{i\omega}{c}(\ell+x)}\right) & \text{in } [0,\ell]\\[2ex] \dfrac{q_1}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(1+R_0e^{\frac{2i\omega\ell}{c}}\right)\left(e^{\frac{i\omega}{c}(x-\ell)}+R_Le^{\frac{i\omega}{c}(2L-\ell-x)}\right) & \text{in } [\ell,L]\end{cases} \qquad (2)$$

$$u_1(x)=\begin{cases}\dfrac{\frac{q_1}{\rho c}}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(1+R_Le^{\frac{2i\omega}{c}(L-\ell)}\right)\left(-e^{\frac{i\omega}{c}(\ell-x)}+R_0e^{\frac{i\omega}{c}(\ell+x)}\right) & \text{in } [0,\ell]\\[2ex] \dfrac{\frac{q_1}{\rho c}}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(1+R_0e^{\frac{2i\omega\ell}{c}}\right)\left(e^{\frac{i\omega}{c}(x-\ell)}-R_Le^{\frac{i\omega}{c}(2L-\ell-x)}\right) & \text{in } [\ell,L],\end{cases} \qquad (3)$$

if the gauze is considered as a monopole source at $x=\ell$ in a tube of length L. ω is the angular frequency. R_0 and R_L are the reflection coefficients at the tube ends $x=0$ and $x=L$. The pressure and velocity field produced by the loudspeaker are

$$p_2(x)=\frac{q_2}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(e^{\frac{i\omega}{c}(L-x)}+R_0e^{\frac{i\omega}{c}(L+x)}\right) \qquad (4)$$

$$u_2(x)=\frac{\frac{q_2}{\rho c}}{1-R_0R_Le^{\frac{2i\omega L}{c}}}\left(-e^{\frac{i\omega}{c}(L-x)}+R_0e^{\frac{i\omega}{c}(L+x)}\right) \qquad (5)$$

The complex quantities q_1 and q_2 are a measure of the strength of the two acoustic sources, hot gauze and loudspeaker, respectively.

The feedback system couples the loudspeaker motion to the pressure field at the microphone. This coupling can be expressed as

$$p_2(\ell_m)=\alpha e^{i\phi}\left(p_1(\ell_m)+p_2(\ell_m)\right) \qquad (6)$$

$\alpha\, e^{i\phi}$ is the transfer function between the pressures $p_2(\ell_m)$ and $p_1(\ell_m)+p_2(\ell_m)$, ϕ being the difference between their phases and α the ratio of their magnitudes.

The stability criterion is

$$\begin{matrix}\text{stable} \\ \text{unstable}\end{matrix} \quad \text{if } \Delta E \lesseqgtr 0, \tag{7}$$

where ΔE is the difference between gain and loss of acoustic energy in the Rijke tube. For a tube with negligible mean flow and a uniform mean density $\bar{\rho}$ and uniform speed of sound c

$$\Delta E = \frac{\gamma-1}{\bar{\rho} c^2}\, \overline{p'(\ell) Q'} - A\left[\overline{p'(L) u'(L)} - \overline{p'(0) u'(0)}\right] \tag{8}$$

where $p'(0)$, $p'(\ell)$, $p'(L)$ are the pressure perturbations at $x=0$, ℓ, L, and $u'(0)$, $u'(L)$ are the velocity perturbations at $x=0$, L. γ is the specific heat ratio and A is the cross-sectional area of the tube.

ΔE can be expressed in terms of α and ϕ with (1) to (6). The numerical results are shown in Fig. 2 for an originally unstable Rijke tube with $\ell=0.3$ m, L=1 m, $\gamma=1.4$, $\bar{\rho}=1.2$ kg m^{-3}, c=358 m s^{-1}, $\beta/A=4.6 \cdot 10^4$ kg m^{-1} s^{-2}, $\tau=2.87 \cdot 10^{-4}$ s.

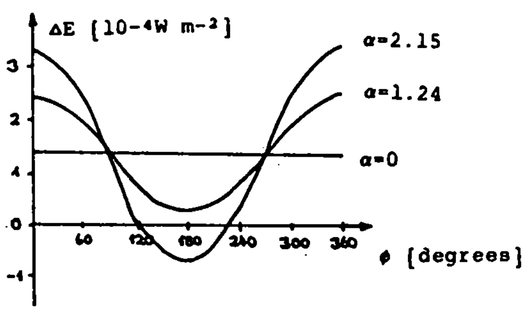

Fig. 2

Stability occurrs if α is sufficiently high, e.g. for $\alpha=2.15$ in a phase range between 150 and 210 degrees (Fig. 2). This agrees with observations.