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INFLUENCE OF CALORICALLY PERFECT GAS ASSUMPTION AND THERMAL DIFFUSION ON INDIRECT NOISE GENERATION

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With the reduction of jet and fan noises over the last decades, combustion noise now emerges and needs to be reduced. It is basically decomposed into direct noise and indirect noise, the latter originating from the acceleration of entropy and vorticity fluctuations by the mean flow. Models used to predict indirect noise rely on several assumptions, among which the calorically perfect gas assumption and the negligible thermal diffusion have never been addressed and are the subject of the study. Investigations are conducted for entropy-generated noise using the flow solver Sunday of ONERA. The effect of varying c_p is evaluated for a nozzle flow with a temperature variation representative of a turbine stage and maximum variations of 10% in generated noise are observed compared to a calorically perfect gas. Thermal diffusion is then evaluated for a Nozzle Guide Vane corresponding to a High Pressure Turbine inlet and for a uniform flow. It is shown to be negligible even with very low velocity. The present study hence validates these two assumptions commonly used for low-order models in combustion noise.

Keywords: combustion noise, entropy noise, thermoacoustics, low-order model

1. Introduction

With the important increase of air traffic occurring nowadays, noise pollution around airports and heliports is becoming a major environmental issue. Among all the noise sources of an aircraft, engine noise remains particularly important. For a long time, it has been widely dominated by jet and fan noise until efforts for their reduction led to the emergence of combustion noise typically between 300 Hz and 1000 Hz [1]. Considering helicopter noise, the contribution of combustion to the total engine noise is even more important due to the absence of jet and fan.

Combustion noise is classically decomposed into direct and indirect noise. Direct noise refers to pressure fluctuations generated by the unsteady heat release of the flame while indirect noise corresponds to additional noise produced when heterogeneities such as vorticity (vorticity noise) or hot/cold spots (entropy noise) are accelerated by the mean flow [1]. Among all the studies performed, a major result is the analytical modelling of indirect noise through a nozzle proposed by Marble & Candel [2], extended to the turbine by Cumpsty & Marble [3]. These models remain the fundamental bricks for all the recent models (see for instance [4, 5, 6, 7]) and rely on the following hypotheses:

- H1 Heat exchanges between the walls and the flow are negligible (adiabatic walls).
- H2 The flow is inviscid.
- H3 The flow is isentropic outside of a possible shock.
- H4 Perturbations are small hence the model is linear.
- H5 The gas mixture has a single component and the effects of heterogeneities in mixture composition are not considered.

H6 The mean flow is quasi-one-dimensional (nozzle flow) or two-dimensional (turbine flow).
H7 The fluid is calorically perfect (the heat capacities at constant pressure c_p and constant volume c_v and the adiabatic coefficient γ are constant).
H8 Thermal diffusion is negligible.

Among the hypotheses listed above, H1 is a reasonable hypothesis for the mean flow in experimental facilities operating at ambient temperature [8, 9, 10, 11]. Its reliability is however questionable for temperature perturbations and for real engines, in particular for the turbine that is cooled to support the very high temperature gas coming from the combustor. For such configurations, wall cooling could be taken into account by considering the associated entropy variation in the jump relations used for the compact models (low-frequency limit). Viscosity is not considered to play a role in indirect noise generation (H2) but its impact has never been quantified and, as noted by Morgans & Duran, this assumption is uncertain in such flow with intensified viscosity [12]. H3 is a consequence of H1 and H2, as wall cooling and viscosity are important sources of entropy variation in the flow. H4 is globally valid for medium to high frequency perturbations, because such perturbations have short wavelengths and are subject to intense turbulent dissipation. For low frequencies, where large temperature fluctuations can be expected, a nonlinear model has been proposed by Huet & Giauque [13, 14] for the nozzle. H5 has been discussed recently by Magri *et al.* [15] and this noise source, dependent on the local mixture composition, was shown to possibly exceed entropy noise for fuel-lean conditions and supercritical nozzle flows. For nozzle flows, H6 indicates that the flow varies in the axial direction because of section reduction or expansion but no radial or azimuthal variations are considered. This restricts the physics to planar acoustic and entropy waves only. Contribution of the azimutal component has been addressed by Stow *et al.* [16], Dowling & Mahmoudi [1] and Duran & Morgans [17] by considering a thin annular duct where radial variations of the flow are neglected. Considering that the radial flow gradients inside the nozzle strongly deform the waves entering the nozzle and thus may modify the generated noise, Zheng *et al.* [18] derived a 2D model for the nozzle taking into account these radial gradients. For the turbine, the flow variation is basically two-dimensional to take into account the circumferential component of the turbomachine. Radial flow variations seem negligible for noise generation but important for an accurate attenuation of the entropy waves [19]. In addition to that, the models assume the convection of entropy perturbations outside of the nozzle or turbine to occur without dispersion, caused for instance by the turbulent mixing or the mean flow gradients. Analytical and numerical analyses performed by Sattelmayer [20] and Morgans *et al.* [21, 22, 23] however showed that entropy waves are likely to disperse before reaching the end of the combustor. To end, H7 and H8 have never been addressed and are the subject of the present study.

2. Methodology

Investigations are conducted for entropy noise. The influence of the calorically perfect gas assumption and of the thermal diffusion on the planar acoustic and entropy modes is considered to be representative of the physical phenomena encountered and the study is conducted in the linear regime. Simulations are performed with the code Sunday of ONERA [5, 13, 14], which solves the quasi-one-dimensional flow equations for the conservative variables, Eqs. (1)-(3):

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = -\rho u \frac{1}{A}\frac{dA}{dx} \tag{1}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 + p}{\partial x} = -\rho u^2 \frac{1}{A}\frac{dA}{dx} \tag{2}$$

$$\frac{\partial \rho e}{\partial t} + \frac{\partial \left(\rho e + p\right) u}{\partial x} = -\frac{\partial q}{\partial x} - \left(q + \left(\rho e + p\right) u\right)\frac{1}{A}\frac{dA}{dx} \tag{3}$$

In these equations, t stands for the time, x for the axial coordinate and A for the section of the geometry. ρ, u, p and e are the density, velocity, pressure and fluid energy, respectively, with $e =$

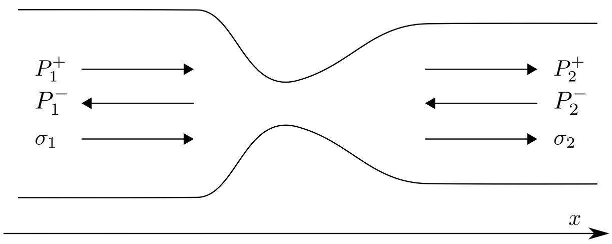


Figure 1: Sketch of the waves present in quasi-one-dimensional nozzle flows.

$e_s + \frac{1}{2}u^2$, e_s being the sensible energy of the fluid defined as:

$$e_s = e_s^{st} + \int_{T^{st}}^{T} c_v(\tau) d\tau, \qquad e_s^{st} = e_s(T = T^{st}) \tag{4}$$

where T is the temperature and $(\cdot)^{st}$ indicates a flow variable taken at the standard state. To end, q is the heat flux, related to the temperature gradient using the Fourier law: $q = -\lambda \partial T/\partial x$ with λ the thermal conductivity of the fluid. The thermal conductivity is evaluated using Eucken's law, $\lambda = \mu\,(c_p + 5r/4)$ with μ the dynamic viscosity and r the ideal gas constant for the considered fluid. The viscosity is obtained using Sutherland's law, $\mu(T) = \mu_{ref}\,(T/T_{ref})^{3/2}\,(T_{ref} + T_S)\,/\,(T + T_S)$. In this equation, T_{ref} is the reference temperature, T_S the Sutherland temperature and μ_{ref} the dynamic viscosity at the reference temperature.

Numerical resolution is performed in Sunday with a finite difference approach using high-order time and space schemes, an adaptative spatial filtering for shock capturing [24] and perfectly non-reflective characteristic boundary conditions [25].

The influence of the calorically perfect gas assumption and thermal diffusion on noise generation is quantified through the evaluation of the transfer functions of the nozzle. These transfer functions represent the ratio between the waves leaving the nozzle and the forcing, as a function of the frequency. In this study nondimensional waves are used, see Fig. 1. They write:

$$P_i^{\pm} = \frac{1}{2}\left(\frac{p'}{\gamma \overline{p}} \pm \frac{u'}{\overline{c}}\right), \qquad \sigma_i = \frac{s'}{c_p} \tag{5}$$

where P^+ and P^- correspond to the progressive and regressive acoustic waves, σ to the entropy wave and the overlined quantities $\overline{(\cdot)}$ to mean flow values. $i = 1$ in the region upstream of the nozzle and $i = 2$ downstream. All the simulations presented in the document are performed with air.

3. The calorically perfect gas hypothesis

A basic assumption of the low-order models is to consider a calorically perfect fluid. This hypothesis is done classically when considering the propagation of acoustic perturbations, as the associated temperature fluctuations remain low even for large amplitude perturbations. However, it can be incorrect when considering a steady mean flow with large temperature variations, as c_v is involved in the evaluation of the sensible energy e_s of the fluid and its variation modifies the mean flow profiles. In particular, Marble & Candel [2] showed that the local indirect noise source term is directly linked to the mean velocity gradient, so that any modification of the mean flow impacts indirect noise generation. In aero-engines, temperature may vary between 600 K and 1500 K. For air, it corresponds to a variation of γ between 1.38 and 1.31. The importance of this variation is assessed in this section. To this end, the noise generated by a flow with a temperature variation representative of the flow through a high-pressure turbine is considered [26]. The geometry is modelled by a supercritical nozzle with an inlet temperature $\overline{T}_1$ = 1200 K, a temperature ratio between outlet and inlet $\overline{T}_2/\overline{T}_1 \simeq 2$ and an outlet

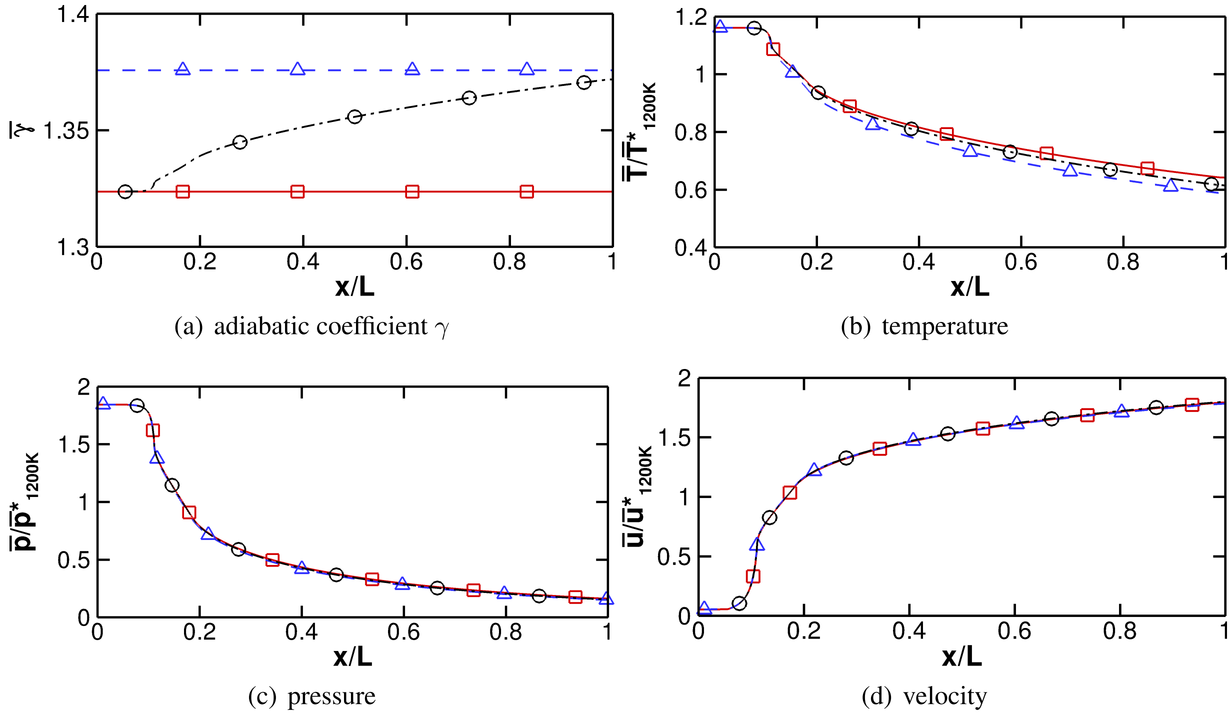
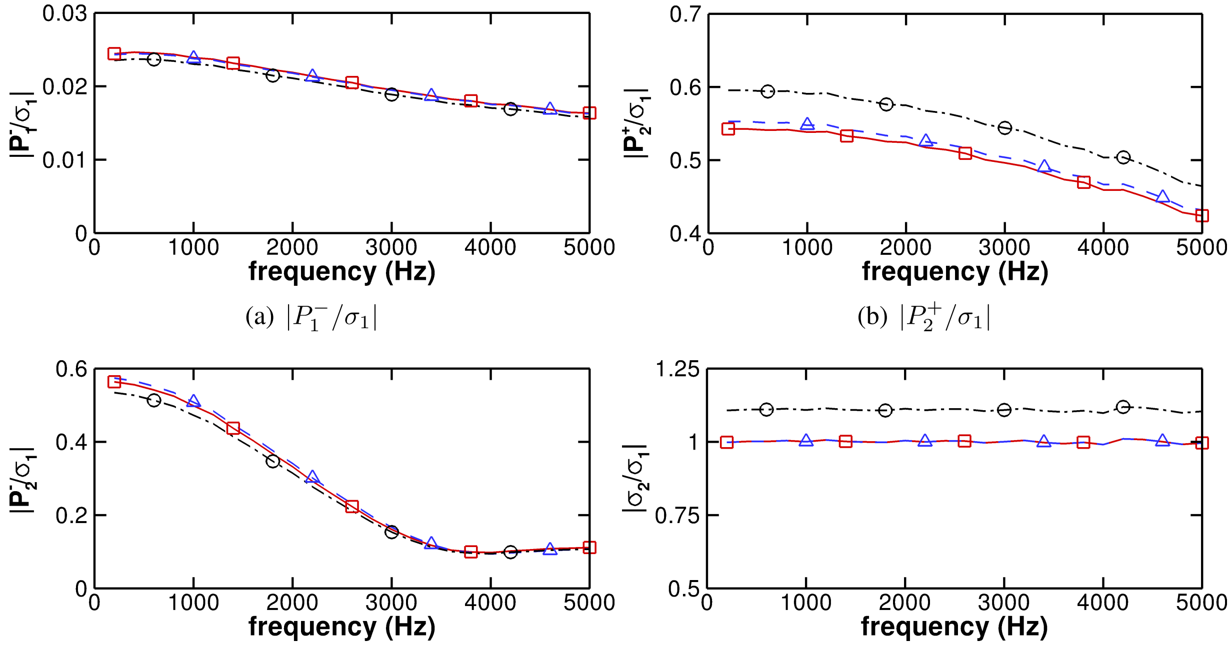


Figure 2: Mean flow fields evolution along the nozzle. —□ : γ_{1200K}; – –△ : γ_{600K}; – · ○ : $\gamma_{variable}$.

static pressure $\overline{p}_2 \simeq 100000$ Pa. It must be mentioned that the mean flow field is not representative of the flow through a turbine but it is sufficient to evidence the effect of variable γ on the generation of indirect combustion noise.

Simulations are performed for 3 calorically different behaviours of the fluid. γ_{600K} and γ_{1200K} correspond to simulations performed with a calorically perfect gas, while the adiabatic coefficient varies with the temperature along the nozzle for the last computation $\gamma_{variable}$. The mean flow evolutions along the nozzle are reproduced in Fig. 2, where the mean flow variables have been nondimensionalised by the values at nozzle throat for configuration γ_{1200K} and the axial distance by the length of the nozzle L. The variation of γ along the nozzle is clearly visible in Fig. 2 (a) for the case $\gamma_{variable}$. Slight differences due to the variation of γ are observed for the temperature field, Fig. 2 (b), where the temperature for the simulation $\gamma_{variable}$ is bounded by γ_{600K} and γ_{1200K}. Finally, the pressure and velocity profiles reproduced in Figs. 2 (c)-(d) are very similar for the 3 simulations, which indicates that the effect of γ is negligible for the evolution of those quantities.

The indirect combustion noise generation being directly linked to the mean velocity gradient of the flow [2], it is expected from the results above that the assumption of constant γ has a limited impact on the generated noise. This is confirmed by the computed thermoacoustic transfer functions, reproduced in Fig. 3 (a)-(c). In this figure, the three acoustic waves generated inside the nozzle by the acceleration of the entropy are very similar using either constant or varying γ. The most important difference is observed for the wave P_2^+, where using a varying γ increases the amplitude of the generated wave by 10% compared to the constant γ hypothesis. This difference remains however low and the calorically perfect gas hypothesis is globally valid for the computation of indirect combustion noise. Finally, Fig. 3 (d) reproduces the entropic transfer function. For inviscid flows, outside of shocks entropy is simply convected by the mean flow without attenuation, so that the modulus of the entropic transfer function is 1 for all frequencies using constant γ. With a varying gamma, however, the decrease of mean temperature from 1200 K to 600 K decreases c_p by 10%, hence increasing σ by 10%. This analytical increase of the nondimensional entropy fluctuation is well reproduced numerically. To end with this section, it has been verified that the variations observed in Fig. 3 do not come from the



(a) $|P_1^-/\sigma_1|$ (b) $|P_2^+/\sigma_1|$

(c) $|P_2^-/\sigma_1|$ (d) $|\sigma_2/\sigma_1|$

Figure 3: Influence of the constant γ hypothesis on the thermoacoustic transfer functions of the nozzle. —□ : γ_{1200K}; – –△ : γ_{600K}; – · ○ : $\gamma_{variable}$.

$T_{ref}(K)$	T_S (K)	μ_{ref} (kg.m^{-1}.s^{-1})	c_p (J.K^{-1}.s^{-1})	r (J.K^{-1}.s^{-1})
300	110.4	1.8 $\times 10^{-5}$	1003.4	286.71

Table 1: Parameters used for computation of the thermal conductivity λ.

non-dimensionalisation of the pertubations. Transfer functions of the dimensional quantities exhibit similar variations with differences limited to 20% at the maximum.

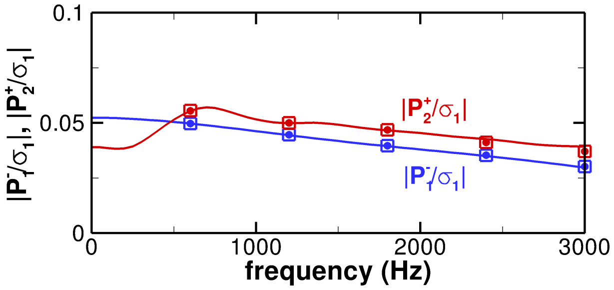
4. The influence of thermal diffusion

The models used to evaluate the indirect combustion noise classically neglect thermal diffusion. The velocity inside the combustion chamber and at the inlet of the nozzle guide vane (NGV) is however low and some thermal diffusion may occur, reducing the amplitude of the entropy fluctuations along the flow path and hence the local noise sources. The role of this diffusion is investigated numerically and analytically in this section.

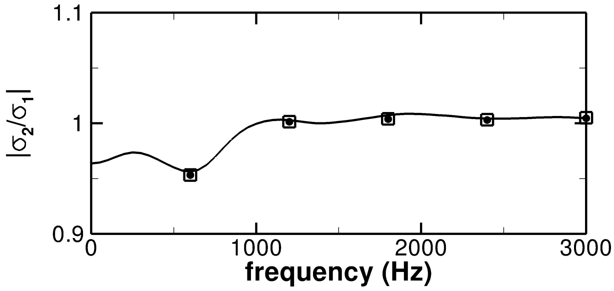
4.1 Transfer functions of a NGV

In a first step, thermoacoustic transfer functions are computed without and with thermal diffusion. The geometry considered is the HAT nozzle designed at DLR [27]. The nozzle has a length L of 39 cm and its shape corresponds to the cross-sectional change of a typical NGV geometry at the high pressure turbine inlet of aero-engines. Flow conditions are chosen to be representative of an aircraft at take-off condition. The flow is choked with a shock located in the diffuser. Boundary conditions are $\overline{T}_1 = 1200$ K, $\overline{p}_1 = 30$ bar and $\overline{p}_2 = 29.7$ bar. Simulations are run using a calorically perfect gas. The Mach number varies between 0.11 at nozzle inlet and outlet to 1.18 upstream of the shock and the thermal conductivity λ varies between 0.053 J.m^{-1}.s^{-1} and 0.061 J.m^{-1}.s^{-1}. The values of the coefficients used to compute the thermal conductivity are given in Table 1.

The transfer functions computed with Sunday without and with thermal diffusion are plotted in



(a) thermoacoustic transfer functions



(b) entropic transfer function

Figure 4: Influence of thermal diffusion on the thermoacoustic transfer functions of the HAT nozzle. — Marcan analytical solution without thermal diffusion; □ : Sunday simulation without thermal diffusion; • Sunday simulation with thermal diffusion.

Fig. 4, where are also reported for clarity the modelled transfer functions computed with Marcan, the low-order code of ONERA for indirect combustion noise [5, 28] (quasi-one-dimensional hypothesis, calorically perfect gas, no thermal diffusion). The presence of thermal diffusion does not modify the thermoacoustic transfer functions, meaning that there is no change in the noise generation inside the nozzle. This result is explained by the transfer function of the entropy fluctuations, Fig. 4 (b). The amplitude of the entropy fluctuations at nozzle outlet is identical for the two simulations. It indicates that the diffusion phenomenon does not have time to occur, hence the identical generated noise.

4.2 Thermal diffusion in a uniform mean flow

The thermal diffusion is negligible inside the NGV because the convection velocity is sufficiently high for the diffusion phenomenon not to take place. This may not be the case in the combustion chamber, where the entropy spots are convected downstream of the flame toward the NGV at a very low velocity, $\overline{u} \sim 10$ m/s. To end the investigation, the thermal diffusion in the downstream part of the combustion chamber is evaluated. Let us consider a 1D uniform flow with a mean velocity $\overline{u}$ and a constant thermal conductivity λ. Assuming that only small entropy perturbations are present, Eqs. (1)-(3) reduce to the convection equation for entropy:

$$\frac{\partial \sigma}{\partial t} - D\frac{\partial^2 \sigma}{\partial x^2} + \overline{u}\frac{\partial \sigma}{\partial x} = 0 \tag{6}$$

where $D = \lambda/\overline{\rho}c_p$ is the thermal diffusivity of the fluid. The harmonic solution of this equation writes $\sigma(x,t) = \sigma_0(x)e^{i\omega t}$ with $\sigma_0(x) = \sigma_0(x=0)\exp\left(\overline{u}\left(1 - \sqrt{1 + 4i\omega D/\overline{u}^2}\right)x/2D\right)$.

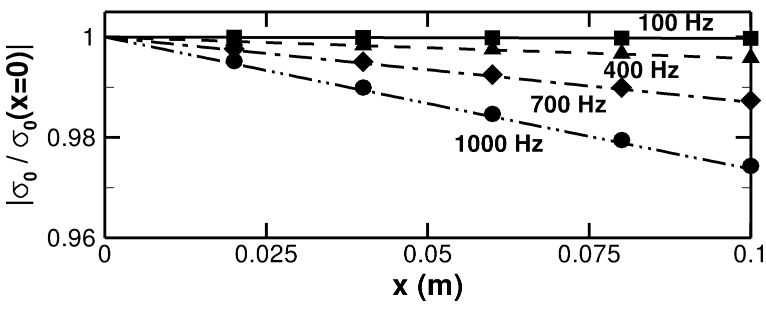


Figure 5: Damping of an entropy wave due to thermal diffusion as a function of the axial distance for different frequencies. Lines correspond to the analytical model and symbols to Sunday simulations.

Figure 5 illustrates the damping of a convected entropy wave as a function of the distance for different frequencies. Numerical results obtained with Sunday are also plotted on the figure. The mean flow is chosen to be representative of the conditions inside the combustion chamber of an aero-engine at take-off: $\overline{u} = 10$ m/s, $\overline{T} = 1200$ K et $\overline{p} = 30$ bar. For air, those conditions correspond to a thermal diffusivity of $D = 6.75 \times 10^{-6}$ m^2/s. The thermal diffusion remains negligible for all the frequencies considered. The maximum damping is observed for $f = 1000$ Hz but stays below 3% for a propagation over 10 cm. Moreover, for such frequencies the turbulent mixing coming from the flow itself or from the dilution holes in the combustion chamber may have a more dominant effect on the entropy damping compared to the thermal diffusion. As a conclusion, thermal diffusion can be safely neglected in the modelling of indirect combustion noise.

5. Conclusion

The low-order models used in the literature to predict indirect combustion noise rely on several hypotheses. In this paper, the calorically perfect fluid and negligible thermal diffusion assumptions are investigated numerically using the code Sunday of ONERA. The first configuration investigated is an accelerated flow with a mean temperature decreasing from 1200 K to 600 K, which is representative of a high-pressure turbine. Temperature-dependent heat capacities slightly modify the mean fields and modifications observed for the generated noise remain below 10%. The second configuration is a nozzle guide vane with a shocked flow, typical of the inlet of a high pressure turbine. Flow velocity is shown to be high enough to neglect thermal diffusion. Additional analytical developments indicate that thermal diffusion remains negligible even inside the combustor, despite the very low convection velocity. The present study hence validates the calorically perfect fluid assumption and the negligible thermal diffusion hypothesis commonly used for low-order models in combustion noise.

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