ASYMPTOTIC THEORY OF CONTRA-ROTATING ADVANCED OPEN ROTOR WAKE INTERACTION NOISE

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The advanced open rotor is a novel aeronautical engine which promises significant reductions in fuel burn relative to current generation turbofan engines. Thrust is produced by two contra-rotating open rotors which must be carefully designed in order to ensure that the noise produced by these engines is an acceptable level. A particularly important source of noise produced by an advanced open rotor engine is the tones produced when the wakes from the upstream rotor impinge on the downstream rotor blades. This paper will present a simple analytical model for predicting this tonal noise. Asymptotic analysis is then used to simplify the expressions and deduce how changes in rotor geometry can be used to reduce noise levels.

Keywords: propeller noise, aeroacoustics

1. Introduction

An advanced open rotor engine is a novel aeronautical propulsor which offers significant reductions in fuel burn relative to a current generation turbofan engine. Thrust is produced by the two contra-rotating coaxial ‘open rotors’. The downstream rotor is used to recover the swirl from the wake of the upstream rotor which improves the efficiency relative to a single rotor engine. The noise spectrum produced by the open rotor consists of a significant broadband level in addition to a multitude of tones. The tones produced by the rotor blades include the usual ‘rotor-alone’ tones which occur at integer multiples of the blade passing frequency of each rotor as well as ‘interaction’ tones produced by the interaction of the rotor blades with the unsteady flow-field from the adjacent rotor. Rotor-alone tones are primarily caused by the steady loading and thickness of the rotor blades. Interaction tones are believed to be primarily produced by the periodic unsteady loading on the rotor blades.

This paper contains a description of a relatively straightforward analytical method for predicting the tones produced by the interaction of the viscous wakes from the upstream rotor with the downstream rotor. Asymptotic theory is then applied to these equations and the resulting expressions are used to show how the ‘sweep’ of the downstream rotor blades can be used to reduce the level of these tones.

2. Analytical noise prediction of interaction tone noise

A number of methods for predicting open rotor noise are available and a summary of these can be found in the review paper by the author (Kingan (2014)) which the reader is referred to for further details. As one might expect these methods involve varying degrees of complexity and computational time and range from high fidelity (but time intensive) CFD methods to simple (but quick) analytical methods. For the purposes of analytical modelling, the unsteady flow-field produced by each rotor blade can be decomposed into (1) the viscous wake, (2) the tip-vortex, and (3) the bound potential field. The velocity perturbation associated with each of these fields can then be decomposed into a Fourier series of ‘harmonic convected gusts’ by making use of the periodicity of the problem in the azimuthal coordinate (the angle through which the blade rotates). The unsteady loading on or 'response' of the adjacent rotor's blades to the each of these gusts is then calculated using well-known 'blade response functions'. The far-field noise radiation can be predicted from the calculated unsteady loading using the analytic 'frequency domain' expressions of Hanson (1985) or Parry (1988).

2.1 Analytic method for predicting viscous wake interaction tones

In this section we develop a model for calculating the tonal noise produced by the unsteady loading on the downstream rotor blades due to their interaction with the viscous wakes of the upstream rotor. This model is a simple extension of the model developed by Parry (1988). For the analysis presented in this paper, it will be convenient to introduce a cylindrical coordinate system, $\{x, r, \phi\}$, where x is the axial coordinate which is collinear with the propeller axis, r is the radial coordinate and ϕ is the azimuthal angle. The rotors are immersed in a uniform airflow with Mach number M_x in the negative x-direction relative to the advanced open rotor and the air has ambient density ρ_0 and speed of sound c_0. The upstream and downstream rotors rotate in the negative and positive ϕ-directions at rotational speeds Ω_1 and Ω_2 respectively. The pitch-change axis of the reference blades on the front and rear rotors are located at $\phi = 0$ rad at time $\tau = 0$ s and are separated by a distance g in the axial direction. Also note that the convention adopted in this paper will be that the subscripts 1 and 2 denote parameters associated with the front and rear rotors. The blades of both rotors have chord $c(r)$, sweep $s(r)$, lean $l(r)$ and sectional drag coefficient $C_D(r)$ and both rotors have B blades and have an equal diameter which is denoted D.

The unsteady loading on the downstream rotor blades at a given radial location is calculated using an equivalent 2D problem where the wakes from an upstream cascade of blades interacts with the blades of a downstream cascade. The formulation presented here will make use of a Cartesian coordinate system $\{x, y\}$, where x is an axial coordinate defined such that the airflow has Mach number M_x in the positive x-direction and y is a tangential coordinate which is parallel to the direction in which the blade rows translate. The upstream and downstream blades translate in the negative and positive y-directions at Mach numbers $\Omega_1 r/_0$ and $\Omega_2 r/c_0$ respectively. At time $\tau = 0$ s the pitch-change axis of the front rotor reference blade is aligned with the pitch change axis of the rear rotor reference blade at $y/D = 0$ and the spacing between the mid-chord positions of the blades on each cascade in the y-direction is equal to $2\pi r/B$. The blades are modelled as infinitely thin flat-plates which are aligned with the local flow direction but otherwise have identical characteristics (such as chord-length, sweep, lean and drag coefficient) to the actual rotor blade at that particular radius. Also, the effect of the flow induced by the rotors is neglected such that the stagger angle, α, of each blade is defined by $\tan\alpha = zM_T/M_x$, where $z = 2r/D$ and $M_T = \Omega D/2c_0$.

In order to describe the development of the wakes from the upstream cascade it is convenient to introduce two coordinate systems which are locked to the upstream blade row and have origins located at the mid-chord of the upstream reference blade. The $\{x_1, y_1\}$ coordinate system has coordinates which a parallel to the global $\{x, y\}$ coordinate system. The $\{X_1, Y_1\}$ coordinate system has coordinates parallel to the chordwise and chord-normal directions and is related to the $\{x_1, y_1\}$ coordinate system by the equations below.

$$X_1 = x_1 \cos\alpha_1 + y_1 \sin\alpha_1, \qquad (1)$$

$$Y_1 = -x_1 \sin\alpha_1 + y_1 \cos\alpha_1, \qquad (2)$$

The reference blade of the upstream blade row produces a wake with mean deficit velocity u' aligned with the negative X_1-direction at the axial location of the leading edge of the downstream rotor which will be modelled using the 'Schlichting' wake profile

$$u' = U_{r_1} \frac{\sqrt{10}}{18\beta} \left(\frac{c_1 C_{D_1}}{L_1}\right)^{\frac{1}{2}} \left[1 - \left(\frac{Y_1}{b}\right)^{1.5}\right]^2, |Y_1| \le b \qquad (3)$$

where $\beta = 2^{\frac{1}{3}} / \left[4\sqrt{10}\left(\sqrt{2}-1\right)^{2/3}\right]$, U_{r_1} is the velocity of the air relative to the blade and L_1 is the length of the wake which is defined as the distance in the X_1-direction between the mid-chord of the reference blade and the axial location of the point of interest, $b = 4\sqrt{10}\beta b_{1/2}$ and $b_{1/2}$ is defined as $b_{1/2} = \frac{1}{4}\sqrt{C_{D_1} c_1 L_1}$. Substituting eq. (2) into eq. (3) gives an expression for the wake deficit velocity produced by the front rotor reference blade in the $\{x_1, y_1\}$ coordinate system.

In order to calculate the unsteady loading on the downstream blade row, it is necessary to express the front rotor wake velocity deficit in a coordinate system fixed to the rear rotor blades. For this purpose we introduce a blade locked axial/tangential coordinate system $\{x_2, y_2\}$ which is parallel to the axial and tangential coordinates and has its origin located at the leading edge of the reference blade on the downstream blade row. The $\{x_2, y_2\}$ coordinate systems are related to the $\{x_1, y_1\}$ coordinate system by eqs. (4) and (5) below.

$$x_1 = x_2 + g - s_1 \cos\alpha_1 + \left(s_2 - \frac{c_2}{2}\right)\cos\alpha_2 - l_1 \sin\alpha_1 + l_2 \sin\alpha_2, \qquad (4)$$

$$y_1 = y_2 + (\Omega_1 + \Omega_2) r\tau + l_1 \cos\alpha_1 - s_1 \sin\alpha_1 - \left(s_2 - \frac{c_2}{2}\right)\sin\alpha_2 + l_2 \cos\alpha_2, \qquad (5)$$

It will also be assumed that the downstream rotor is located sufficiently far downstream of the upstream rotor that the wake development (increase in wake width and decrease in the wake centreline velocity deficit) in the axial direction can be neglected in the vicinity of the downstream rotor. Thus we set L_1 equal to its value at the leading edge of the downstream rotor blades.

The mean velocity deficit produced by the upstream blade row, v', is assumed to be equal to the sum of the velocity deficit produced by all the blades on the upstream cascade (which are evenly spaced and identical) which, making use of Poisson's summation theorem can be written as

$$v' = \sum_{n_1=-\infty}^{\infty} \frac{B_1 C_{D_1} c_1 U_{r_1}}{4\pi r \cos\alpha_1} G(k_b) \exp\left\{i\frac{n_1 B_1}{r} y_1 - i\frac{n_1 B_1}{r} x_1 \tan\alpha_1\right\}, \qquad (6)$$

where

$$k_b = \frac{n_1 B_1 b}{r\cos\alpha_1}, \qquad (7)$$

and

$$G(k_b) = \frac{40}{3k_b^4}\left\{(1 - \cos k_b - k_b \sin k_b) + \frac{k_b^2}{2}\left(\frac{\pi}{2k_b}\right)^{0.5} C\left[\left(\frac{2k_b}{\pi}\right)^{0.5}\right]\right\}, \qquad (8)$$

where $C[\quad]$ is the Fresnel cosine integral.

One final coordinate transformation is required in order to express the upstream rotor wake velocity deficit incident onto the reference blade of the downstream blade row in a chordwise/chordnormal coordinate system, $\{X_2, Y_2\}$, which is defined by eqs. (9) and (10) below.

$$x_2 = X_2 \cos\alpha_2 + Y_2 \sin\alpha_2, \qquad (9)$$

$$y_2 = -X_2 \sin\alpha_2 + Y_2 \cos\alpha_2. \qquad (10)$$

Substituting eqs. (9) and (10) into eqs. (7) and (8) and then substituting the resulting expressions into eq. (4) gives

$$v' = \sum_{n_1=-\infty}^{\infty} \frac{B_1 C_{D_1} c_1 U_{r_1}}{4\pi r \cos\alpha_1} G(k_b) \exp\{\mathrm{i} n_1 B_1 (\Omega_1 + \Omega_2)\tau\} \times$$
$$\exp\left\{-\mathrm{i}k_X X_2 - \mathrm{i}k_Y Y_2 - \mathrm{i}k_X\left(s_2 - \frac{c_2}{2}\right) - \mathrm{i}k_Y l_2 - \mathrm{i}k_{Y_1}(g\sin\alpha_1 - l_1)\right\}, \qquad (11)$$

where

$$k_X = \frac{2n_1 B_1}{DM_{r_2}}\left[M_{T_1} + M_{T_2}\right], \qquad (12)$$

$$k_Y = -\frac{2n_1B_1}{DM_{r_2}}\left[\frac{M_x}{z} - z\frac{M_{T_1}M_{T_2}}{M_x}\right], \quad (13)$$

and

$$k_{Y_1} = \frac{2n_1B_1M_{r_1}}{zDM_x}. \quad (14)$$

Note that the upstream rotor wake deficit velocity is aligned with the $-X_1$ direction and therefore the upwash velocity (which is the component of velocity in the Y_2 direction) onto the downstream reference blade is given by

$$w = -\sin(\alpha_1 + \alpha_2)\, v'. \quad (15)$$

We therefore have the following expression for the upwash on the chordline of the reference blade of the downstream blade row (on which $Y_2 = 0$)

$$w = \sum_{n_1=-\infty}^{\infty} w_{n_1} \exp\{\mathrm{i}k_X(U_{r_2}\tau - X_2) - \mathrm{i}k_Y Y_2\}, \quad (16)$$

where U_{r_2} is the velocity of the downstream blade relative to the air and

$$w_{n_1} = -\sin(\alpha_1 + \alpha_2)\frac{B_1 C_{D_1} c_1 U_{r_1}}{4\pi r\cos\alpha_1} G(k_b) \exp\left\{-\mathrm{i}k_X\left(s_2 - \frac{c_2}{2}\right) - \mathrm{i}k_Y l_2 - \mathrm{i}k_{Y_1}(g\sin\alpha_1 - l_1)\right\}. \quad (17)$$

The total unsteady lift force per unit area acting on the chordline of the reference blade in the $-Y_2$ direction can be expressed as the sum of the unsteady lift on the blade due to its interaction with each upwash harmonic i.e.

$$\Delta p = \sum_{n_1=-\infty}^{\infty} \Delta p_{n_1} \exp\{\mathrm{i}n_1B_1(\Omega_1 + \Omega_2)\tau\}, \quad (18)$$

where $\Delta p_{n_1} \exp\{\mathrm{i}n_1B_1(\Omega_1 + \Omega_2)\tau\}$ is the response of the reference blade to a gust of the form $w_{n_1} \exp\{\mathrm{i}k_X(U_{r_2}t - X_2) - \mathrm{i}k_Y Y_2\}$ and Δp_{n_1} is given by (see, for example, Goldstein (1976)).

$$\Delta p_{n_1} = \frac{2\rho_0 U_{r_2} w_{n_1}}{[\pi\sigma_2(1+M_{r_2})\bar{X}_2]^{0.5}} \exp\left\{-\mathrm{i}\frac{\pi}{4} - \mathrm{i}\frac{\sigma_2 M_{r_2}}{1+M_{r_2}}\bar{X}_2\right\},\ n_1 \neq 0 \quad (19)$$

where $M_{r_2} = U_{r_2}/c_0$ is the Mach number of the airflow relative to the downstream rotor blade, $\bar{X}_2 = 2X_2/c_2$ is a dimensionless chordwise coordinate and $\sigma_2 = k_X c_2/2$ is the reduced frequency of the gust harmonic interacting with the rear rotor blade. We do not consider the $n_1 = 0$ terms in the analysis presented here as these correspond to the steady component of loading on the rotor blades.

Following the derivation presented by Hanson (1985), the far-field tonal sound pressure produced by the periodic lift forces on the downstream rotor of a contra-rotating open rotor due to the wakes shed by the upstream rotor is given by the following expression

$$p = \frac{\mathrm{i}\rho_0 c_0^2 B_2 D}{8\pi R_e(1 - M_x\cos\theta_e)} \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} \exp\left\{\mathrm{i}\omega\left(t - \frac{R_e}{c_0}\right) - \mathrm{i}\nu\left(\phi - \frac{\pi}{2}\right)\right\} I_{n_1,n_2}, \quad (20)$$

where

$$I_{n_1,n_2} = \int_{z_h}^{1} M_{r_2}^2 \exp\{-\mathrm{i}(\phi_l + \phi_s)\} J_\nu\left(\frac{\nu}{z^*}z\right) k_y \frac{C_{Ln_1}}{2}\psi_{L_{n_1}}(k_x)\, dz, \quad (21)$$

$$\omega = n_1B_1\Omega_1 + n_2B_2\Omega_2, \quad (22)$$

$$\nu = n_2B_2 - n_1B_1, \quad (23)$$

$$k_x = \frac{2}{M_{r_2}}\left[\frac{(n_1B_1M_{T_1} + n_2B_2M_{T_2})M_x\cos\theta_e}{(1 - M_x\cos\theta_e)} + \nu M_{T_2}\right]\frac{c_2}{D}, \quad (24)$$

$$k_y = -\frac{2}{M_{r_2}}\left[\frac{(n_1B_1M_{T_1} + n_2B_2M_{T_2})M_{T_2}z\cos\theta_e}{(1 - M_x\cos\theta_e)} - \nu\frac{M_x}{z}\right]\frac{c_2}{D}, \quad (25)$$

$$\phi_s = \frac{2}{M_{r_2}}\left[\frac{(n_1 B_1 M_{T_1} + n_2 B_2 M_{T_2}) M_x \cos\theta_e}{(1 - M_x\cos\theta_e)} + \nu M_{T_2}\right]\frac{s_2}{D}, \qquad (26)$$

$$\phi_l = \frac{2}{M_{r_2}}\left[\frac{(n_1 B_1 M_{T_1} + n_2 B_2 M_{T_2}) M_{T_2} z \cos\theta_e}{(1 - M_x\cos\theta_e)} - \nu\frac{M_x}{z}\right]\frac{l_2}{D}, \qquad (27)$$

$$z^* = \frac{(1 - M_x\cos\theta_e)\nu}{(n_1 B_1 M_{T_1} + n_2 B_2 M_{T_2})\sin\theta_e}, \qquad (28)$$

and

$$\frac{1}{2}\rho_0 U_{r_2}^2 c_2 C_{Ln_1}\psi_{Ln_1}(k_x) = \int_0^{\square_2}\Delta p_{n_1}\exp\left\{-\mathrm{i}\frac{k_x}{2}\left(\frac{2\square_2}{\square_2} - 1\right)\right\}\mathrm{d}X_2, \qquad (29)$$

where Δp_{n_1} is the n_1^{th} Fourier harmonic of the unsteady pressure jump on the downstream blade row.

The value z^*, defined in eq. (28) is an important parameter; it represents the point at which the argument of the Bessel function becomes equal to its order and, significantly, is close to the point at which the Bessel function achieves its maximum value.

Substituting eq. (19) into eq. (29) and evaluating the integral yields for $n_1 \neq 0$

(1) $$C_{Ln_1}\psi_{Ln_1}(k_x) = \frac{w_{n_1}}{U_{r_2}}\frac{2\sqrt{2}\exp\left\{\mathrm{i}\frac{k_x}{2} - \mathrm{i}\frac{\pi}{4}\right\}}{\sigma_2^{0.5}(1+M_{r_2})^{0.5}\left[\frac{\sigma_2 M_{r_2}}{(1+M_{r_2})} + \frac{k_x}{2}\right]^{0.5}} E^*\left(\frac{2}{\sqrt{\pi}}\left[\frac{\sigma_2 M_{r_2}}{(1+M_{r_2})} + \frac{k_x}{2}\right]^{0.5}\right), \qquad (30)$$

where E^* is the conjugate of the complex Fresnel integral.

Having described the full equations for the unsteady response of the downstream blade row to the front rotor wakes, and the resultant sound radiation to the far-field, we turn to asymptotic analysis of the formulae to aid interpretation of the underlying physics.

We start by noticing that the Bessel function in eq. (21) originates from an integration of the noise sources over the propeller disc and we can return to the original form by replacing it with Bessel's integral

$$J_\nu\left(\frac{\nu}{z^*}z\right) = \frac{1}{2\pi\mathrm{i}^\nu}\int_{-\pi}^{\pi}\exp\left\{\mathrm{i}\nu\left(\frac{z}{z^*}\cos u + u\right)\right\}\mathrm{d}u, \qquad (31)$$

which gives

$$I_{n_1,n_2} = \frac{1}{2\pi\mathrm{i}^\nu}\int_{z_h}^{1}\int_{-\pi}^{\pi} g(z)\exp\{\mathrm{i}\nu\Phi(u,z)\}\,\mathrm{d}u\mathrm{d}z, \qquad (32)$$

where $g(z)$ is an amplitude function which is defined as

$$g(z) = -\frac{(1-\mathrm{i})G(k_b)B_1 C_{D_1} c_1 M_{r_1}^2\sin(\alpha_1+\alpha_2)M_{r_2}k_y}{4\pi r M_x\sigma_2^{0.5}(1+M_{r_2})^{0.5}\left[\frac{\sigma_2 M_{r_2}}{(1+M_{r_2})} + \frac{k_x}{2}\right]^{0.5}} E^*\left(\frac{2}{\sqrt{\pi}}\left[\frac{\sigma_2 M_{r_2}}{(1+M_{r_2})} + \frac{k_x}{2}\right]^{0.5}\right), \qquad (33)$$

and $\Phi(u,z)$ is a phase function which is defined as

$$\Phi(u,z) = \frac{z}{z^*}\cos u + u - \Gamma(z), \qquad (34)$$

with

$$\Gamma(z) = \frac{1}{M_{r_2}}\left[\frac{M_x\cot\theta_e}{z^*} + M_{T_2}\right]\bar{s}_L + \frac{1}{M_{r_2}}\left[\frac{M_{T_2} z\cot\theta_e}{z^*} - \frac{M_x}{z}\right]\bar{l}_2$$
$$+ \frac{n_1 B_1}{\nu}\left[\frac{(M_{T_1}+M_{T_2})}{M_{r_2}}\bar{s}_L - \frac{1}{M_{r_2}}\left[\frac{M_x}{z} - z\frac{M_{T_1}M_{T_2}}{M_x}\right]\bar{l}_2 + \bar{g}\frac{M_{T_1}}{M_x} - \frac{M_{r_1}}{zM_x}\bar{l}_1\right]. \qquad (35)$$

where $\bar{l}_1 = 2l_1/D$, $\bar{l}_2 = 2l_2/D$, $\bar{s}_L = 2s_L/D$, $s_L = (s_2 - c_2/2)$ and $\bar{g} = 2g/D$.

2.2 Asymptotic analysis and quiet rotor design

A number of authors including Chako (1965) and Cooke (1982) have considered evaluating double integrals of the form given by eq. (32) asymptotically for the case where $|\nu| \to \infty$. These studies all demonstrate that the *principle contributions* to I_{n_1,n_2} arise from small regions of the integrand around certain *critical points* which for the purposes of the problem considered here can be divided into two general types: (a) Stationary points of the phase function which occur either within the 'source annulus' or on the bounding curve of the annulus. (b) Points on the source annulus boundary where the tangential derivative of the phase function vanishes.

In the following sections expressions for the leading order terms in the asymptotic expansion of I_{n_1,n_2} will be presented. It will be assumed that $B_1 \to \infty, B_2 \to \infty$ and that linear combinations of B_1 and B_2 can also be regarded as being infinitely large (i.e. $|\nu| \to \infty$, $|\omega| \to \infty$). One final assumption which will be made is that the absolute value of a ratio of large parameters can be regarded as being $O(1)$. Note that these assumptions will not hold valid for all possible tones and observer angles produced by the open rotors considered later. This work will require the partial derivatives of the phase function. Note that we have adopted the notation

$$\Phi_{p,q}(u,z) = \frac{\partial^{p+q}}{\partial u^p \partial z^q}\Phi(u,z). \qquad (37)$$

We consider first the case of stationary points which occur within the source annulus and, adopting the terminology of Chako (1965), will refer to these points as *interior stationary points*. It is assumed that each of these interior stationary points lie within the source annulus sufficiently far away from the inner and outer edge of the annulus and also separated from other critical points by a sufficient distance such that the principle contribution to I_{n_1,n_2} from each point can be considered in isolation. An interior stationary point occurs at $\{u,z\} = \{\tilde{u},\tilde{z}\}$ when $\Phi_{1,0} = \Phi_{0,1} = 0$. For now it will also be assumed that $\Phi_{2,0}\Phi_{0,2} \neq \Phi_{1,1}^2$ at any of these points. From the definition of the partial derivatives we can determine the location of the stationary points to be the solution to the following two equations,

$$\sin\tilde{u} = \frac{z^*}{\tilde{z}},\ \cos\tilde{u} = \Gamma'(\tilde{z})z^*, \qquad (38)$$

or, on eliminating $\tilde{u}$, when

$$\tilde{z}^2 = {z^*}^2[[\tilde{z}\Gamma'(\tilde{z})]^2 + 1]. \qquad (39)$$

We follow the method of Cooke (1982) to evaluate the contribution to I_{n_1,n_2} from an interior stationary point for $|\nu| \to \infty$ which gives the following expression

$$I_{n_1,n_2} \sim \frac{\tilde{g}_{0,0}}{|\nu||\widetilde{\Phi}_{2,0}\widetilde{\Phi}_{0,2} - \widetilde{\Phi}_{1,1}^2|^{\frac{1}{2}}} \exp\left\{i\nu\left(\widetilde{\Phi}_{00} - \frac{\pi}{2}\right) + i\frac{\pi}{4}\operatorname{sgn}(\nu)\operatorname{sgn}(\widetilde{\Phi}_{2,0})\left[1 + \operatorname{sgn}(\widetilde{\Phi}_{2,0}\widetilde{\Phi}_{0,2} - \widetilde{\Phi}_{1,1}^2)\right]\right\}. \qquad (40)$$

where the hat on a parameter indicates that it is evaluated at the critical point. Contributions from the hub region can be evaluated using the same method which yields a similar expression.

A plot of the sound pressure level spectrum calculated using the expressions given above for a straight-bladed rotor is shown in figure 10 below. Results calculated using a full numerical calculation of I_{n_1,n_2} (circles) are plotted along with the levels calculated using the asymptotic expressions derived in this paper (dots). There is generally reasonable agreement between the exact and asymptotic results which gives confidence in the accuracy (and usefulness) of the asymptotic expressions. The other point of note with these results is that tones associated with interior critical points (blue) are generally of a much higher level than tones associated with boundary critical points (red). Based on these results, one approach to reduce the overall level of noise produced by this noise source would be to design a rotor for which no interior critical points occur for all significant tones at important observer positions.

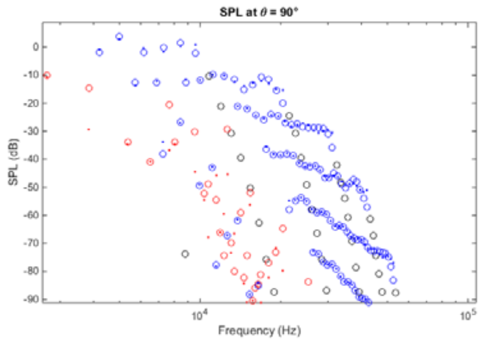

Figure 1. Plot of SPL vs frequency. Circles denote a numerical solution and dots denote an asymptotic solution. Normal interior critical point (blue), boundary critical point (red), $|z^*| > 1$ (black).

Recall that for $|z^*| > 0$, interior critical points are located at

$$\tilde{z}^2 = {z^*}^2[[\tilde{z}\Gamma'(\tilde{z})]^2 + 1]. \qquad (43)$$

For no lean and $|z^*| > 0$ we have $\Gamma(z)$ defined as

$$\Gamma(z) = \frac{1}{z^* \sin\theta_e} \frac{\bar{s}_L}{M_{r_2}} + \frac{n_1 B_1}{\nu} \bar{g} \frac{M_{T_1}}{M_x}. \qquad (44)$$

It will also be useful to define $\Theta(z) = \bar{s}_L / M_{r_2} \sin\theta_e$ such that

$$\Gamma'(z) = \frac{1}{z^*}\Theta'(z). \, (45)$$

Substituting into about eq. yields the following expression for ϕ for a ‘critical design’

$$\Theta'(z) = \sqrt{1 - \left(\frac{z^*}{z}\right)^2}, \; z \geq |z^*|. \qquad (46)$$

Rotors which have $\Theta'(z)$ larger than the ‘critical design’ defined by the equations above will have no interior critical points. Such a design can be attained by selecting an appropriate profile for the downstream blade leading edge sweep, $\bar{s}_L$. Examination of the asymptotic expression for the boundary critical points reveals that increases in blade sweep past the critical design will result in reductions in the level of the radiated sound (via the $\widehat{\Phi}_{0,1}$ term). Also note that increasing the sweep of the downstream blade row has the added effect of increasing the spacing between the two rotors which increases the wake width at the downstream rotor and decreases the amplitude of certain high frequency tones (via reducing the magnitude of the $G(k_b)$ term).

Figure 2 below plots the sound pressure level of 4 individual tones against a ‘sweep parameter’ λ, which is defined such that the critical design corresponds to $\lambda = 1$. Values of λ less than one have interior critical points on the blade whilst values of λ greater than one do not have interior critical points on the blade. It is clearly observed that, as expected, further increases in blade sweep past the critical design reduce the level of these tones.

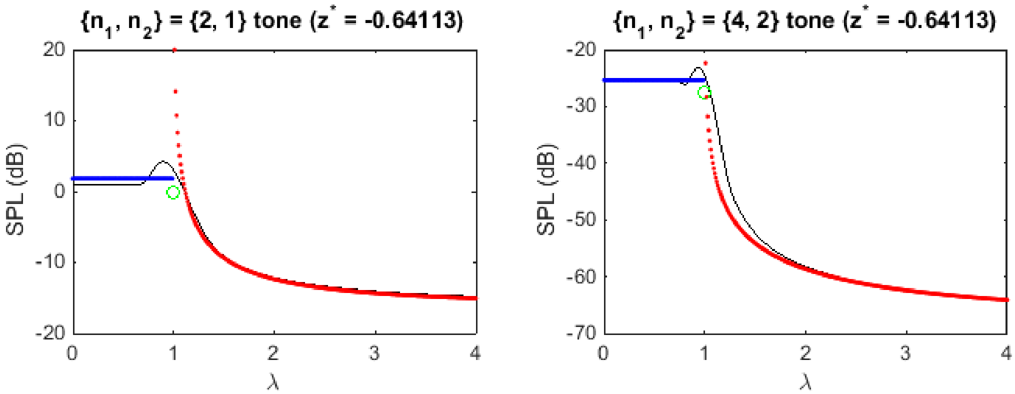

Figure 11: Plot of SPL versus ‘sweep parameter’, λ for 2 different tones. ‘Exact’ numerical solutions (black curve), Interior critical asymptotic solutions (blue dots), boundary critical asymptotic solutions (red dots), asymptotic solution for the critical design (green circle).

3. Conclusions

This paper has summarised a number of low- and high-speed experimental wind tunnel experiments undertaken using a model-scale open rotor test rig. Some of the issues encountered and the findings from these tests were described. A method was then presented for predicting the tonal noise produced by the interaction of the viscous wake from the upstream rotor with the downstream rotor. The paper concluded with an asymptotic analysis of the analytical equations which yielded insight into how the noise from this noise source could be reduced.

REFERENCES

Parry, A. B. (1988) Theoretical prediction of counter-rotating propeller noise, PhD thesis, University of Leeds

Hanson, D. B. (1985) Noise of counter-rotation propellers, Journal of Aircraft, 1985, 22, pp. 609–17

Kingan, M. J. (2014) Advanced open rotor noise prediction, The Aeronautical Journal, vol. 118, issue 1208, pp. 1125-1135

Goldstein, M. E., (1976) Aeroacoustics, McGraw-Hill

Chako, N., (1965) Asymptotic expansions of double and multiple integrals occurring in diffraction theory, Journal of the Institute of Mathematics and its Applications, vol. 1, pp. 372-422

Cooke, J. C., (1982) Stationary phase in two dimensions, IMA Journal of Applied Mathematics, vol. 29, pp. 25-37