24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

THEORETICAL AND EXPERIMENTAL STUDY OF VIBRATION TRANSMISSION IN AN EXPONENTIAL HORN FILLED WITH SOLID MATERIAL

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There are many kinds of audio-bases on the market and they are used for trying to improve the reproduced sound of audio equipment. On the other hand, their effects seem not always better one for reproduced sound quality, because of uncertain mechanism of audio-base on acoustic and electric signal. Furthermore, considering its usage, the working of audio-base seems to be limited to a change in the vibration state of loaded audio equipment. In previous study, we had clarified that an audio-base is effective to reduce inevitable vibration and electrical noise caused on stereophonic equipment and to improve the sound quality and summing localization under standard stereophonic representation. In this study, to describe the working of audio-base from theoretical viewpoint, first, we derived a driving impedance of exponential horn at excitation point "throat" or "mouth". Then, the difference in transmission characteristics owing to the transmitting direction is shown as difference in driving impedance based on the impedance matching theory. Consequently, we could clarified that the audio-base has alternating frequency point to passing through or to insulate the vibration related with material and shape. This tendency is also confirmed experimentally by measuring the difference in vibration velocity between input and output surfaces of audio-base.

Keywords: Audio equipment, Exponential horn, Audio-base, Wave equation, Vibration transmission

1. Introduction

Recently, digital technique provides high quality sound source and its reproducing audio equipment. On the other hand, the analogue devices should be improved to realise true sound same as at recording cite and faithful to the source. Especially, induced noise in stereophonic equipment affect to minor information on circumstance of recording cite or sensitivity contained in music source and prevent faithful representation on three-dimensional sound field and high quality sound. The most of noise is originated in electrical factor like as fluctuation of DC voltage, pulse noise coming from external power supply and caused noise in electric devices and *etc*. While, vibration caused on equipment induces electromagnetic noise in the electric circuit from different sphere with mechanical mean.

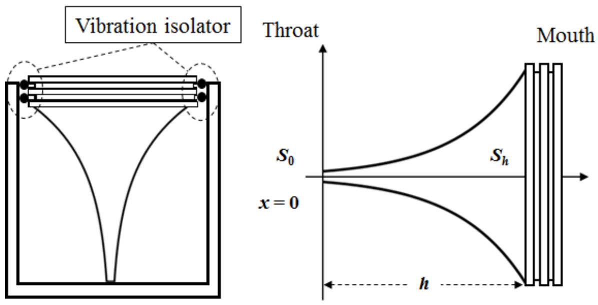
The former noise should be reduced by electrical method like as choice of low noise devices, use of stabilized power supply, noise cut technique from power supply and adjusting the tracking of CD driver *etc*. The latter noise is originated from vibration on equipment and it could not be dealt with an electric method. Vibration caused on audio equipment is originated from excitation by power transformer, CD driving system and airborne sound come from loudspeaker. Especially, vibration on loud speaker system affect directly the magnet of speaker unit which should be absolutely fixed point and its vibration will be directly radiated.

On the other hand, there are many kinds of audio-bases on the market and they are used to improve the reproduced sound of audio equipment. Indeed, the fact that the reproduced sound and its quality can be changed or improved by using audio-bases, have been confirmed experimentally. But, their effects on changing or improving represented sound or its mechanism has not been confirmed. In a previous study [1], we had confirmed experimentally that an audio-base shaped in exponential horn is more effective to reduce inevitable vibration and electrical noise induced on stereophonic equipment.

In this study, to confirm the above fact that the audio-base shaped in exponential horn shows a vibration elimination characteristic to loaded equipment from theoretical viewpoint, the solutions of a wave equation have been derived under connection of load impedance to the output of horn. The difference in vibration transmission characteristic owing to transmitting direction was illustrated as driving impedance at “throat” or “mouth” of the horn. Because the amount of transmitted vibration energy into horn can be evaluated by ratio of driving impedance to inner impedance of exciting source. As the vibration transmission characteristic is influenced by shape factor like as flare constant, density, Young’s modulus and wave velocity of material, we had carried out numerical calculation to illustrate the turning point of insulation or elimination of vibration on loaded equipment. Finally, an experimental measurement was carried out and we could confirm an approximate correspondence of turning point between theoretical and experimental ones.

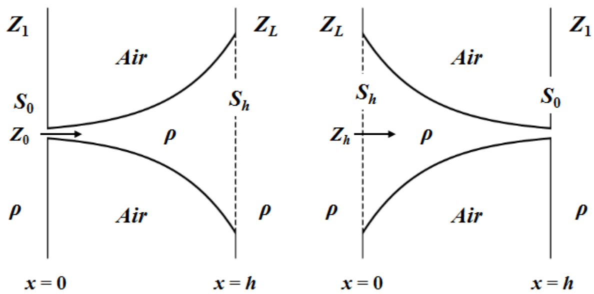
2. Analysis of vibration transmission characteristics shaped in the exponential horn

A conical horn type is most popular one among many kinds of audio-bases on the market. On the other hand, we proposed a new type of audio-base shaped in exponential horn in previous study [2]. Here, we employ an exponential horn shaped audio-base. As shown in Fig. 1(a), the horn is put in a cup and the “throat” is connected to its bottom and followed a column to “mouth” made by same material with horn itself. The column and the cup are isolated by O-ring made by rubber. Fig. 1 (b) shows the cross-section of the horn, and we defined as sectional area at $x = 0$ is "throat" and area at $x = h$ is "mouth". Then, the horn is excited by external vibration origin through bottom of cup or column connected with horn. By considering this structure, in any case the horn is connected to impedance Z_0 of the cup at "throat" side or impedance Z_L of column at "mouth" side (see Fig. 2 (a) and (b)) made by same material with the horn.



(a) Structure of audio-base. (b) Cross section of exponential horn.

Figure 1. Structure and cross section of audio-base.



(a) Throat to mouth. (b) Mouth to throat.

Figure 2. Input and output of exponential horn.

2.1 Wave equation in the horn

In the case of audio-base, inner of horn is filled with solid material of density ρ. Then, the pressure wave $p(x)$ propagating in the horn satisfies following wave equation as shown in literacy [3],

$$\frac{\partial^2 p(x)}{\partial x^2} + \frac{1}{S(x)}\frac{dS(x)}{dx}\frac{\partial p(x)}{\partial x} = \frac{1}{c^2}\frac{\partial^2 p(x)}{\partial t^2}, \tag{1}$$

where c is the propagating velocity and $S(x)$ is cross-section area of the horn at x given by

$$S(x) = S_0 e^{mx}. \tag{2}$$

With flare constant m. Then Eq. (1) becomes as

$$\frac{\partial^2 p(x)}{\partial x^2} + m\frac{\partial p(x)}{\partial x} = \frac{1}{c^2}\frac{\partial^2 p(x)}{\partial t^2}. \tag{3}$$

When the horn is used as audio-base, the vibration transmission characteristics is most important to describe its mechanism to reduce vibration of loaded equipment, because their effect on the stereophonic equipment should be limited in mechanical mean. On the other hand, as the usage of horn is selective to connect the load with "mouth" or "throat", the difference in vibration transmission characteristic should be compared the "throat" excitation case (i.e. normal direction) with "mouth" excitation case (i.e. inverse direction). For normal direction, we have following general solutions of Eq. (3):

i). for $1-(2k/m)^2 < 0$ ($f < f_c : f_c = mc/4\pi$)

$$p(x) = e^{-\frac{m}{2}x}\cdot\left\{Ae^{-\frac{m}{2}x\sqrt{1-\left(\frac{2k}{m}\right)^2}} + Be^{\frac{m}{2}x\sqrt{1-\left(\frac{2k}{m}\right)^2}}\right\}\cdot e^{j\omega t} \tag{4}$$

or

ii). for $1-(2k/m)^2 > 0$ ($f > f_c$)

$$p(x) = e^{-\frac{m}{2}x}\cdot\left\{Ce^{-j\frac{m}{2}x\sqrt{\left(\frac{2k}{m}\right)^2-1}} + De^{j\frac{m}{2}x\sqrt{\left(\frac{2k}{m}\right)^2-1}}\right\}\cdot e^{j\omega t}, \tag{5}$$

where $f_c = mc/4\pi$ is cut off frequency, $k = \omega/c$ is wave number and A, B, C and D are constants depend on boundary condition of horn. To evaluate the difference in vibration transmission characteristic due to transmission direction, the driving impedance at exciting point is useful because of impedance matching theory.

2.2 Driving impedance at "throat" toward "mouth"

Here, let us suppose simply the connected impedances Z_1 and Z_L as ρc of material to solve Eqs. (4) and (5). Then, boundary conditions become as $\dot{u}(0) = \dot{u}_0 e^{j\omega t}$ at $x = 0$ and $p(h) = \rho c\dot{u}(h)$ at $x = h$, and we have

i). for $f < f_c$, $p(x)$ and $\dot{u}(x)$ become as

$$p(x) = \frac{2jk\rho c}{m}\dot{u}_0 e^{-\frac{m}{2}x}\frac{\left\{1 - j\frac{m}{2k}(\varepsilon-1)\right\}e^{-\frac{m}{2}\varepsilon(x-h)} - \left\{1 + j\frac{m}{2k}(\varepsilon+1)\right\}e^{\frac{m}{2}\varepsilon(x-h)}}{\left\{1 + j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{-\frac{m}{2}\varepsilon x} + \left\{1 - j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{\frac{m}{2}\varepsilon x}} \tag{6}$$

and

$$\dot{u}(x) = \dot{u}_0 e^{-\frac{m}{2}x}\frac{\left\{1 - j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{-\frac{m}{2}\varepsilon(x-h)} + \left\{1 + j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{\frac{m}{2}\varepsilon(x-h)}}{\left\{1 + j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{-\frac{m}{2}\varepsilon x} + \left\{1 - j\frac{m}{2k}(\varepsilon^2-1)\right\}e^{\frac{m}{2}\varepsilon x}}, \tag{7}$$

here, $\varepsilon = \sqrt{1-(2k/m)^2}$. Then, the driving impedance Z_0 at "throat" is given as

$$Z_0 = \frac{p(0)}{S_0\dot{u}(0)}$$

$$= \frac{1}{S_0}\cdot\frac{\frac{4k\rho c}{m}\cdot\left[\frac{m}{k}\cdot\left\{1-\left(\frac{2k}{m}\right)^2\right\} + j\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right)^2\right]}{\left\{-\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right) + \varepsilon\cdot\left(e^{-\frac{m}{2}\varepsilon h} + e^{\frac{m}{2}\varepsilon h}\right)\right\}^2 + \left\{\frac{2k}{m}\cdot\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right)\right\}^2}, \tag{8}$$

by considering the impedance at output for a cross-section area S_0 of "throat".

ii). For $f > f_c$, the driving impedance Z_0 becomes as

$$Z_0 = \frac{p(0)}{S_0\dot{u}(0)} = \frac{1}{S_0}\cdot\frac{\frac{4k\rho c}{m}\cdot\left[\frac{m}{4k}\cdot\left\{\left(\frac{2k}{m}\right)^2 - 1\right\} + j\sin^2\left(\frac{m}{2}\mu h\right)\right]}{\left[\frac{2k}{m}\cdot\sin\left\{\frac{m}{2}\mu h\right\}\right]^2 + \left[\sin\left\{\frac{m}{2}\mu h\right\} + \mu\cdot\cos\left\{\frac{m}{2}\mu h\right\}\right]^2} \quad (9)$$

with $\mu = \sqrt{(2k/m)^2 - 1}$.

2.3 Driving impedance at "mouth" toward "throat"

For the "Inverse" direction, Eq. (4) and (5) can be rewritten by substituting the flare constant m to $-m$ and S_0 to S_h by referring Fig.1 (b). Then, "mouth" is at $x = 0$ and "throat" is at $x = h$. The driving impedance Z_h is given as follows

i). for $f < f_c$, we have

$$Z_h = \frac{p(0)}{S_h\dot{u}(0)} = \frac{1}{S_h}\cdot\frac{\frac{4k\rho c}{m}\cdot\left[\frac{m}{k}\cdot\left\{1 - \left(\frac{2k}{m}\right)^2\right\} - j\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right)^2\right]}{\left\{\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right) + \varepsilon\cdot\left(e^{-\frac{m}{2}\varepsilon h} + e^{\frac{m}{2}\varepsilon h}\right)\right\}^2 + \left\{\frac{2k}{m}\cdot\left(e^{-\frac{m}{2}\varepsilon h} - e^{\frac{m}{2}\varepsilon h}\right)\right\}^2} \quad (10)$$

and

ii). for $f > f_c$, we have

$$Z_h = \frac{p(0)}{S_h\dot{u}(0)} = \frac{1}{S_h}\cdot\frac{\frac{4k\rho c}{m}\cdot\left[\frac{m}{4k}\cdot\left\{\left(\frac{2k}{m}\right)^2 - 1\right\} - j\sin^2\left(\frac{m}{2}\mu h\right)\right]}{\left[\frac{2k}{m}\cdot\sin\left\{\frac{m}{2}\mu h\right\}\right]^2 + \left[\sin\left\{\frac{m}{2}\mu h\right\} - \mu\cdot\cos\left\{\frac{m}{2}\mu h\right\}\right]^2}. \quad (11)$$

2.4 Evaluation method of transmitted vibration energy into the horn

The transmitted and consumed vibration energy in audio-base can be evaluated by analogy between mechanical vibration and electric circuit. Now consider the pressure $p(x)$ and vibration velocity $u(x)$ at $x = 0$ as voltage and current respectively in an electric circuit. Let us consider the loaded equipment on the audio-base as vibration source having internal impedance $Z_0 = r + jx$ and driving impedance of the audio-base as $Z_1 = R + jX$ as shown Fig. 3. The vibration energy P transmitted into the horn is given by

$$P = Re[p\dot{u}] = \rho c\dot{u}^2, \quad (12)$$

under $Z_0 = \rho c$. In Fig. 3, the current $|I|$ is given as

$$|I| = \frac{E}{\sqrt{(r+R)^2 + (x+X)^2}}. \quad (13)$$

The consumed energy P is given as

$$P = E^2\frac{R}{(r+R)^2 + (x+X)^2}. \quad (14)$$

Fig. 4 illustrates the consumed energy P at R for R/r. From this figure, the nearer Z_1 to Z_0, the more vibration energy is transmitted to Z_1. This fact is well known as impedance matching.

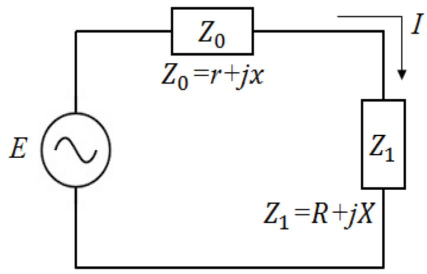


Figure 3. An electric circuit model to describe the connection of impedance and energy transmission.

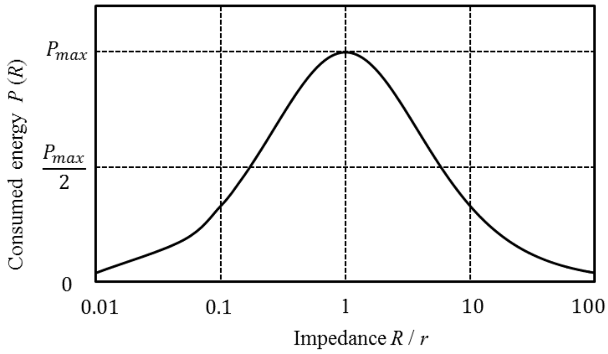


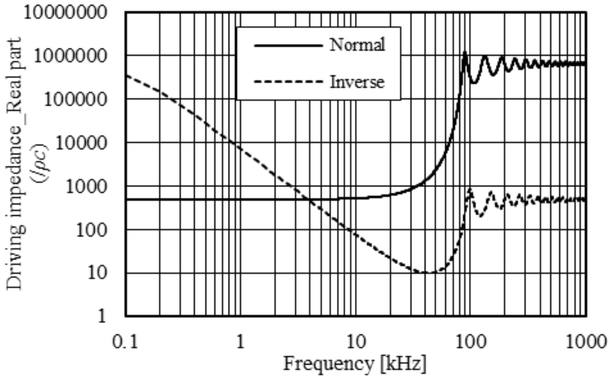
Figure 4. Frequency characteristic of vibration energy consumed at real part of input impedance $\boldsymbol{Z}_1$.

3. Numerical calculation of driving impedance of horn

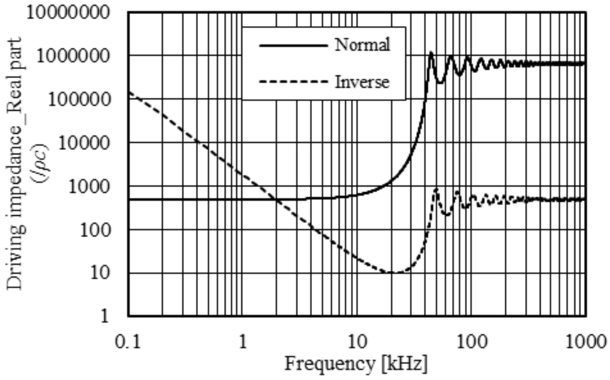
Based on the experimental result of working of audio-base to a loaded equipment [1], we carried out numerical calculation to evaluate transmitted vibration energy from equipment to audio-base. As mentioned in previous section, the ability to transmit the vibration energy into the horn can be evaluated its driving impedance at input of horn by making a comparison given in normal direction and in inverse direction. The energy transmission characteristics were evaluated for some parameters like as shape and material of horn by using Eqs. (8) to (11). The used parameters are shown in Table 1.

Fig. 5 illustrates the comparisons of driving impedances for normal direction and inverse direction under several flare constants of $m = 100$, 200 and 400 with fixed cross sections S_0 and S_h as shown in Table 1. In these figures, driving impedances for normal direction keep approximately fixed value related with cross section S_0 and ρc in lower frequency region. On the other hand, the driving impedances for inverse direction decrease with the increase of frequency. Based on the previous discussion on impedance matching, the closer driving impedance to ρc, the more vibration energy is transmitted to the horn. In these figures, the driving impedance for inverse direction approach ρc than that for normal direction after specific frequency point approximately 2 kHz for $m = 100$, 4 kHz for $m = 200$ and 8 kHz for $m = 400$. That is, the more flare constant increase, the more the specific frequency alternate point increase under fixed cross sections S_0 and S_h. Then, we could reach to conclusion that audio-base prevents well vibration in lower region of the specific frequency but eliminates it well in higher region of it for inverse direction comparing with normal direction. Contrasting with exponential one, a column shaped one keeps fixed value of driving impedance over all frequency region related with cross section and ρc but without the direction.

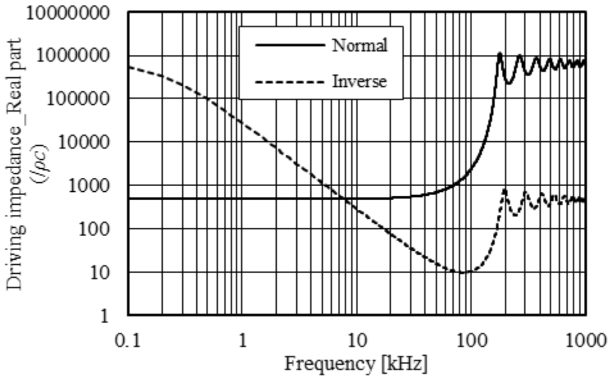
Next, to illustrate the influence of material of horn, we carried out numerical calculation for three kinds of materials for example Brass, Nylon 6,6 and Magnesium with use of parameters shown in Table 2. Results are shown in Fig. 5(a), Figs. 6 (a) and (b). Comparing these results, the wave propagating velocity influences the specific frequency point of alternating the vibration transmitting characteristic between normal and inverse directions. In these figures, they are in order of increasing the wave propagating velocity as Nylon6,6, Brass and Magnesium.



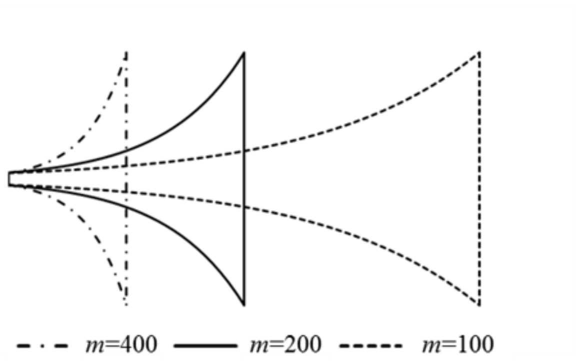
(a) Material: Brass, Flare constant: m =200.



(b) Material: Brass, Flare constant: m =100.

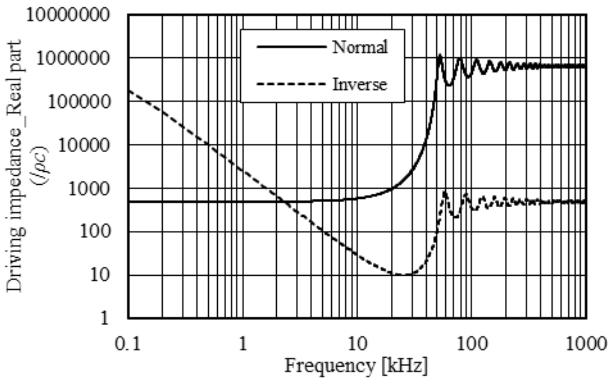


(c) Material: Brass, Flare constant: m=400.

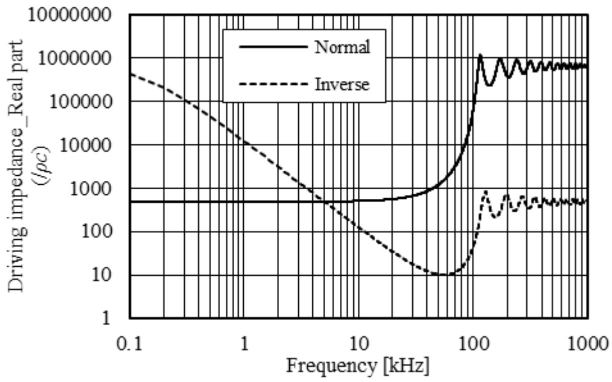


(d) Shapes of employed horns by difference of flare constant under fixed S_0 and S_h.

Figure 5. Comparisons of driving impedance for exponential horn related to transmitting direction for three kinds of flare constants.



(a) Material: Nylon 6,6, ρ =1110 kg/m^3, c =2620 m/s.



(b) Material: Magnesium, ρ =1540 kg/m^3, c =5770 m/s.

Figure 6. Comparisons of driving impedance for exponential horn related to transmitting direction for two kinds of material of Nylon 6,6 and magnesium.

Table 1. Parameters used in numerical calculation to evaluate the specific frequency point alternating the driving impedance for normal direction and inverse direction owing to flare constants.

Properties And Parameters		Values
Density (Brass)	ρ [kg/m^3]	8500
Velocity (Brass)	c [m/s]	4430
Radius of Throat	r_0 [m]	0.0007
Radius of Mouth	r_h [m]	0.0256
Cut-Off Frequency	f_c [Hz]	70505

Flare Constant	m	100	200	400
Height	h [m]	0.072	0.036	0.018
Alternating Frequency	f [Hz]	1900	3800	7700

Table 2. Parameters used in numerical calculation to evaluate the specific frequency point alternating the driving impedance for normal direction and inverse direction for several materials.

Shaping Parameters		Values		
Flare Constant	m		200	
Height	h [m]		0.036	
Radius of Throat	r_0 [m]		0.0007	
Radius of Mouth	r_h [m]		0.0256	
Properties of Material		Brass	Nylon 6,6	Magnesium
Density	ρ [kg/m^3]	8500	1110	1540
Velocity	c [m/s]	4430	2620	5770
Alternating Frequency	f [Hz]	3800	2300	5000

Fig. 7 shows a comparison of transmitted energy P into audio-base for normal direction with inverse direction. Calculation was carried out for a horn formed under $m = 200$ with parameters given in Table 1. Here, P was calculated by Eq. (14) under $E = 1$ and illustrated in dB values. As shown in this figure, in the higher frequency region than the specific frequency decided by driving impedance, the transmitted vibration energy for normal direction decreases less than that for inverse direction.

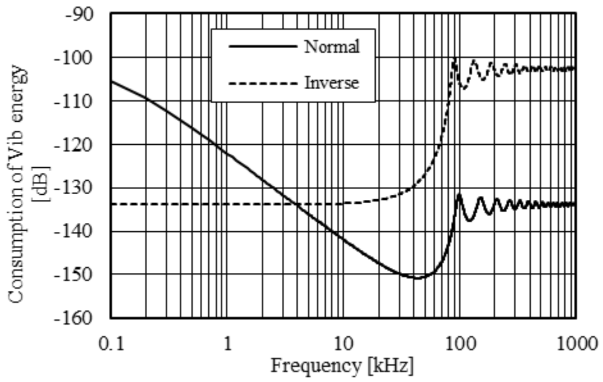


Figure 7. A comparison of transmitted vibration energy into audio-base between inverse and normal directions.

4. Experimental study on vibration transmission characteristic

Next, to confirm experimentally the theoretical results, we carried out an experiment to observe a vibration transmission characteristic of audio-base. Here, we employed an audio-base shaped in exponential horn under $m = 200$ made by brass with surrounding cup as shown in Fig. 1(a) whose parameters are given in Table 1. The diagram of measuring system for vibration transmission characteristics is shown in Fig. 8. The surface of “throat” or “mouth” was excited by a piezo exciting unit with several frequencies of sinusoidal signal and vibration velocity amplitude on exciting point (input) and transmitted side (output) were measured by laser Doppler vibrometers and a FFT analyser.

Fig. 9 shows the result of difference of vibration velocity amplitude level between input and output surfaces for both direction. Here, the difference between “Normal” and “Inverse” directions means the ability to insulate or eliminate the vibration toward output. In this figure, in higher frequency region than about 4 kHz, the vibration level difference for “Inverse” direction become smaller that for “Normal” direction. This frequency point 4 kHz is approximately corresponds to the alternate points given in Figs. 5 and 7. And the same tendency in Fig. 9 is also shown in Fig. 5 and 7. This means that the audio-base works as insulator to prevent vibration from floor for “Normal” direction but discharger to eliminate the vibration to floor for “Inverse” direction in higher frequency region of specific frequency point alternating the transmission characteristic between both directions. So, an audio-base is expected to reduce vibration on the loaded equipment.

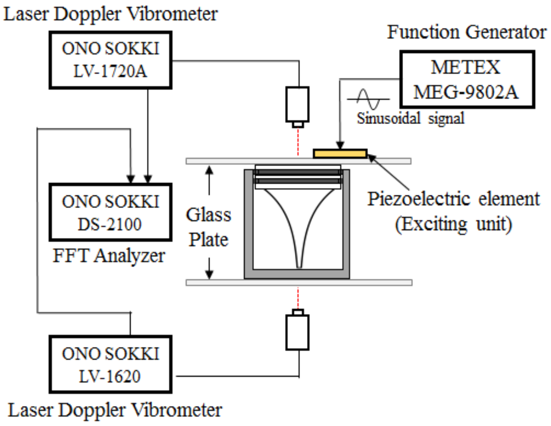


Figure 8. Schematic diagram of experimental arrangement to measure the vibration reduction by a horn.

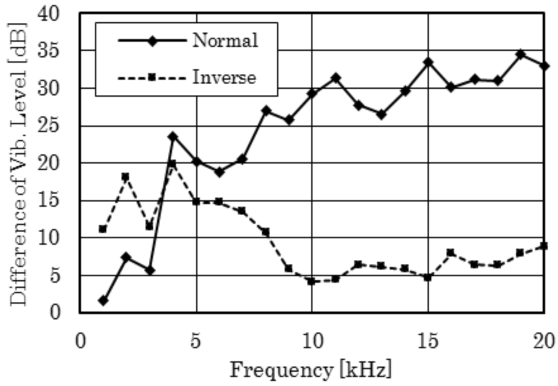


Figure 9. Experimental result of vibration transmission characteristics for exponential horn shape audio-base related to transmitting direction.

5. Conclusions

In this study, we have tried to clarify the working mechanism of audio-bases especially shaped in exponential horn by theoretical analysis on vibration transmission in the horn. The vibration transmission characteristics of the audio-base was evaluated by driving impedance of horn under excitation at “throat” or “mouth”. Based on the numerical result, it was clarified that a horn shaped audio-base has an alternating frequency point. In higher frequency region than this specific frequency point, the horn passes through vibration energy well from “mouth” to “throat” direction (i.e. inverse direction) than that for opposite direction and in lower frequency region than it the horn prevent vibration transmission. Especially, this specific frequency point is controllable by flare constant and transmission velocity of material of horn. Furthermore, these theoretically results of vibration transmission characteristics show same tendency of experimentally observed results for audio-base with exponential horn. Based on above theoretical and experimental results, we could draw a conclusion that the audio-base is effective in discharging vibration of audio equipment put on it and in insulating it from floor in a specific frequency region.

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