24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

IDENTIFICATION OF FOUNDATIONS IN ROTATING MACHINERY WITH NON-PROPORTIONAL HYSTERETIC DAMPING ASSUMPTION

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The overall dynamic behaviour of rotating machinery is significantly affected by its foundation. Therefore, foundation parameters are necessarily identified in the recent parametric model based rotor dynamic study, including rotor fault diagnosis and machine structure design. This paper presents a procedure to identify the modal parameters of rotor foundations with non-proportional hysteretic damping assumption. By introducing quasi-modal parameter, the modal analysis equation can be decoupled under physical coordinate. Hence, parameter identification equation with multiple solution sets is derived and each independent solution set corresponds to the modal parameters of each vibration mode. A double iteration algorithm is applied to solve the derived non-linear identification equation with complex unknowns. The excitation forces and displacements of the foundation, which are obtained when the rotating machinery is in operation, serve as input data to solve the identification equation. Here, the foundation excitation forces are calculated by the absolute displacements of the rotor at the bearing supports, based on the known rotor model and rotor unbalances. The technique developed in this paper is validated by identifying the equivalent foundation of a 162 degree of freedom numerical rotor-bearing-foundation system (RBFS). It is shown that an accurate and reliable foundation model with non-proportional hysteretic damping assumption can be identified, which reproduces the rotor responses correctly when substituted into the original RBFS. However, further experimentation is needed to properly evaluate the applicability of the proposed identification procedure to the foundation of actual rotating machinery.

Keywords: system identification, rotor foundation, complex mode shape

1. Introduction

The overall dynamic behaviour of rotating machinery systems is significantly affected by its supporting structure or foundation which, in order to enable rotor dynamic system studies [1-3], generally needs to be modelled by appropriate parameters to form an equivalent foundation (defined as a foundation which, when substituted for the actual foundation, reproduces the vibration behaviour of the rotating machinery system over the operating speed range of interest). Obtaining these appropriate parameters is still an area for research and one promising technique uses the measurements of the foundation motions of actual rotating machinery [4] to identify relevant modal parameters for the equivalent foundation. If successful, such an identification technique would be applicable to identify the supporting structures of existing rotating machinery installations, using readily available monitoring instrumentation.

Such modal parameter identification procedures usually solve a modal analysis equation to identify the modal parameters of all vibration modes simultaneously, which include natural frequencies, mode shapes, modal masses and damping ratios/factors [5, 6]. Serving as the measurement data input are the harmonic excitation forces and displacement responses on the foundation, caused by the existing rotor unbalance when the machinery is in operation. Under laboratory environment, the displacement responses of the foundation at any given rotor operating speed can be measured by accelerometers; however the foundation excitation forces need to be determined indirectly. Nevertheless, a few different techniques to estimate the excitation forces have been experimentally validated by different researchers [7, 8], providing different levels of accuracy in force estimation.

In earlier modal approaches, the foundation was commonly assumed to have proportional damping as this assumption reduced the complexity of the modal analysis and provided a general approximation [9, 10]. Hence, a few modal parameter identification techniques with proportional damping assumption have been developed, enabling acceptable identification results with experimental validation [11, 12]. However, in real applications, the structures to be identified do not necessarily have proportional damping and may even involve nonlinearity [13]. Consequently, a more accurate assumption of non-proportional damping is preferred when modelling large and complex structures in industry [14-15].

In our previous research, we developed a quasi-modal parameter based identification technique to identify an equivalent system for 5 degree of freedom (DOF) systems, under the assumption of non-proportional hysteretic damping [16]. In this paper, this technique will be further evaluated, via numerical experiments, to identify equivalent foundations in rotor machinery. Experimental evaluation of the technique under laboratory environment will be left for future work.

2. Identification theory

An n DOF equivalent foundation with hysteretic damping, which is expected to accurately represent the original foundation, has the equation of motion:

$$\mathbf{M}\ddot{\tilde{\mathbf{x}}} + \mathbf{C}\dot{\tilde{\mathbf{x}}} + \mathbf{K}\tilde{\mathbf{x}} = \tilde{\mathbf{f}}. \tag{1}$$

Here, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the assumed symmetric mass, damping and stiffness matrices of the equivalent foundation respectively. The hysteretic damping matrix is further represented as $\mathbf{C} = \mathbf{H}/\Omega$ since it is assumed to be frequency dependent [17]. The elements in vector $\tilde{\mathbf{x}}$ are the n independent displacements chosen to coincide with the convenient measurement locations on the original foundation, and include the application points of the excitation forces. The elements of vector $\tilde{\mathbf{f}}$ are the excitation forces acting at those selected locations. Assuming the structure is under harmonic excitation with frequency Ω and the response of the structure at steady state is periodic, $\tilde{\mathbf{f}} = \tilde{\mathbf{F}}e^{i\Omega t}$ and $\tilde{\mathbf{x}} = \tilde{\mathbf{X}}e^{i\Omega t}$; one can obtain from Eq. (1):

$$-\Omega^2\mathbf{M}\tilde{\mathbf{X}} + i\mathbf{H}\tilde{\mathbf{X}} + \mathbf{K}\tilde{\mathbf{X}} = \tilde{\mathbf{F}}. \tag{2}$$

Defining complex stiffness $\mathbf{K}^* = \mathbf{K} + i\mathbf{H}$, equation (2) is written as

$$-\Omega^2\mathbf{M}\tilde{\mathbf{X}} + \mathbf{K}^*\tilde{\mathbf{X}} = \tilde{\mathbf{F}}. \tag{3}$$

The homogenous form of Eq. (3) leads to a complex eigenproblem, whose solutions are diagonal complex eigenvalue matrix $\boldsymbol{\lambda}^*$ and complex modal matrix $\boldsymbol{\Phi}^*$. The orthogonality property of the complex modal matrices $\boldsymbol{\Phi}^*$ holds; therefore matrices $\mathbf{M}$ and $\mathbf{K}^*$ are diagonalised by the complex modal matrix to obtain complex modal mass matrix and complex modal stiffness matrix:

$$\mathbf{m}^* = \boldsymbol{\Phi}^{*\mathbf{T}}\mathbf{M}\boldsymbol{\Phi}^*, \tag{4}$$

$$\mathbf{k}^* = \mathbf{\Phi}^{*\mathbf{T}}\mathbf{K}^*\mathbf{\Phi}^*. \tag{5}$$

Thereby, complex eigenvalue matrix is defined as

$$\boldsymbol{\lambda}^* = \mathbf{m}^{*-\mathbf{1}}\mathbf{k}^*. \tag{6}$$

Rewriting Eq. (3) in modal coordinate by letting $\widetilde{\mathbf{X}} = \mathbf{\Phi}^*\widetilde{\mathbf{Q}}$ and pre-multiplying both sides of the equation by the transpose of complex modal matrix $\mathbf{\Phi}^{*\mathrm{T}}$, one obtains:

$$(-\Omega^2\mathbf{\Phi}^{*\mathbf{T}}\mathbf{M}\mathbf{\Phi}^* + \mathbf{\Phi}^{*\mathbf{T}}\mathbf{K}^*\mathbf{\Phi}^*)\widetilde{\mathbf{Q}} = \mathbf{\Phi}^{*\mathbf{T}}\widetilde{\mathbf{F}}. \tag{7}$$

Substituting $\widetilde{\mathbf{Q}} = \mathbf{\Phi}^{*-1}\widetilde{\mathbf{X}}$ and pre-multiplying both sides of the equation by $\mathbf{m}^{*-1}$, Eq. (7) is re-written as:

$$(-\Omega^2\mathbf{I} + \mathbf{m}^{*-\mathbf{1}}\mathbf{k}^*)\mathbf{\Phi}^{*-\mathbf{1}}\widetilde{\mathbf{X}} = \mathbf{m}^{*-\mathbf{1}}\mathbf{\Phi}^{*T}\widetilde{\mathbf{F}}. \tag{8}$$

Introducing complex quasi-modal matrix, which has transformation relationship with complex modal matrix $\mathbf{\Phi}^*$:

$$\mathbf{A}^{*\mathbf{T}} = \mathbf{\Phi}^{*-\mathbf{1}}, \tag{9}$$

and substituting Eq. (6) into Eq. (8), one obtains:

$$(-\Omega^2\mathbf{I} + \boldsymbol{\lambda}^*)\mathbf{A}^{*\mathbf{T}}\widetilde{\mathbf{X}} = \mathbf{m}^{*-\mathbf{1}}\mathbf{\Phi}^{*T}\widetilde{\mathbf{F}}. \tag{10}$$

Eq. (10) comprises *n* identification equations (*k=1...n*):

$$(-\Omega^2 + \boldsymbol{\lambda}_k^*)\sum_{j=1}^{n}\boldsymbol{a}_{jk}^*\boldsymbol{X}_j = \boldsymbol{m}_k^{*-1}\sum_{j=1}^{n}\boldsymbol{\Phi}_{jk}^*\boldsymbol{F}_j. \tag{11}$$

Providing the amplitudes of excitation force and displacement response on the foundation as input data, the modal parameters to be identified in each of Eq. (11) are λ_k^*, a_{jk}^*, Φ_{jk}^* and m_k^* (*j=1...n*), which correspond to the modal parameters of each individual vibration mode. Hence, it can be considered there are maximum *n* independent sets of solution to Eq. (11). The procedure to solve Eq. (11) for the unknown parameters are as presented in Ref [16].

After all required complex modal parameters, which include λ_k^*, Φ_{jk}^* and m_k^* (*j = 1...n; k = 1...n*), are identified, they are substituted into the frequency response function to reproduce the displacement response of each DOF. For an *n* DOF system with non-proportional hysteretic damping, the displacement response at each DOF is (*i=1...n*) [17]:

$$\boldsymbol{x}_i = \sum_{k=1}^{n}\sum_{j=1}^{n}\frac{\boldsymbol{\Phi}_{ik}^*\boldsymbol{\Phi}_{jk}^*\boldsymbol{F}_j}{\boldsymbol{m}_k^*(-\Omega^2+\boldsymbol{\lambda}_k^*)}. \tag{12}$$

By convention, the eigenvalues λ_k^* (k = 1…n) are further represented in term of complex quantities [17]:

$$\boldsymbol{\lambda}_k^* = \boldsymbol{\omega}_k{}^2 + \boldsymbol{i}\boldsymbol{\omega}_k{}^2\boldsymbol{\eta}_k, \tag{13}$$

where ω_k is the natural frequency and η_k is the damping loss factor of the *k*th vibration mode.

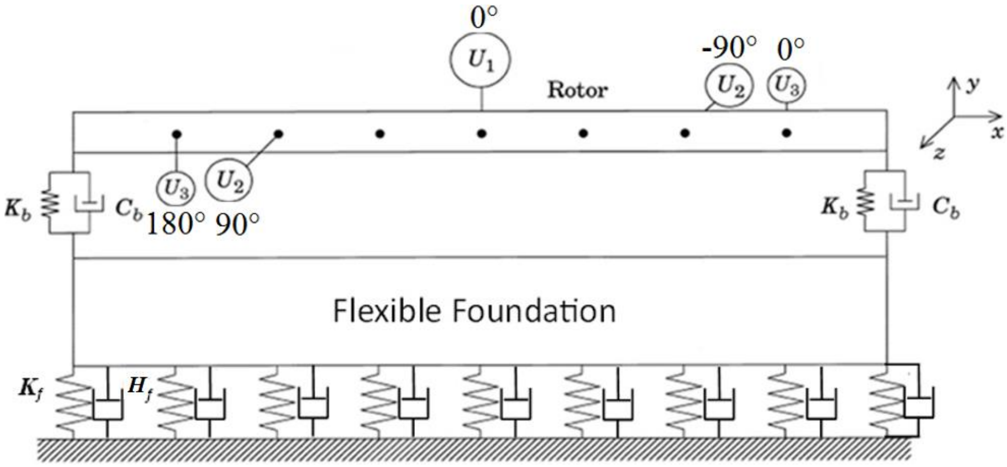
3. Illustrative example

3.1 Model setup of the RBFS

The developed identification theory was applied to identify the equivalent foundation of an RBFS via numerical experiment. The schematic of the RBFS is shown in Fig. 1, which represents an unbalanced rotor running in two hydrodynamic bearings mounted on a damped flexibly supported flexible foundation block.

The rotor is assumed to be 1270 mm long and have a uniform diameter of 101.6 mm. The rotor model is divided into 8 equal segments so that each segment is 158.75 mm long. Each end of the rotor is supported by narrow plain hydrodynamic journal bearings. Each bearing is assumed to have a length/diameter ratio of 1/4, a radial clearance of 0.0508 mm and a mean oil viscosity of 6.9 $\times$ 10^{-3} N s/m^2. The unbalances are assumed to be lumped on the rotor as shown in Fig. 1. The magnitude and phase of each unbalance are also indicated in Fig. 1.

The dimension of the foundation block model is shown in Fig. 2, being L=1270 mm long, W=317.5 mm wide and H=158.75 mm high. The block is assumed to be isotropic with density 7850 kg m^{-3}, Young's modulus 210 GPa, shear modulus 80 GPa and Poisson's ratio 0.3. It is divided into 16 cubical finite elements, resulting in 54 nodes. Points B_1 and B_2 are the connection points between the foundation and the bearings, where the excitation forces are transmitted to the foundation. Every node will have 3 DOFs, viz. the x, y and z translational displacements. Hence, this foundation has a total of 162 DOFs and 162 vibration modes. Each cubical element in the foundation model has the same isotropic spring and damper supports at its bottom nodes as shown in Fig. 2; but different cubical elements have different values of spring stiffness and hysteretic damping, which are the same as those used in Ref [18].



U_1=10^{-4} kg m, U_2=10^{-5} kg m, U_3=10^{-6} kg m.

Figure 1: Unbalanced rotor mounted on a flexible foundation block with spring and damper support; with unbalance distribution No. 1

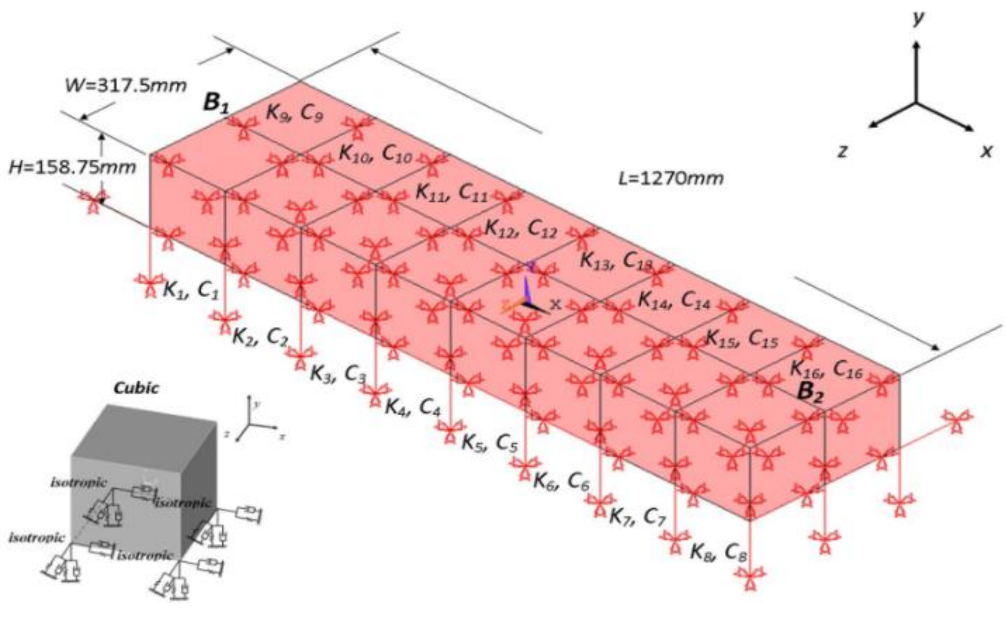


Figure 2: Finite element model of the foundation with spring and damper support.

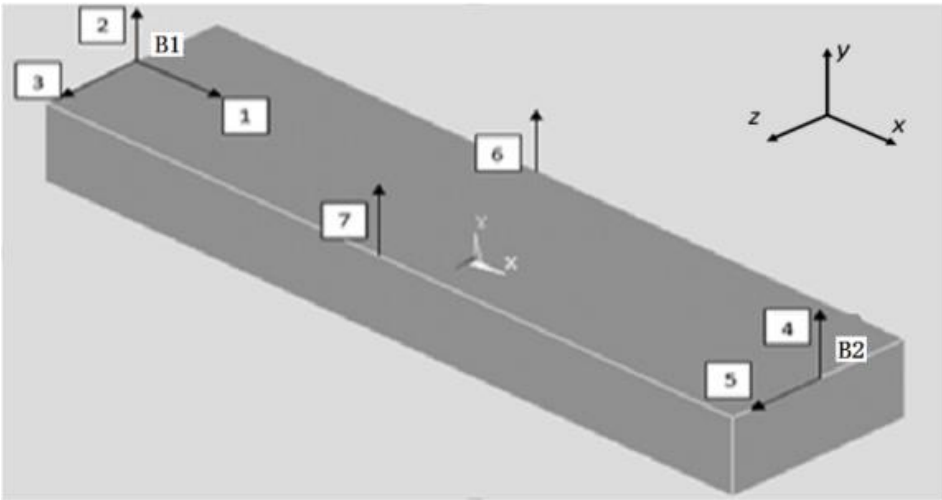


Figure 3: The 7 DOFs and measurement locations on the foundation block.

The equation of motion for such a foundation model, assuming a lumped mass formulation for each cubical element, results in a 162 by 162 diagonal mass matrix, a 162 by 162 diagonal damping matrix and a 162 by 162 symmetric stiffness matrix. ANSYS is used to assemble these matrices and find the foundation natural frequencies and mode shapes. The foundation model has 162 vibration modes. According to the mode shapes simulated by ANSYS [18], the first 6 modes are rigid body modes and the 7^{th} onwards are flexural modes. The first 8 natural frequencies are 517.41 rad/s, 697.43 rad/s, 786.28 rad/s, 797.84 rad/s, 886.56 rad/s, 1009.3 rad/s, 2730.6 rad/s and 4767.5 rad/s.

Assuming the operating speed range of the RBFS to be from 300 to 1250 rad/s, the maximum excitation frequency will be 1250 rad/s. In this system since the 8^{th} natural frequency of the foundation is around 4 times of the maximum measurement frequency, the contribution of the 8^{th} and higher vibration modes to the vibration behaviour of the foundation was assumed to be negligible, and hence an equivalent foundation with 7 DOFs was expected to properly reproduce the vibration behaviour of the RBFS over the operating speed range. Fig. 3 shows the 7 selected measurement locations and directions on the foundation surface. Measurements in directions 1, 2, and 3 are taken at *B1*; in direction 4 and 5 are taken at *B2*; and in directions 6 and 7 are taken at the two sides halfway along the block. It is assumed that all external forces on the foundation are transmitted through the bearings. These forces act in directions 2, 3 at *B1* and 4, 5 at *B2*. These selected DOFs are expected to sufficiently describe the 1^{st} to 7^{th} mode shapes, hence forming a condensed 7 DOF equivalent foundation.

The vibration behaviour of the RBFS was simulated by an in-house transfer matrix software [19], in which the rotor, bearing and foundation models as discussed above were implemented. The numerically calculated forces and displacements at the selected locations (DOFs) served as the ‘measurement data’, which were used as the input data to identify the modal parameters of the equivalent foundation. The input data at these locations resulted from the unbalance distribution shown in Fig. 1, and was calculated at 50 rad/s intervals over the rotor speed range of 300 to 1250 rad/s, resulting 20 measurement speeds. The ‘measurement’ data was simulated with 16 digit accuracy at this stage in order to validate the proposed identification technique in principal.

3.2 Identification result

The identified natural frequencies are compared with the ‘actual’ natural frequencies obtained from numerical experiment in table 1. As can be seen, all 7 natural frequencies are identified to within an accuracy of 1 percent. The identified damping loss factors are also shown in table 1. The identified mode shapes and modal masses are complex quantities. Their moduli and arguments are presented in tables 2 and 3 respectively.

Table 1: Actual and identified natural frequencies (rad/s); identified damping loss factors (dimensionless)

Mode	1st	2nd	3rd	4th	5th	6th	7th
Actual ω_k	517.41	697.43	786.28	797.84	886.56	1009.3	2730.6
Identified ω_k	514.49	702.39	782.61	797.77	887.33	1011.8	2743.2
Identified η_k	9.3e-3	4.8e-3	2.7e-2	1.5e-2	1.4e-2	1.5e-2	1.1e-2

Table 2: Moduli of identified mode shapes and modal masses

Mode	Φ_{1k}^*	Φ_{2k}^*	Φ_{3k}^*	Φ_{4k}^*	Φ_{5k}^*	Φ_{6k}^*	Φ_{7k}^*	m_k^*
1st	0.001	0.004	0.399	0.010	0.475	0.323	0.309	78.59
2nd	0.507	0.558	0.106	0.970	0.028	0.242	0.177	192.4
3rd	0.064	0.106	0.559	0.044	0.523	0.129	0.124	113.4
4th	0.211	0.396	0.038	0.390	0.050	0.397	0.384	102.2
5th	0.289	0.938	0.075	0.594	0.108	0.169	0.235	184.6
6th	0.001	0.012	0.285	0.004	0.009	0.360	0.373	96.40
7th	0.106	0.348	0.065	0.308	0.064	0.181	0.187	15.31

Table 3: Arguments of identified mode shapes and modal masses (degrees)

Mode	Φ_{1k}^*	Φ_{2k}^*	Φ_{3k}^*	Φ_{4k}^*	Φ_{5k}^*	Φ_{6k}^*	Φ_{7k}^*	m_k^*
1st	159.69	-178.54	-2.26	173.44	-2.19	177.7	-2.22	-5.05
2nd	-2.40	177.88	-129.93	-0.97	-27.88	15.48	-19.94	-7.84
3rd	-122.99	60.88	179.19	106.28	3.54	18.77	131.58	33.70
4th	0.24	-178.74	49.19	-178.54	-148.48	-177.60	-179.73	6.22
5th	2.17	0.29	-103.12	177.41	66.01	-20.49	22.68	4.91
6th	-116.38	0.97	-174.92	-79.39	79.80	-174.06	5.08	2.85
7th	169.1	-7.58	-108.40	9.56	-30.65	-178.69	177.43	-2.71

The modal parameters presented in tables 1 to 3 were used to reproduce the displacement responses of the foundation. Both the amplitudes and phases of the reproduced responses are compared in Fig. 4 with those obtained numerically from the actual foundation in Fig. 2. The response comparisons are in the directions 3, 5 and 7. As can be seen, there is good agreement between the actual and reproduced responses. Similar level of agreement was obtained in others directions, although they are not presented in this paper.

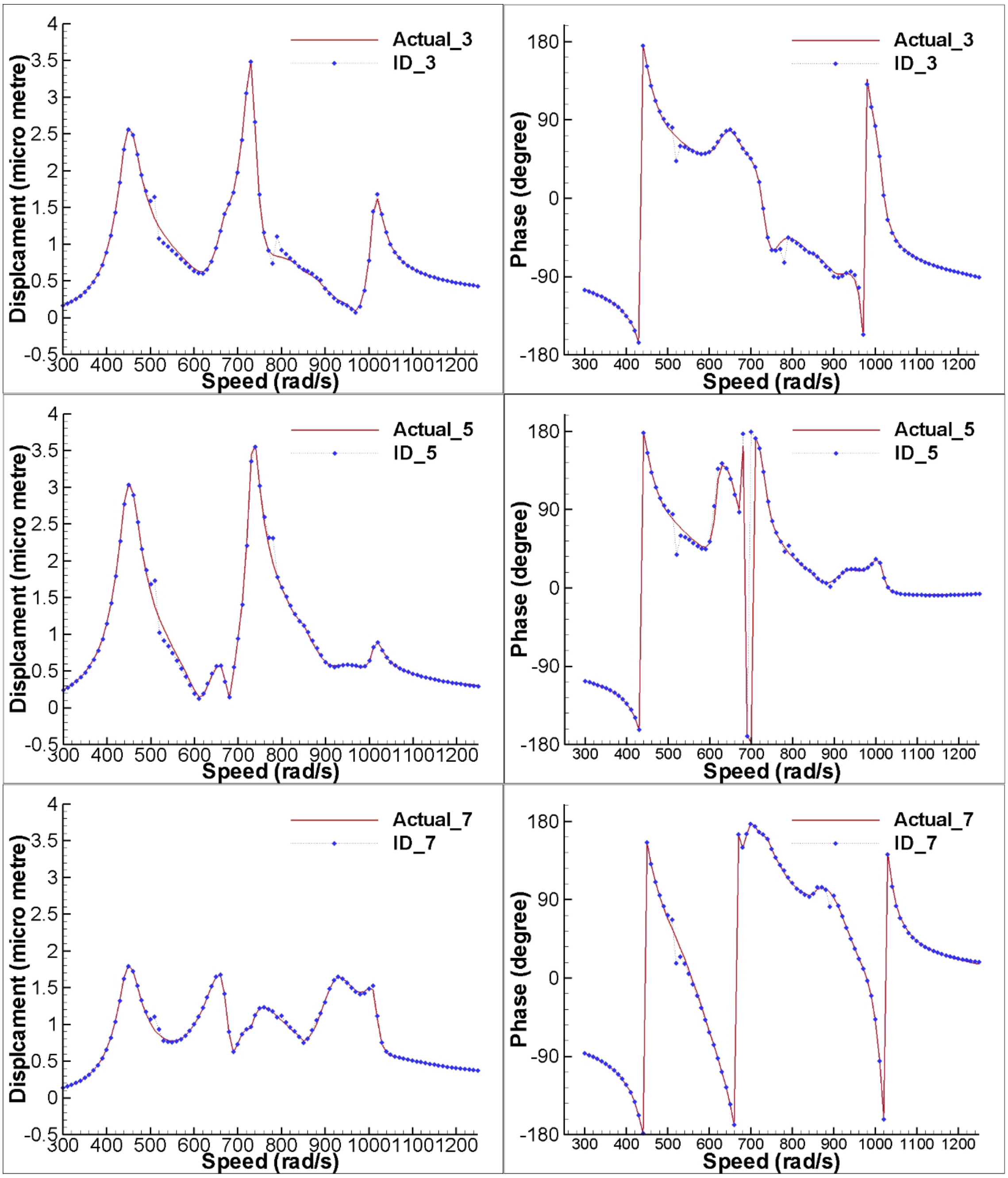


Figure 4: 'Actual' and reproduced responses of the foundation; in the directions 3, 5 and 7.

4. Conclusion

This paper evaluates a quasi-modal parameter based identification technique by identifying the equivalent foundation of an RBFS, under the assumption of non-proportional hysteretic damping. The identification results show that, the identified natural frequencies of the equivalent foundation agree well with the natural frequencies of the actual foundation; and the identified equivalent foundation is able to reproduce displacement responses correctly over the operating frequency range of the rotor machinery. Hence, the identified 7 DOF equivalent foundation is considered to provide proper substitution for the original foundation. However, the conclusions are based on numerical experiments. In an actual rotating machinery, the error in the input measurement data will be larger; therefore verification requires further laboratory experimentation.

5. Acknowledgments

This work is supported by the National Nature Science Foundation of China (11602309, 11572356).

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