EFFECT OF THE POROUS MATERIAL MODELING ON THE EXTERNAL SOUND FIELD

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Porous materials are widely used in noise control applications for reducing the reverberation acoustic field in closed space, the noise propagation inside dissipative mufflers or the panel vibration. One difficulty in noise control problems is the prediction of the real sound-absorbing capacity of porous materials before installing them.
Usually, to reduce computational time, porous materials are simulated in numerical models (such as FEM, BEM, ray tracing) as an impedance plane or by choosing an absorption coefficient value.
In previous studies, however, the authors investigated about the influence of sound source position and non-acoustic properties of a porous material on its surface acoustic impedance and its sound absorption coefficient. The main result is that, for a given frequency, its acoustic properties change and therefore is generally wrong to assume a single value for the surface acoustic impedance and/or the sound absorption coefficient to characterize the behaviour of the whole porous material.
In this work, it will be reported some features about the errors in choosing a single value for the surface acoustic impedance instead of a suitable modelling of the porous material. The differences depend on the behaviour of the porous material (local or non-local reactive) and the height of the sound source from its surface.
Keywords: porous material, sound field, numerical simulation, local or non-local reaction

1. Introduction

The knowledge of the sound field is important in many studies of acoustics. It allows researchers to study acoustical condition in open and closed spaces, to identify hidden sound sources or to compare different solutions for noise control.

Often the sound field must be numerically simulated such as when the object of study is a new project or it is impossible to mount different solutions. In this case, numerical techniques help to assess all descriptors of the sound field such as the sound pressure, the particle velocity, and so on. The knowledge of boundary conditions is essential to have results close to real conditions. Therefore, the more the boundary conditions are correct the more the results are close to real values. Usually boundary conditions are expressed in terms of the surface acoustic impedance Z_s that in general depends on porous material properties such as its air flow resistivity, thickness or bending stiffness. In this paper, we study the influence of different boundary conditions applied to a porous material layer supposed to be rigid and hard baked by a rigid and impervious surface. The simpler way to model a porous material is to give a constant value of the surface acoustic impedance $Z_s(f)$ for a given frequency on the entire porous material surface. However, if a plane wave impinges on the porous material surface with different incidence angle ϑ_i , the surface acoustic impedance $Z_s(f,\vartheta_i)$ is also affects by the latter parameter [1].

In real situations, any plane wave exists and the sound field is more complicated. Considering a spherical wave is, of course, a subsequent approximation. In this case the surface acoustic impedance

$Z_s(f,\vartheta_i)$ changes over the porous material surface x_p and it also depends on the sound source positions x_s [2]. It is easy to understand that considering $Z_s(f,\vartheta_i,x_p,x_s)$, in numerical simulations, implies complications in designing the model and requires more computation time. In the latter case, appropriate models should be used [3]. In many cases, porous material is modelled with $Z_s(f)$ hypothesis without considering any further complications and therefore sound field is affected by this condition. Aim of this work is to study what are the main consequences of this simplification.

A simple case of sound field above a porous material is taken into account and different values of frequency, thickness, airflow resistivity and sound source position are analysed.

2. Surface acoustic impedance models

The simplest model that can be used to predict $Z_s(f)$ starting from the acoustic properties of the porous material can be obtained by combining ingoing and outgoing plane waves that travel in the porous material along the normal direction. According to this model the surface acoustic impedance is given by:

$$Z_s(f) = -j\frac{Z_m}{\phi}cot(k_m d) \tag{1}$$

where Z_m and k_m are the characteristic impedance and wave number of the porous material, ϕ is the open porosity and d the thickness. $Z_s(f)$ is constant above the surface of the porous material and it does not depend on the angle of sound incidence ϑ_i.

$Z_s(f,\vartheta_i)$ model considers a plane-wave propagation inside the porous material but along a different direction with respect to the normal at the porous material-free air interface. As for the previous model, by combining ingoing and outgoing plane waves in the porous material, it is possible to obtain a similar equation for the surface acoustic impedance:

$$Z_s(f,\vartheta_i) = -j\frac{Z_m}{\phi}\frac{k_m}{k_{mz}}cot(k_{mz} d) \tag{2}$$

where k_{mz} is the component of the complex wave number vector of the porous material orthogonal to its surface. It can be obtained from Snell's law [1]:

$$k_{mz} = \sqrt{k_m^2 - k_{mr}^2} = \sqrt{k_m^2 - k_{0r}^2} = \sqrt{k_m^2 - k_0^2\, sin^2\,\vartheta_i} \tag{3}$$

where k_0 is the wave number in air, k_{mr} and k_{0r} are the components of the wave number vector parallel to the material surface in the porous material and in free air that are equal and will be reported as k_r. In both cases the reflection coefficient can be obtained by the following equation:

$$R = \frac{Z_S - \frac{Z_0}{cos(\vartheta_i)}}{Z_S + \frac{Z_0}{cos(\vartheta_i)}} \tag{4}$$

It is worth pointing out that this equation is valid only if the wave front, that impinges on the porous material surface, is planar otherwise differences can be observed [4]. Finally, the absorption coefficient can be obtained by:

$$\alpha = 1 - R^2 \tag{5}$$

3. Sound field models

For the case of constant surface acoustic impedance $Z_s(f)$ or $Z_s(f,\vartheta_i)$ over the porous material surface different models can be considered to assess the sound pressure field p.

By previous studies [5] it has been found that the Di and Gilbert model [6] is consistent with numerical simulations compared with other models and therefore it has been used in this study. According to it, total sound pressure, in a point R above the material surface, is given by Eq. (6):

$$p(R) = D\left(\frac{e^{-jk_0 r_1}}{r_1} + \frac{e^{-jk_0 r_2}}{r_2} - 2\rho_0 k_0 c\beta \int_0^\infty e^{-k_0\beta q}\frac{e^{-jk_0 r_q}}{r_q}dq\right) \tag{6}$$

where r_1 and r_2 are the distances between the receiver and the sound source and the receiver and the image sound source respectively and $r_q = \sqrt{r^2 + (z_s + z_r - jq)^2}$. $\beta = 1/Z_s(f)$ is the surface acoustic admittance, q is the integration parameter that appears after using Laplace transform of the reflection coefficient and $Z_s(f)$ is given by Eq. (1). D is a factor that considers the sound source level, r is the horizontal distance between the sound source and the receiver, z_r and z_s are, respectively, the heights of the receiver and the sound source with respect to the porous material surface. The first term of Eq. (6) represents the incident sound pressure while the last two terms represent the reflected sound pressure.

The propagation model over a porous material surface proposed by Allard *et al.* [7], instead, is more general because it does not require any assumption on the porous material surface impedance. The total sound pressure given by this model can be considered as the real values. Numerical simulations [5] and measurement results [4] underline that this model is adequate to predict the sound field above a porous layer. The sound pressure can be obtained by the following equation:

$$p(R) = \frac{e^{-jk_0 r_1}}{r_1} - \frac{e^{-jk_0 r_2}}{r_2} + \int_0^\infty e^{-\nu_0(z_s+z_r)}\frac{2\rho_{m,E}}{\rho_{m,E}\nu_0 + \rho_0\nu_1 tgh(\nu_1 d)}J_0(rk_r)k_r dk_r \tag{7}$$

where $\nu_0^2 = (k_r^2 - k_0^2)$ and $\nu_1^2 = (k_r^2 - k_m^2)$, with $Re(\nu_0^2) > 0$ and $Re(\nu_1^2) > 0$, and J_0 is the zero-order Bessel function. $\rho_{m,E}$ is the complex density of the equivalent fluid medium, that is $\rho_{m,E} = \rho_m/\phi$ where ρ_m is the complex density of the porous material. Also in this case the first term of Eq. (7) represents the incident sound pressure while the last two terms represent the reflected sound pressure. When the porous material becomes highly reflective the last term in Eq. (7) tends to become twice as large as the second term.

4. Numerical simulations

A simple case of a porous material layer is considered as reported in Fig. 1. It is supposed to be infinitely extended in order to avoid the edge effects and a spherical sound source is posed over its surface at a given position.

The sound pressure field is numerically obtained by solving both Eq. (6) and Eq. (7) with a Matlab code verified both by FEM numerical simulations [5] and experimentally [4]. Porous material properties (i.e. ρ_m, Z_m and k_m) are obtained by considering the Miki model [8] to simplify the porous material analysis because only one parameter, the airflow resistivity σ, should be considered. By fixing the frequency, the sound source position, the thickness and the airflow resistivity the sound pressure depends only on the receiver position that is recursively changed to study an extend region of space over the porous material surface.

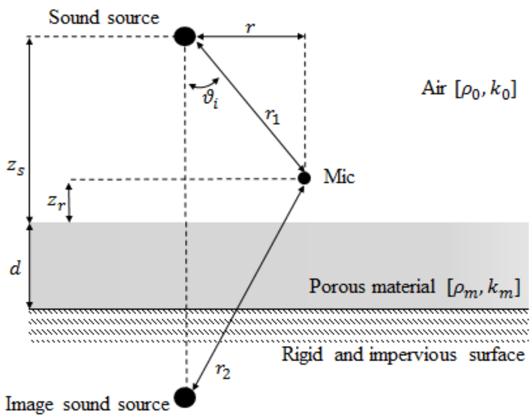

Figure 1: Sketch of the system composed with a porous material backed by a rigid and impervious surface and of the sound source and receiver positions.

5. Results

Several simulations are carried out by considering three values of frequency (100, 500 and 1000 Hz), three values of thickness (3, 100 and 200 cm), three values of airflow resistivity (5000, 10000 and 100000 Rayl/m) and three values of sound source position (30, 100 and 200 cm). Hereafter the guidelines of the analysis method are reported and main results are shown.

As explained in the introduction, the aim of the work is to find differences between sound pressure field given by Eq. (6) and the one given by Eq. (7) that is considered as a benchmark.

The results shown in Figs. 2 and 3 are obtained for a porous layer having an airflow resistivity of 5000 Rayl/m, a thickness of 100 cm and they refer to a frequency $f = 1000Hz$. The sound source height is 30 cm. In particular, the sound pressure field reported in Fig. 2 is given by Eq. (6) and that reported in Fig. 3 is given by Eq. (7). Figures 2.a and 3.a show the sound pressure level referred to 20 μPa.

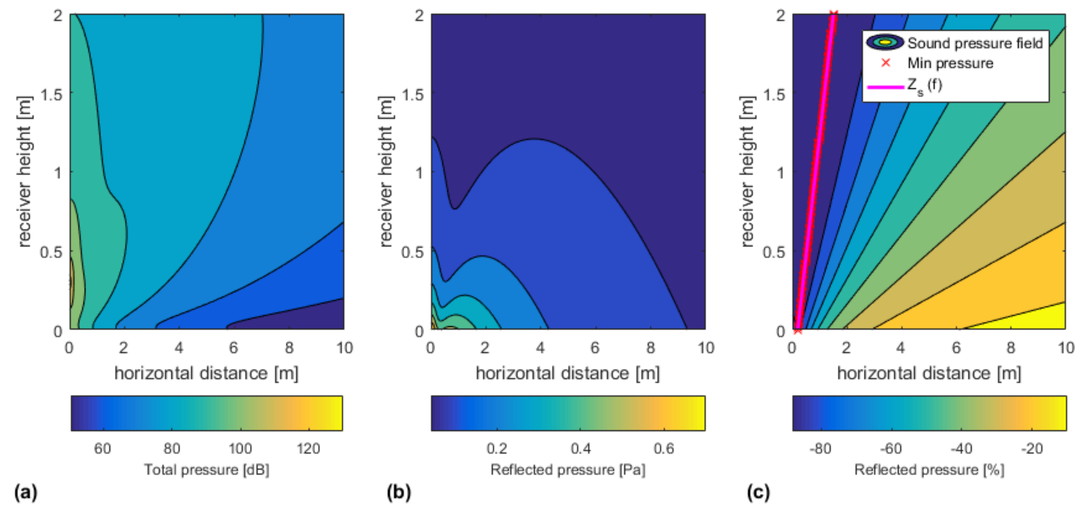

Figure 2: Sound pressure field above porous material obtained by the Di and Gilbert model. Porous material thickness is 100 cm.

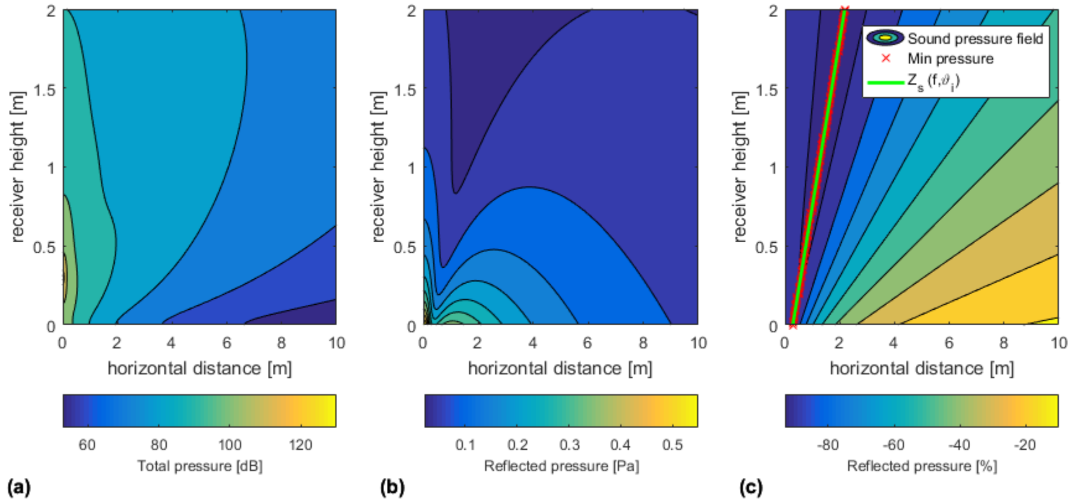

Figure 3: Sound pressure field above porous material obtained by the propagation model proposed by Allard *et all*. Porous material thickness is 100 cm.

Finding a criteria to assess differences is very difficult, therefore it is possible exclude the direct sound field because is equal in both models (i.e. the first term in Eq. (6) and Eq. (7)). Figure 2.b and 3.b report the values of the reflected sound field $p_r(R)$ where a clear difference between two models cannot yet be seen. A way to overcome these difficulties is to refer to the reflected sound pressure (i.e. the last two terms in Eqs. (6) and (7)) with respect to the specular reflection, without attenuation, given by Eq. (8) reported below:

$$p_{sr}(R) = D\left(\frac{e^{-jk_0 r_2}}{r_2}\right) \tag{8}$$

Figure 2.c and 3.c report the absolute value of the perceptual value of reflected sound pressure field expressed as:

$$p_{\%}(R) = 100\frac{p_r(R)-p_{sr}(R)}{p_{sr}(R)} \tag{10}$$

In the same figures the minimum values of $|p_{\%}(R)|$ are reported by crosses. It is possible to see that they are roughly aligned along a straight line but with different orientation. A careful inspection reveals that the orientation of minimum values is strictly related to the angle of maximum sound absorption given by Eq. (4) and reported with a solid line in Figs. 2.c and 3.c. In Figure 2.c this angle is equal to 33.11° and it is computed by using $Z_s(f)$ given by Eq. (1) while in Figure 3.c it is equal to 43.49° and it is computed by using $Z_s(f,\vartheta_i)$, given by Eq. (2). The thickness is chosen large enough to neglect multiple reflections coming from the porous material (i.e. the limit thickness, in the present case is 54 cm). Results underline that the choose of boundary conditions has a significant effect on the reflected sound field. If the multiple reflections inside the porous material are taken into account, the analysis becomes even more interesting.

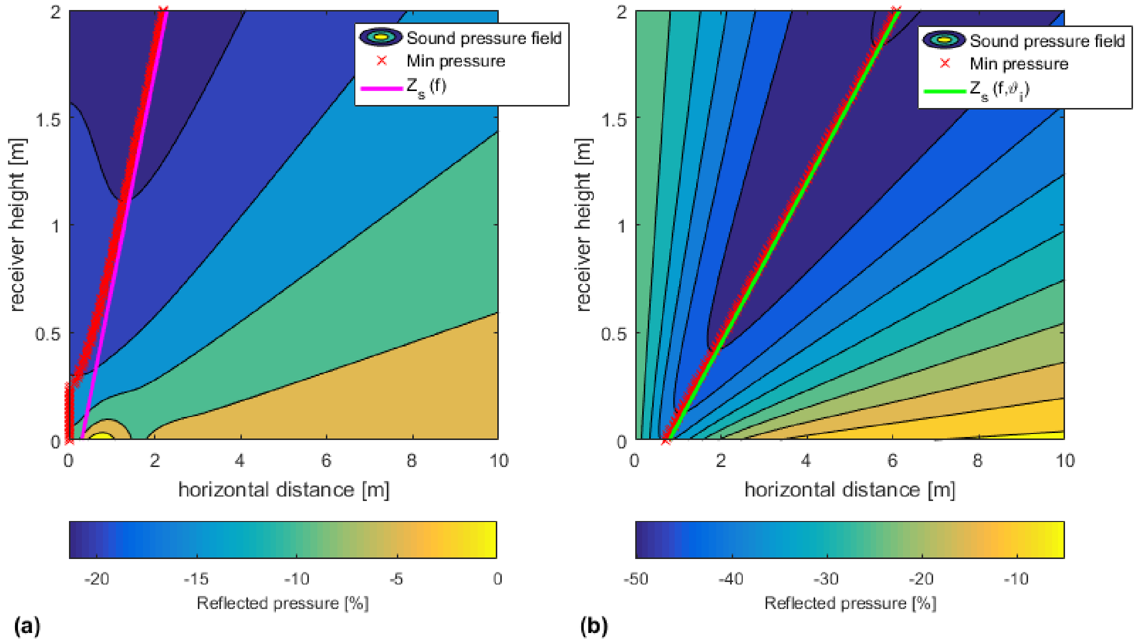

Figure 4: Sound pressure field above porous material obtained by the Di and Gilbert model (a) and the Allard model (b). Porous material thickness is 3 cm.

Figures 4.a and 4.b report values of $|p_{\%}(R)|$ for a thickness of 3 cm for the same frequency and sound source height of Figs. 2 and 3. It is possible to note that the values of minimum of $|p_{\%}(R)|$ are aligned only after a certain distance that can be supposed the distance after that the sound pressure values given by the multiple reflections becomes smaller than the values of the sound pressure reflected directly by the porous material.

By changing the frequency, the position of the source and/or the airflow resistivity a similar behaviour can be observed.

6. Conclusions

In this paper the effects of the porous material modelling on the external sound pressure field are discussed. Even if it is evident that different boundary conditions affect differently the sound pressure distribution over a porous material surface, in this work the real effect was analysed. By choosing a simpler geometry it was observed that the area of the minimum reflected sound pressure field changes considerably if a simpler boundary condition is chosen. In real situations, for which the geometry is more complicated the effect should be amplified and the resulting sound pressure field, especially when the contribution of the direct sound pressure field is neglected (e.g. decay of sound pressure), is wrongly estimated if a constant impedance condition is chosen.

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