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MEASUREMENTS OF TRANSFER MATRICES OF DUCT ELEMENTS AND SOURCE IMPEDANCES, USING THE PAIR-MICROPHONES TECHNIQUE

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## INTRODUCTION

As well known, acoustic characteristics of ducts and silencers are usually analyzed by the transfer matrix method in the frequency region, where plane waves propagate. In this method, it is imperative to estimate the transfer matrix of each duct element and the source impedance as well, in order to estimate the acoustic characteristics of a total duct system. Theoretical estimations are feasible for simple duct elements such as straight ducts and expansion chambers, but real duct elements are often too complicated to be analyzed theoretically. Source impedances are usually assumed to be infinite, but this assumption is not always proper. Therefore, it is indispensable to measure transfer matrices of complicated duct elements and source impedances and to analyze acoustic characteristics of a total duct system using these data. Seybert & Ross<sup>[1]</sup> and Blaster & Chung<sup>[2]</sup> proposed a pair-microphones technique by which acoustic impedances and transmission losses were measured. In this paper, a method based on this technique is studied by which transfer matrices of duct elements and source impedances can be measured.

## MEASUREMENT PRINCIPLE

### Transfer matrix

An acoustic duct and notation are shown schematically in Fig.1. Acoustic pressure  $P_1, P_3$  and volume velocity  $U_1, U_3$  at sections ①, ③ are related as follows.

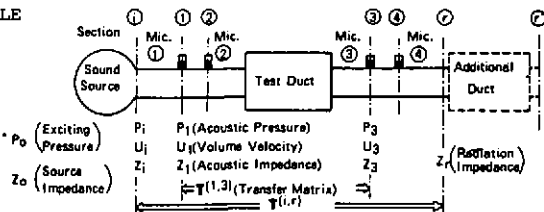


Fig.1 Acoustic duct and notation

$$\begin{pmatrix} F(P_1) \\ F(U_1) \end{pmatrix} = \mathbf{T}^{(1,3)} \begin{pmatrix} F(P_3) \\ F(U_3) \end{pmatrix} \quad (1)$$

where  $F$  denotes the Furier transform and  $\mathbf{T}^{(1,3)} = \begin{pmatrix} T_{11}^{(1,3)} & T_{12}^{(1,3)} \\ T_{21}^{(1,3)} & T_{22}^{(1,3)} \end{pmatrix}$  is the transfer matrix of the section ① - ③. When an additional duct is attached to the end, the above relation remains unchanged. Therefore,

$$\begin{pmatrix} F(P_1') \\ F(U_1') \end{pmatrix} = \mathbf{T}^{(1,3)} \begin{pmatrix} F(P_3') \\ F(U_3') \end{pmatrix} \quad (2)$$

where ' indicates the condition with the additional duct. From equations (1) and (2), the following relations are obtained.

$$T_{11}^{(1,3)} = \frac{H_{31}Z_3 - H_{31}'Z_3'}{Z_3 - Z_3'}, \quad T_{12}^{(1,3)} = -\frac{(H_{31} - H_{31}')Z_3Z_3'}{Z_3 - Z_3'} \quad (3)$$

$$T_{21}^{(1,3)} = \frac{H_{31}Z_3Z_1' - H_{31}'Z_3'Z_1}{(Z_3 - Z_3')Z_1Z_1'}, \quad T_{22}^{(1,3)} = -\frac{(H_{31}Z_1' - H_{31}'Z_1)Z_3Z_3'}{(Z_3 - Z_3')Z_1Z_1'}$$

where  $Z_1 = F(P_1)/F(U_1)$ ,  $Z_3 = F(P_3)/F(U_3)$  denote the acoustic impedances at ① and ③, and  $H_{31} = F(P_1)/F(P_3)$  denotes the transfer function of acoustic pressures.  $Z_1, Z_1', Z_3$ , and  $Z_3'$  can be measured by the pair-microphones technique, and  $H_{31}, H_{31}'$  are also measurable data. Then it is found that the transfer matrix  $\mathbf{T}^{(1,3)}$  can be obtained from the data measured at two different operative conditions, i.e., with or without an additional duct.

#### Source impedance

The sound insertion loss of an additional duct is expressed as follows.

$$IL = 10 \lg \frac{|Z_i' + Z_o| |T_{21}^{(1,3)}(i, r) Z_r' + T_{22}^{(1,3)}(i, r)|^2 \operatorname{Re}(Z_r')}{|Z_i + Z_o| |T_{21}^{(1,3)}(i, r) Z_r + T_{22}^{(1,3)}(i, r)|^2 \operatorname{Re}(Z_r)} \quad (4)$$

where  $\operatorname{Re}(\ )$  denotes the real part of the argument. This relation reveals that source impedance is one of the important factors which influence the noise reducing performance of silencers. On the other hand, the following relations are obtained because the exciting pressure  $P_o$  does not change at both work conditions, i.e., with or without an additional duct.

$$F(P_o) = \frac{F(P_i)}{Z_i} (Z_i + Z_o) = \frac{F(P_i')}{Z_i'} (Z_i' + Z_o) \quad (5)$$

Therefore,

$$Z_o = -\frac{(H_{oi} - H_{oi}')Z_iZ_i'}{H_{oi}Z_i' - H_{oi}'Z_i} \quad (6)$$

where  $H_{oi} = F(P_i)/F(P_{oo})$  and  $H_{oi}' = F(P_i')/F(P_{oo})$ .  $P_{oo}$  is an physical quantity which corresponds to the exciting pressure and does not change at any duct conditions. In the case of an acoustic driver, the input voltage may be measured as  $P_{oo}$ . Because  $Z_i, Z_i', H_{oi}$  and  $H_{oi}'$  can be measured, it is found that the source impedance  $Z_o$  can be obtained from the data measured at two different conditions, i.e., with or without an additional duct.

## EXPERIMENT

## Test equipments

Test equipments are shown in Fig.2. Test ducts were set in an anechoic room, and insertion losses were measured by Mic.⑤.

## Results

The transfer matrix of a simple duct with acoustic lining is shown in Fig.3. The measured results are well consistent with the plane wave theory. The source impedance of an acoustic driver is shown in Fig.4. The insertion loss of a horn calculated by using the measured data of source impedance matches well with the measured result of insertion loss as shown in Fig.5, compared with the result calculated on the assumption that the source impedance is infinite.

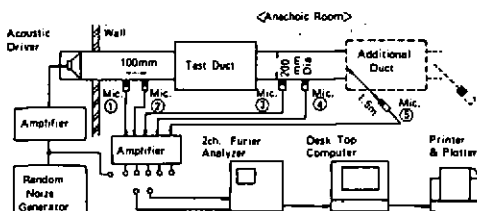


Fig.2 Test equipments

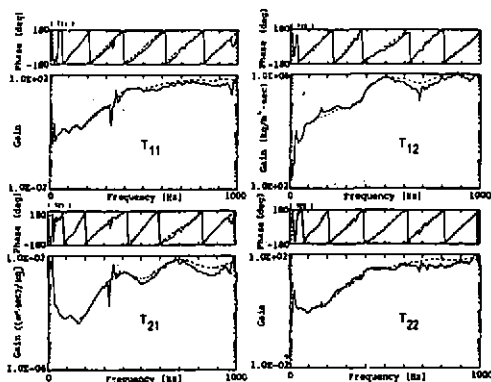


Fig.3 Transfer matrix of the duct with acoustic lining

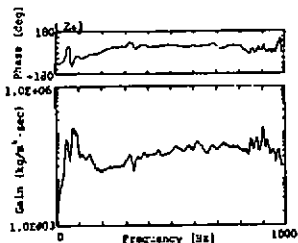


Fig.4 Measured source impedance of the acoustic driver

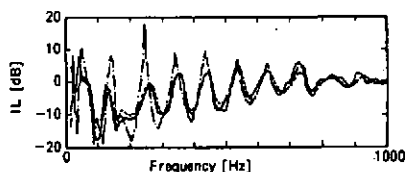
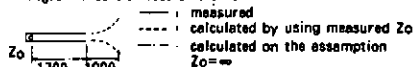


Fig.5 Insertion loss of the horn



The insertion loss of acoustic lined duct was calculated by using the measured data of transfer matrix and source impedance. The result coincides well with the measured insertion loss as shown in Fig.6. These results explain that transfer matrices and source impedances can be measured properly by the above mentioned method. This method was applied to a fan-duct-silencer system. The results is shown in Fig.7. In this case, the insertion loss is calculated based on such a parallel acoustic circuit model as shown in Fig.8, because both sides of the sound source are connected to ducts. The calculated insertion loss well corresponds to measured one, then this method is found to be useful.

#### CONCLUSION

By this research, a specific method was developed by which transfer matrices of duct elements and source impedances were measured. And acoustic characteristics of duct systems were found capable to be analyzed tout a fait for increased accuracy based on these measured data. This method was applied to a fan-duct-silencer system and successful results were obtained. It is expected that this method can be applied to not only air duct systems but also hydraulic piping systems.

#### REFERENCES

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- [2] D.A.Blaster and J.Y.Chung, Proceedings of Inter Noise '78, PP901-908 (1978)

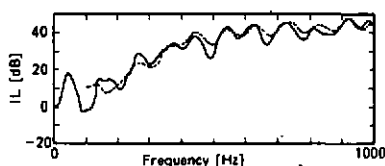


Fig. 6 Insertion loss of the lining duct

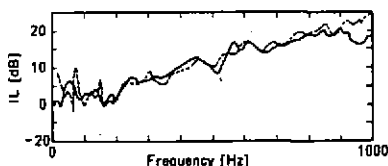
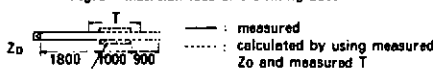


Fig. 7 Insertion loss of the silencer

—: measured  
---: calculated by using measured T and measured  $Z_r$  on the assumption  $Z_o = 0$

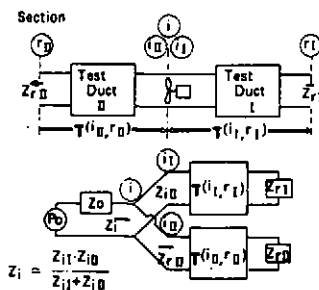
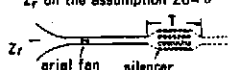


Fig. 8 Parallel acoustic circuit model