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HARMONIC VIBRATION OF A COLUMN STRUCTURE WITH TRANSVERSE BEAMS*

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Introduction

Structural vibration and transmission of waves in the vertical direction, from floor to floor, often give rise to low frequency vibration and noise problems in multi-storey buildings of lightweight construction (Figure 1). This paper discusses harmonic vibration and wave propagation characteristics at the lower audio frequencies, up to about 300 Hz, of a simplified model of a <u>single</u> vertical transmission path of a modular building structure. The model, which has a two-dimensional configuration similar to that of Figure 1a, consists of a tall uniform column loaded at regular intervals (L) with identical transverse beams. These represent in an idealized form parts of multi-modal floor structures.

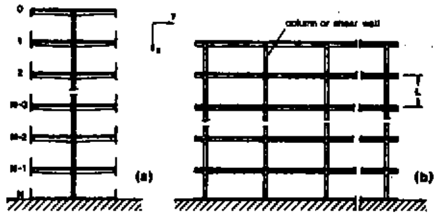

Fig. 1. Examples of idealized building structures.

The one-dimensional transmission path may be devided into a number of identical structural units or "periodic elements" joined together to form a so-called "periodic structure" (Figure 2a). The periodic element consists of the wave-carrying component, column (C) of length L, and of the load component, the transverse beam, which is devided into two "half-loads" (B) and (D) for convenience in analysis [1].

Outline of the theory

It is well-known that free harmonic wave propagation in an infinite, periodic structure is possible only in certain frequency bands known as "propagation zones", e.g. [2,3]. The frequency ranges in which wave propagation and associated transport of vibrational energy is <u>not</u> possible are known as "attenuation zones". These characteristics are described by a complex frequency-dependent quantity, the "propagation constant" $\mu=\mu_R+i\mu_I$, which relates the harmonic displacement vectors $\bar{q}e^{i\omega t}$ (and forces) at positions

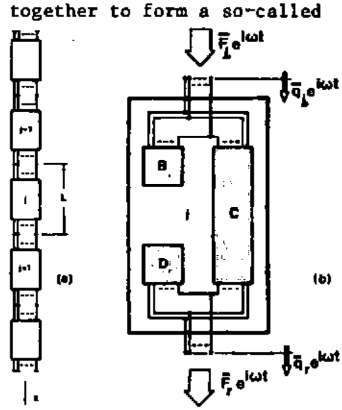

Fig. 2. (a) Block diagram of a periodic structure with multi-coupled elements; (b) a single element.

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(ξ) in adjacent elements

$$\bar{q}(\xi+L)e^{i\omega t} = e^{-\mu}\bar{q}(\xi)e^{i\omega t} \quad (1)$$

The variation with frequency of the real part (μ_R) and the imaginary part (μ_I) of the propagation constant evaluated for longitudinal (i.e. vertical) wave motion in the structure is shown in Figure 3.

The "attenuation constant" μ_R expresses the decay rate in wave amplitude per element and the "phase constant" μ_I, describes the phase change of the wave per element. From equation (1) it is clear that a propagation zone exists provided the propagation constant is purely imaginary, i.e. when $\mu_R=0$.

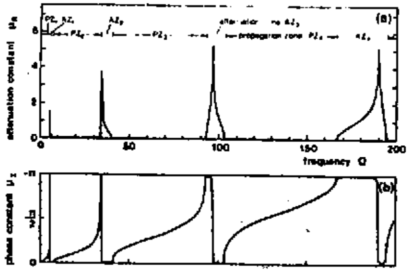

Fig. 3. (a) The real part μ_R and (b) the imaginary part μ_I of the propagation constant for longitudinal waves, $\eta = 0.001$.

The vibration characteristics of the element (Figure 2b) can be defined by its "receptance" functions. Transfer receptance $\alpha_{\ell r}$, for example, is the harmonic displacement $q_\ell e^{i\omega t}$ at end(ℓ) per unit harmonic force $F_r e^{i\omega t}$ at end(r). With n degrees of freedom at the ends the displacement vector $\bar{q}$ is thus given by the matrix products of the nxn receptance matrices $\bar{\bar{\alpha}}$ and the force vectors $\bar{F}$, i.e.

$$\begin{aligned} \bar{q}_\ell &= \bar{\bar{\alpha}}_{\ell\ell}\bar{F}_\ell + \bar{\bar{\alpha}}_{\ell r}\bar{F}_r \\ \bar{q}_r &= \bar{\bar{\alpha}}_{r\ell}\bar{F}_\ell + \bar{\bar{\alpha}}_{rr}\bar{F}_r \end{aligned} \quad (2)$$

Continuity of coordinates, equilibrium of force and the periodic property (equation (1)) can be applied to relate $\bar{q}_\ell$ and $\bar{q}_r$ as well as the forces, which yield

$$\bar{q}_r = e^{-\mu}\bar{q}_\ell \quad \text{and} \quad \bar{F}_r = -e^{-\mu}\bar{F}_\ell \quad (3)$$

From equations (2) and (3) a general system equation can be derived, [3]

$$\left[\bar{\bar{\alpha}}_{\ell\ell}+\bar{\bar{\alpha}}_{rr}-(\bar{\bar{\alpha}}_{\ell r}+\bar{\bar{\alpha}}_{r\ell})\cosh(\mu) - (\bar{\bar{\alpha}}_{\ell r}-\bar{\bar{\alpha}}_{r\ell})\sinh(\mu)\right]\bar{F}_\ell = \left[f(\mu,\bar{\bar{\alpha}})\right]\bar{F}_\ell = 0 \,. \quad (4)$$

The solutions of this are, at any frequency, given by the particular values of μ which satisfy the equation $|f(\mu,\alpha)| = 0$, from which n pairs ($\pm\mu_n$) of propagation constants can be found.

Discussion of numerical results

Free harmonic longitudinal waves can propagate (i.e. without attenuation) in the major part of the entire frequency range considered. The corresponding prop. zones (PZ_i) are individually succeeded by attenuation zones of a distinct "resonant" appearance. The high attenuation in these relatively narrow zones is created by the modal characteristic of the transverse beams vibrating in their symmetric, midpoint-fixed modes. At these frequencies the midpoints are therefore effectively "locked" and the wave motion is suppressed within a distance of one or two elements from the source. Free flexural waves can be shown to propagate only in relatively narrow zones [1,4]. However, with realistic values of damping, corresponding to loss factors η of the order 10^{-2}, these "propagating" flexural waves are found to be significantly attenuated (with about 5 dB/element). The non-dimensional frequency Ω is defined with reference to the column, $\Omega = \omega(S\rho/EI)^{\frac{1}{2}}L^2$. For typical dimensions used in concrete buildings a value of $\Omega = 200$ corresponds approximately to 300 Hz.

Response of a finite periodic structure

The infinite-periodic-structure assumption is well justified in the analysis of lightly damped structures with a large number of elements N. When $N\mu_R$ becomes less than approximately 1.5, however, reflections from the ends of the structure begins to influence the wave motion, thereby creating finite system vibration. Consider a structure (Figure 4) with a termination of receptance α_B. The vibration of the system expressed as the junction receptance α_{mj} ($=\alpha_{jm}$) is for $m \geq j$ given by, [1],

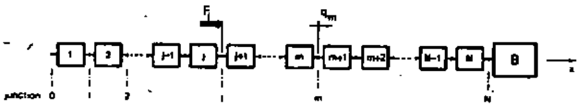

Fig. 4. Block diagram of a periodic structure with arbitrary termination B.

$$\alpha_{mj}=q_m/F_j=\alpha_w\cosh(j\mu)[\alpha_B\cosh((N-m)\mu)+\alpha_w\sinh((N-m)\mu)]/[\alpha_B\sinh(N\mu)+\alpha_w\cosh(N\mu)] \quad (5)$$

where $\alpha_w=\alpha_0\sinh(\mu)$ is the "characteristic wave receptance" introduced by Mead, e.g. [3]. When the freely propagating waves are reflected sufficiently at the ends and the "overall phase change" conditions are satisfied, finite system resonances are built up. This is illustrated in Figure 5 by the spectrum of the forced longitudinal vibration $\alpha_{oo}=q_o/F_o$ of an 8-element structure terminated by a semi-infinite uniform column, or an equivalent infinite plate of the same input receptance. The number of longitudinal modes "within" a propagation zone corresponds to the number of periodic elements (8). Modes occuring well within the propagation zones are highly damped due to the dissipation at the termination. In the second propagation zone the modes can hardly be identified because the receptance of the termination is nearly equal to the characteristic wave receptance, i.e., the input receptance of a semi-infinite periodic structure (dotted line).

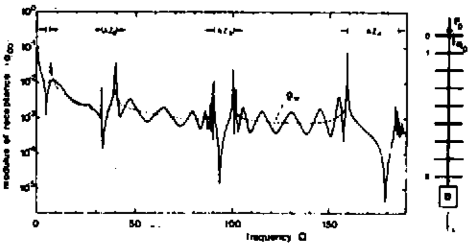

Fig. 5. Modulus of junction receptance α_{oo}. Semi-infinite column termination. η=0.001.

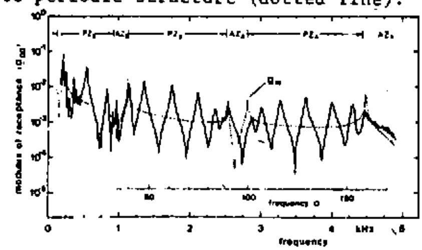

Fig. 6. $|\alpha_{oo}|$ determined experimentally. Short column termination embedded in sand. η(beams) $\sim$ 0.02.

Experimental results

Results from experiments on a nominally identical periodic structure, but with a more reflective termination, show a close overall correspondence with α_w (Figure

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6) . The "end modes" in a group are very sensitive to damping [5]. By adding damping to the beams (obtaining $\eta \sim 0.02$) the response of the first mode in group three, for example, was reduced by 35 dB, whereas the modes 2, 3 and 4 were reduced by only 3 dB. This underlines the strong receptance miss-match between α_w and α_B near the bounds of a propagation and attenuation zone, already apparent from Figure 5. Mode shapes of the flexurally vibrating beams are shown in Figure 7 for one of the longitudinal modes, i.e. a mode with purely longitudinal displacements in the column. The longitudinal displacement mode shapes for all 8 modes in group four are shown in Figure 8. Very similar results are found for band two and three, showing a pronounced decrease in the longitudinal phase velocity as the frequency is increased from mode 1 to 8. The small changes in the mode shape of one of the beams as a function of mode number are illustrated in Figure 9.

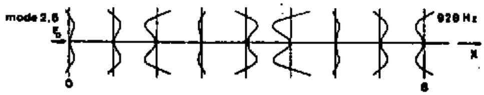

Fig. 7. Junction mode shapes of longitudinal modes.

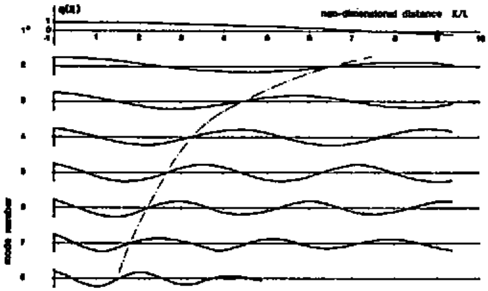

Fig. 8. Mode shapes 1 to 8 in band four, longitudinal displacement of the column. *No added damping.

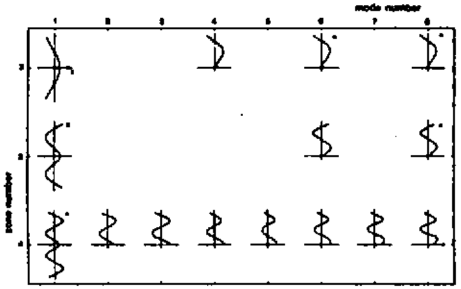

Fig. 9. Mode shapes of the transverse beam at junction two for longitudinal modes in zone 2 to 4. *No added damping.

Conclusion

The modal characteristic of the transverse beam components in the periodic structure has been shown to have a strong influence on the wave motions. Narrow, resonant attenuation zones for longitudinal waves have only a small influence on a broad band attenuation. Their presence, however, is not to be neglected because they define individual propagation zones. The implication of these was demonstrated for a finite system, showing the existence of a large number of longitudinal resonances, controlled mainly by the transverse beams, and occuring at frequencies much below the fundamental frequency $\Omega = 272$ of a column of length L.

References

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