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GEAR MESH RESONANCE AND STIFFNESS VALIDATED IN CALCULATION AND EXPERIMENT

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The design of gearboxes is driven not only by requirements of load carrying capacity but also by the aim of low noise behaviour. Both targets are dependent on an exact estimation of the main gear mesh resonance frequency already at design state. Together with the gear mesh excitation the dynamic overload and the noise emission may be taken into account in the gear mesh layout. The mesh stiffness is of central influence on these design decisions. Various approaches exist to calculate and simulate the resulting stiffness in the mesh of cylindrical gears. These theoretical approaches range from standard methods to complex numerical algorithms. In this paper the background of some of these methods is shown. The derivation of current standard methods is documented. To support the decision, which method of what complexity has to be selected to design adequate gear behaviour, experimental results are shown to support the validity of the design.

Keywords: Gear Noise, Mesh stiffness, resonance frequency

1. Introduction

Gearbox noise emissions are related to the gear meshes in many cases. Especially the power transmitting gear stages may be the main source of noise that the engineer can influence during design. A usual design approach would be to define an advantageous main gear geometry and to design appropriate gear microgeometry to optimize the noise behaviour (see [7], [9] and [11]). The microgeometry is the result of the grinding process of the tooth flanks and can be modified in late design stages, since it is the last manufacturing step. Results of noise emission measurements of the final gearbox can be easily considered by improving the microgeometry. A parameter that is determined already in early design phase and cannot be influenced later is the main resonance of the gear mesh. An adequate estimation of the main resonance frequency on the basis of general data that is available early on is necessary [12].

2. Gearbox noise sources

Gearbox noise excitation related to the gear mesh is determined by the following influence parameters:

- Changing mesh stiffness during engagement
- Deviations of the flank geometry from the nominal shape
- Deformations of the Teeth in the mesh
- Displacements of the gears relative to each other
- Surface conditions of the meshing flanks
- Friction influences

The Mesh excites oscillations of the gearbox structure that propagate to the housing and are transferred to airborne noise. The resonance behaviour of the structure plays an important role in the propagation of these excitations. The most critical influence is the resonance of the gear pair itself.

Any excitation of this resonance frequency by the gear mesh has to be prevented to ensure low noise behaviour and low dynamic overloads.

2.1 Geometric properties of the gear mesh

Considering involute teeth, the acting tooth force is oriented tangentially to the base circle of both gears. The rules of engagement theoretically predict a smooth change between the tooth pairs rolling through contact. Gear geometry determines the profile contact ratio which is the mean number of teeth that is in contact over time, for helical gears the overlap ratio has a similar meaning in direction of the tooth width. Since the loaded gear contact is governed by deformations of the tooth pairs in contact, the changing number of teeth in the mesh results in changing mesh stiffness.

2.2 Parametric mesh excitation

Many practical gear designs for spur gears feature contact ratios between 1 and 2, which means alternating single and double tooth contact while rolling through the mesh. The acting mesh stiffness c_s results from the superposition of the tooth pair stiffness of the tooth pairs in contact (Fig. 1). The mesh stiffness of a spur gear mesh shows significant variations of time. To have a reference, the mean value c_γ is defined.

More common in practical applications are helical gears. An advantage is in the first place a higher contact ratio, that means usually two or more tooth pairs in contact at the same time. This results in a lower variation of mesh stiffness around c_γ than for spur gears.

The changing stiffness of the gear contact is acknowledged as main source of gear mesh excitation. Generally, the variation in stiffness leads to a variation in transmission error and in the acting force. Excitation is not limited to the engagement frequency of the teeth but also consists of higher harmonic frequencies in most cases.

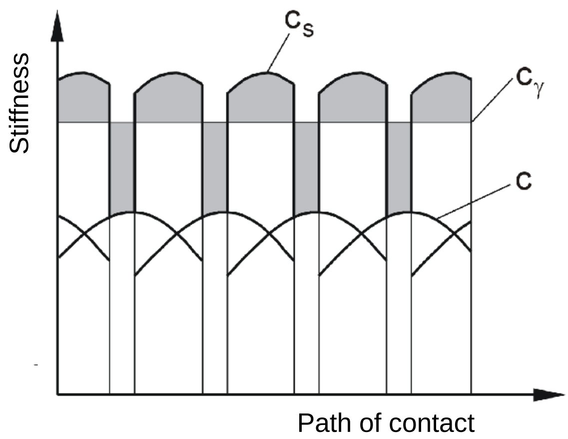

Figure 1: Tooth stiffness c_s, tooth pair stiffness c, mean meshing stiffness c_γ [7]

2.3 Gear main Resonance

To determine the resonance frequencies of the gears, shafts and further elements in the gearbox multi body models may be established and analysed. In the early design stage, a full analysis is not yet possible since most data is still to be defined. A simple model is proposed to estimate the frequency that leads to oscillation of the gears against each other. That resonance frequency is seen as main resonance of the gear stage. It is critical not only in respect of noise but also in respect of load carrying capacity. An excitation in the same frequency by the tooth engagement or its harmonics shall be recognized in design phase and has to be prevented by changing the main gear geometry.

3. Calculation of mesh stiffness

3.1 Method of calculation

Several different approaches to the calculation of gear mesh stiffness exist. The calculation methods are mainly used to achieve a reasonable value for the mean stiffness to determine the effects of any misalignment of the tooth flanks and the influence of gear resonance concerning load carrying capacity. In the following section calculation methods for the mesh stiffness will be presented.

3.2 Resonance frequency, mesh stiffness and dynamic overload

In AGMA 2001 [2], a standard by the American Gear Manufacturers Association, a dynamic overload factor is determined according to gear quality, no indication is given concerning gear resonance.

For the estimation of the gear mesh resonance a simple approach is the formulation as a single mass oscillator. The mass is determined by the moments of inertia of the gears, the coupling stiffness between the gears is given by the mean meshing stiffness of the stage. This simple model describes the critical resonance frequency between pinion and gear. The oscillation around the static nominal position is governed by the meshing stiffness under working conditions. The main resonance for a cylindrical gear stage according to this model is estimated with Eq. 1 acc. to ISO 6336 [5,6], a standard by the International Standardization Organisation.

$$N = \frac{n_1}{n_{E1}} = \frac{n_1 \pi z_1}{30\,000} \sqrt{\frac{m_{red}}{c_{\gamma\alpha}}} \quad (1)$$

n_1 rotational speed of pinion in the gear stage
n_{E1} resonance speed of stage
z_1 number of teeth on pinion
m_{red} reduced mass of gear stage
$c_{\gamma a}$ Mesh stiffness to calculate the resonance ratio N= n_1/n_{E1}

In the standard calculation the result is used to determine a dynamic overload factor that is introduced into the load carrying capacity calculation. The method is not to be applied in a region between N = 0,85 and N = 1,15. The impact of the dynamic factor in the standard methods for flank load capacity and for tooth root capacity [5] and [6] are shown in Eq. 2 and Eq. 3:

For tooth flank capacity (pitting)

$$\sigma_{H1,2} = Z_{B,D} \cdot \sigma_{H0} \cdot \sqrt{K_A \cdot K_v \cdot K_{H\beta} \cdot K_{H\alpha}} \quad (2)$$

$\sigma_{H1,2}$ contact pressure
$Z_{B,D}$ single pair tooth contact factor pinion/gear
σ_{H0} nominal contact stress at pitch point
K_A Application factor
K_v Dynamic factor
$K_{H\beta}$ Face load factor for contact stress
$K_{H\alpha}$ Transverse load factor for contact stress

For tooth root capacity (tooth root breakage)

$$\sigma_{F1,2} = \sigma_{F0} \cdot K_A \cdot K_v \cdot K_{F\beta} \cdot K_{F\alpha} \quad (3)$$

$\sigma_{H1,2}$ tooth root stress
σ_{F0} nominal tooth root stress
$K_{F\beta}$ Face load factor for tooth root stress ($K_{F\beta} = f(K_{H\beta}, b/h)$)
$K_{F\alpha}$ Transverse load factor for tooth root stress ($K_{Fa} = K_{Ha}$)

No further details on noise behaviour are given by ISO 6336. To get reliable results for the resonance frequency, a valid value for the mesh stiffness has to be introduced in Eq. 1.

Standard calculations offer several solutions for the mesh stiffness. In ISO 6336 a simple method was used in recent issues [5] (method C) that proposed a rough value of 20 N/(mm μm) for the mean mesh stiffness. That method was not transferred to newer issues [6].

AGMA 927-A01 [1] defines a load distribution factor and uses a mesh stiffness of $C_{\gamma m}$ of 11 N/(mm μm). The standard does not provide an approach to use that value in any estimate for the resonance speed.

ISO 6336 defines a more elaborate calculation method that will be covered below. All methods mentioned have in common that the underlying idea is oriented at a spur gear mesh. For helical gear meshes ISO 6336 defines a theoretically similar spur gear mesh for the calculations of mesh stiffness.

3.3 Mechanical approach to mesh stiffness

Loading elastic material results in deformations. Dividing load and displacement yields a stiffness value at the location of the acting load. This is also valid for the gear mesh stiffness.

Weber and Banaschek [15] provide a method to calculate the load dependent deformation of a spur gear stage. They divide the tooth deformation in three parts: contact deformation Eq. 4, tooth deformation Eq. 5 and deformation of the surrounding area of the gear body Eq. 6.

$$w_{Hertz} = \frac{2 \cdot W_0 \cdot (1-\nu^2)}{\pi \cdot E} \cdot \left(ln \frac{4 \cdot s_{dn1} \cdot s_{dn2}}{b_H^2} - \frac{\nu}{1-\nu} \right) \tag{4}$$

$$w_{Zahn} = \frac{W_0}{E} \cdot cos^2\alpha' \left[10{,}92 \cdot \int_0^{h_P} \frac{(h_p - y)^2}{2 s_n(y)^3} dy + 3{,}1 \cdot (1 + 0{,}294 tan^2\alpha') \cdot \int_0^{h_P} \frac{dy}{sn(y)} \right] \tag{5}$$

$$w_{einspg} = \frac{W_0}{E} \cdot cos^2\alpha' \left[5{,}2 \frac{h_P^2}{s_{FW\,B}^2} + 1{,}0 \frac{h_P}{s_{FW\,B}} + 1{,}4(1 + 0{,}294 tan^2\alpha') \right] \tag{6}$$

w_{Hertz}	contact deformation of tooth pair
w_{Zahn}	Tooth deformation of a single tooth
w_{einspg}	deformation of surrounding parts of gear body
W_0	acting load per width in the mesh
E	Young's modulus of material
ν	Poisson's ratio
s_{dni}	distance contact point – tooth centerline i=1,2 (pinion, gear)
b_H	half of Hertzian contact width
α'	pressure angle of acting load
h_P	tooth tip height
s_n	tooth tangent length of tooth at height y
$s_{FW\,B}$	tooth tangent length at the tooth root

With these formulas the deformation of a gear pair in the mesh can be determined by summing up the influences. More elaborate methods are documented in literature which are based on this approach but may consider additional influences like special properties of helical gears in more detail [3].

Different methods that are not based on these formulas are available as well [4], [10] which may be used with certain experience in their application.

3.4 Calculation method for mesh stiffness derived from the mechanical method

In ISO 6336 the mesh stiffness is calculated by the following equations:

$$c' = c'_{\mathrm{th}}\, C_{\mathrm{M}}\, C_{\mathrm{R}}\, C_{\mathrm{B}} \cos\beta \tag{7}$$

$$c'_{\mathrm{th}} = \frac{1}{q'} \tag{8}$$

$$q' = C_1 + \frac{C_2}{z_{\mathrm{n1}}} + \frac{C_3}{z_{\mathrm{n2}}} + C_4\, x_1 + \frac{C_5\, x_1}{z_{\mathrm{n1}}} + C_6\, x_2 + \frac{C_7\, x_2}{z_{\mathrm{n2}}} + C_8\, x_1^2 + C_9\, x_2^2 \tag{9}$$

With the constants acc. to table 1

Table 1: Constants for Eq. 9

C1	C2	C3	C4	C5	C6	C7	C8	C9
0,04723	0,15551	0,25791	–0,00635	–0,11654	–0,00193	–0,24188	0,00529	0,00182

These equations are based on a series expansion that Schäfer [13] proposed for results calculated with the method of Weber/Banaschek [15]. Schäfer calculated the stiffness of various gear geometries under the load condition of F/b = 294 N/mm, a tool tip radius of the generating cutter of ρ_{a0} = 0,2· m_n and a tool tip height of h_{a0} = 1,2 · $m_{n.}$ He compared the results of his series expansion with the results by Weber/Banaschek for loads ranging between 94 N/mm < F/b < 980 N/mm and reached compliance within 8 %.

In ISO 6336 further factors are introduced that are based on additional analysis:

Factor $C_{\mathrm{M}} = 0{,}8$ accounts for differences between results according to Weber/Banaschek and measurement results later collected .

The factor $C_{\mathrm{R}} = 1{,}0$ accounts for different gear body designs that are not fully cylindric. These may reduce the mesh stiffness. The shape factor

$$C_{\mathrm{B}} = [1{,}0 + 0{,}5\left(1{,}2 - h_{\mathrm{fP}}/m_{\mathrm{n}}\right)]\,[1{,}0 - 0{,}02\left(20^\circ - \alpha_{\mathrm{Pn}}\right)] \tag{10}$$

is determined for both gears and then averaged ($C_{\mathrm{B}} = 0{,}5\,(C_{\mathrm{B1}} + C_{\mathrm{B2}})$). The factor accounts for different tooth shapes.

3.5 Conclusion for calculation method to be validated

The previous sections presented the calculation method for mesh stiffness according to ISO 6336 and some mechanical background on how the method is derived. To transfer the equations into a format that is compatible with the requirements of the standard ISO 6336 a series expansion has been made that approximates the original set of formulas. In the following section the gear mesh resonance is used to validate a calculation of the resonance speed that uses the ISO 6336 mesh stiffness versus measurement results.

4. Experimental validation

The application of the formulas to determine the resonance frequency of a gear stage shall be validated by experimental data. Measurement of the circumferential acceleration of the gears are used as reference. In Fig. 2 an overview over the test gears and their transverse tooth profiles are shown. Table 2 shows the gear geometry of the test gears. The profile shape of both gear meshes are the same, Gear mesh 1 is designed as spur gear stage, Gear mesh 2 is designed as helical mesh.

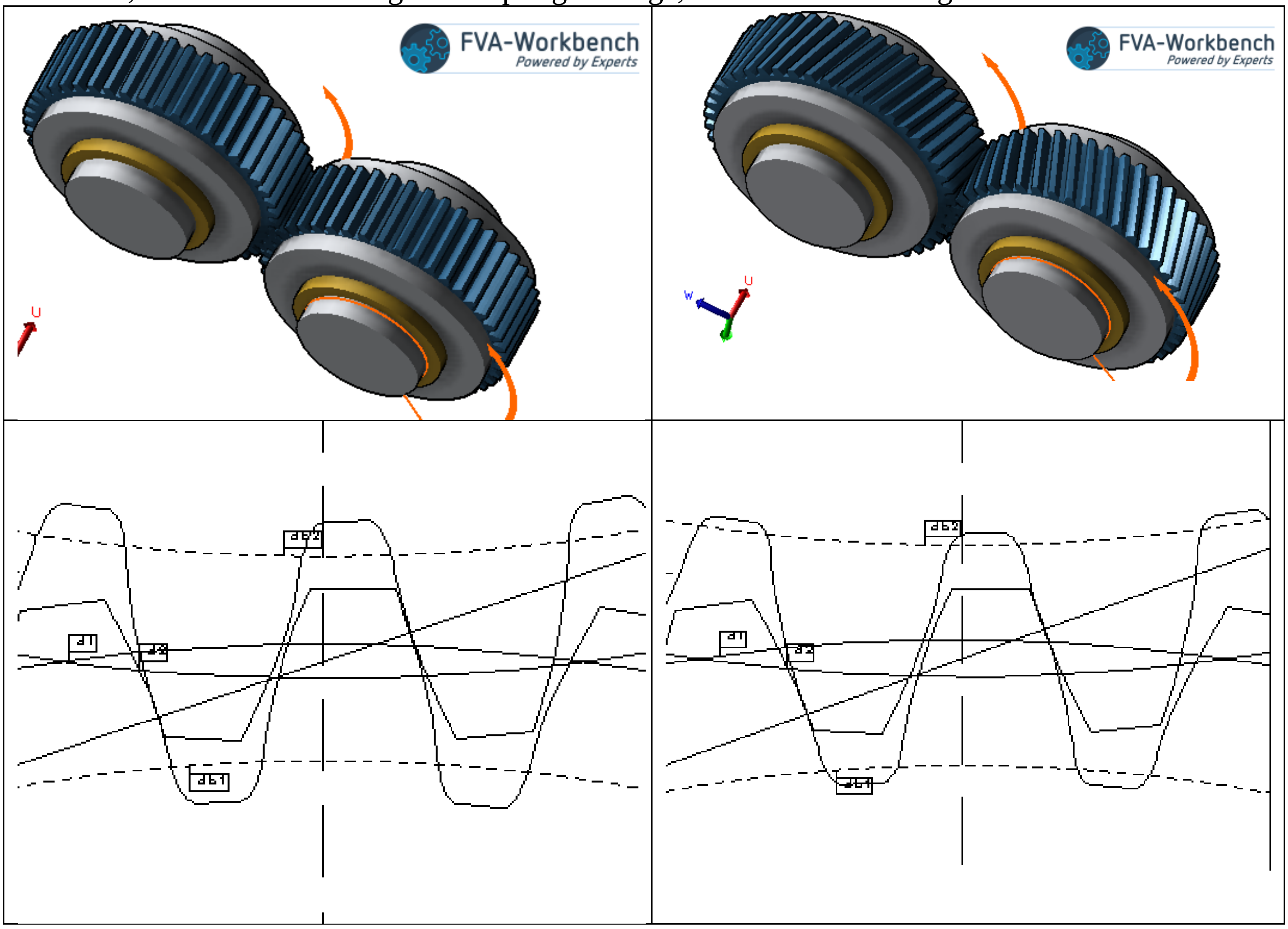

Figure 2: Overview over the gears used for validation, 3D-view and gear mesh transverse profile

Table 2: Main gear geometry and load used for validation

			Gear mesh 1		Gear mesh 2	
			Pinion	**Gear**	**Pinion**	**Gear**
Number of teeth	z	-	43	45	43	45
Normal module	m_n	mm	3,21		3,0	
Normal pressure angle	α_n	°	20		20	
Helical angle	β	°	0	0	21	-21
Profile shift coefficient	x	-	-0,1747	-0,1984	-0,21	-0,2382
Center distance	a	mm	140		140	
Profile contact ratio	ε_α	-	1,53		1,5	
Overlap ratio	ε_β	-	-		1,5	
Torque moment	T	Nm	1000		1000	

Fig. 3 shows the measurement results in circumferential direction directly at the gear body of gear mesh 1. The frequency axis is divided by the pinion rotation frequency which results in order dimension [14]. The main resonance of the gear mesh can be located at about 4880 U/min, recognizable by the red area that indicates high acceleration levels. The calculation according to the ISO 6336 method yields a resonance speed of 5030 U/min. Measurement and calculation results differ by only 4 %,

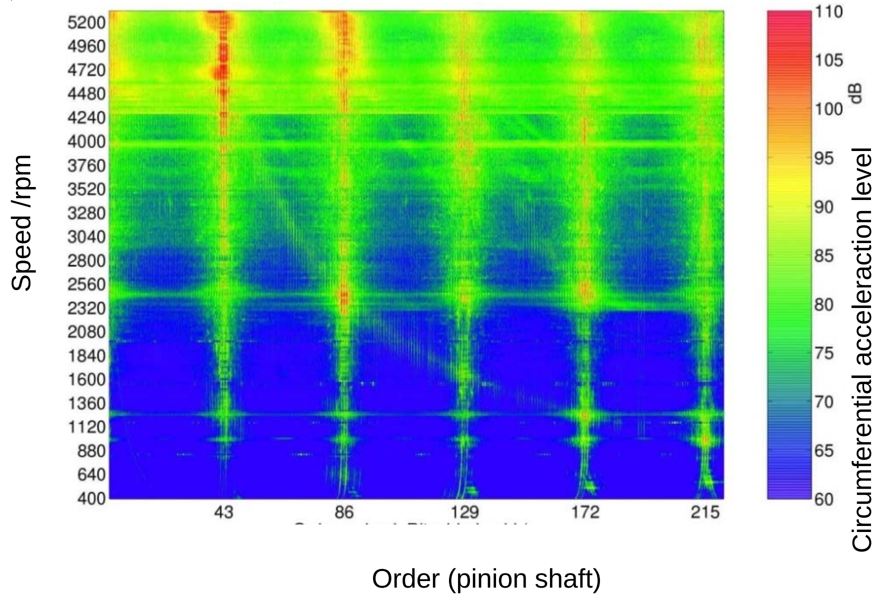

Figure 3: Measured circumferential acceleration level for gear mesh 1 (spur gear mesh)

Fig. 4 shows the measurement result of an acceleration measurement in circumferential direction directly at the gear body of the helical gear mesh 2 [8]. Gear main resonance may be located at about 4400 U/min, which may be recognized at the red area of high acceleration levels. Also the second order at about 2200 U/min meets the resonance frequency and yields high levels. A calculation according to ISO 6336 method above results in a resonance speed of 4820 rpm. The calculated resonance speed and the resonance speed determined by measurement differ only about 10%, which is a quite reasonable agreement.

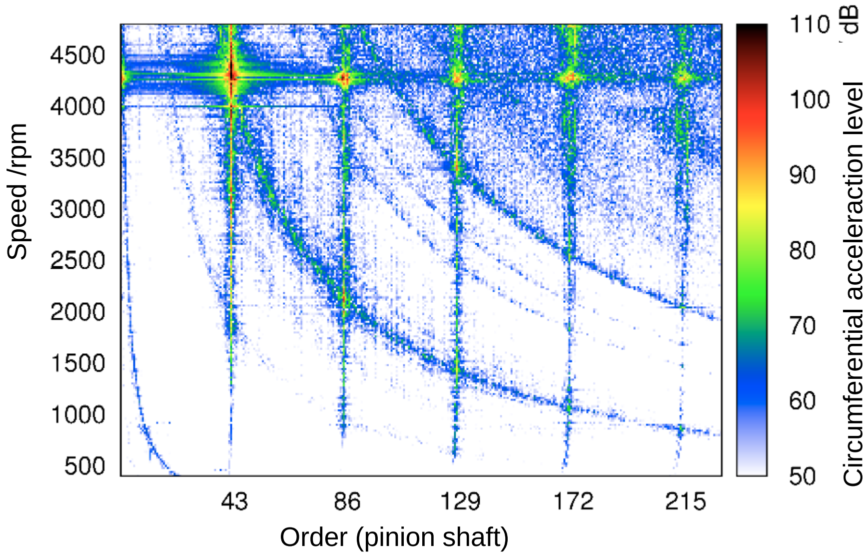

Figure 4: Measured circumferential acceleration level for gear mesh 2 (helical gear mesh)

Comparison of both measurements and the related calculation shows that the resonance speed for the helical mesh is estimated about 10% higher by ISO 6336-2006 than measured. The calculation result meets the measurement not as well as for the spur gear stage but it is still valid for early design purposes.

Using a more complex algorithm (e.g. like mentioned in [3]) to determine the mesh stiffness may yield values that differ from the results acquired with ISO 6336. To ensure valid results in this case also the approach to determine the inertia of the gears has to be reconsidered. Fig. 2 shows that the gear body is wider than the gear teeth. This has to be considered in the used value for the reduced mass when using a more detailed method to determine the mesh stiffness.

5. Conclusion

The main resonance frequency of a gear pair is determined by main geometry parameters that are fixed early in the design process. The Coincidence of excitation frequencies of the gear mesh with the main resonance frequency has to be prevented. That makes an early and reasonable estimation of the gear main resonance frequency necessary. The formulas of ISO 6336 provide an approach for such an estimated value. Comparison of the calculation results and measurement results for a spur gear mesh and a helical gear mesh show reasonable agreement in the area of about 10 %.

The Overall results indicate that the standard calculation from ISO 6336 is well suited for an early estimate of the main gear resonance frequency.

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