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NUMERICAL MODELLING OF UNDERWATER ACOUSTIC PROPAGATION

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1. INTRODUCTION

It is unusual for sound to travel underwater in such a way as to produce a simple intensity versus range-from-source relation. It is more usual for areas of high and low intensity to be formed.

The accurate prediction of sound level with position in the ocean is very important for the detection and ranging of vessels by acoustic means.

Modelling underwater sound prediction is a complicated task and there are a variety of methods available. Factors which complicate the situation are:

a) Sound speed in sea water varies with depth, temperature and salinity. In turn, temperature and salinity can vary significantly with range and depth in the ocean. Thus, sound travelling underwater is refracted into complicated paths.

b) Interactions between sound in the ocean and the sea surface and sea bed are far from simple, and the mechanisms involved are not totally understood.

c) Successful comparisons between measured and modelled acoustic data are difficult to achieve because, generally, only sparse environmental information is available.

Underwater acoustic propagation can be predicted using empirical models, simple analytical models or by using numerical methods to solve reduced wave equations. These simplified wave equations are produced by making assumptions about the nature of the ocean or the nature of the sound source.

Each assumption made by a particular approach reduces the generality of the resulting wave equation, and restricts the range of problems to which the approach can be applied with validity. For instance, a common simplification made is to assume that the ocean is only horizontally stratified, i.e. the sound speed varies with depth but not with range, and the water depth is constant. This allows the use of a separation of variables technique, which greatly simplifies the situation, but which means that the approach cannot reasonably be used to predict propagation in areas containing ocean fronts and seamounts.

The four approaches to acoustic propagation loss modelling described below have two simplifying assumptions in common; namely,

i) A sound source of constant frequency is assumed. This reduces the wave equation to the Helmholtz Equation.

ii) Cylindrical symmetry is assumed to be present in the ocean, and consequently in the acoustic field. This reduces the three-dimensional problem to a more soluble two-dimensional problem.

2. RAY ACOUSTICS

Ray acoustics uses the geometrical acoustics approximation to produce a simplified equation. This approximation says that nothing in the ocean changes significantly over the distance of one wavelength. Strictly, this requires that the sound source be of infinite frequency, i.e. zero wavelength. Practically, the approach can be used for a range of frequencies of interest to ocean acousticians, but at lower frequencies it is not valid.

Ray acoustics in its simplest form does not allow for diffraction, and so, as the source frequency decreases and diffraction becomes more widespread, the results of ray-based models become less believable.

Propagation loss is calculated in ray models by assuming that the acoustic energy contained between two rays at the source remains trapped between then. Thus, the intensity at a given range due to a given pair of rays is inversely proportional to the area enclosed by the rays.

A problem with this method is that sound rays may cross, as the sound is refracted in the ocean. At a crossing point, the simple formula for intensity then predicts an infinite result, as the rays enclose zero area. Such a point is known as a caustic.

Some models using ray theory avoid predicting infinite intensity by noting the location of caustics and using some form of wave theory to predict the intensity at and around the caustic.

Ray theory is a very popular approach to acoustic propagation loss modelling because it gives a good intuitive feel for the paths taken by sound. Many modelling techniques exist which can be classified as ray theory techniques but which are quite different from each other. For instance, some models represent the ocean as being divided into sectors within which the sound speed is constant. The physics of the problem is then reduced to a repeated application of Snell's Law to model the refraction of the rays as they cross sector boundaries. An equally popular approach is to allow sound speed to vary linearly with depth. In such a situation, it can be shown that ray paths form circular arcs.

These different ray-tracing techniques have their own advantages and drawbacks, and thus two different models, both using ray theory, can give signficantly different answers when applied to the same situation.

Other areas in which different ray-based models use differing approaches include the way corrections are made to model diffraction effects, to calculate intensities at caustics, and to model sound reflection from the sea bed and the sea surface.

3. PARABOLIC EQUATION APPROACH

The parabolic equation is an approximation to the wave equation. Its use imposes a restriction on the nature of the sound which can be modelled. The equation can be solved by methods that can be easily implemented on a computer.

To derive the parabolic equation, the acoustic field, U(r,z) is firstly assumed to be of the form

$$U(r,z) = V(r,z) \times S(r).$$

S(r) contains all of the rapid range variation of U(r,z) and V(r,z) varies more slowly with range.

When this form of the field is substituted into the Helmholtz equation with no source term, two equations are produced. One equation involves S(r) and r, and the other involves S(r), V(r,z), r and z. The equation involving S(r) and r can be solved analytically, to give S(r) [S is the zero order Hankel function] and this may be substituted into the other equation to give an equation in V(r,z), r and z.

To solve this equation, the following assumptions are made:

i) S(r) is expressed as a far-field approximation to its true form. This means that the field will be found at distances greater than a few wavelengths.

ii) The paraxial approximation is applied. This is equivalent to saying that only the sound emitted in a narrow angular beam about the horizontal is considered important.

These assumptions give the standard parabolic equation:

$$\frac{\partial^2 V}{\partial z^2} + 2iK_o\frac{\partial V}{\partial r} + K_o^2(n^2-1)V = 0$$

where Ko is a reference number and n is the refractive index.

The parabolic equation can be solved numerically by splitting the ocean environment up into steps within which n does not vary. This method does not produce significant errors is n is small enough. The numerical solution can be performed using either a Fast Fourier Transform technique or by a finite difference technique.

The Fourier Transform approach involves Fourier transforming in the depth variable to give an equation involving V(r,s), where s is a unit of reciprocal space. This equation can be solved if the starting field, V(r,s) is known. The solution to the equation can then be inverse Fourier transformed to give the field as a function of depth. The Fourier and inverse Fourier transforms can be implemented on a computer using the Fast Fourier Transform technique.

Finite difference techniques are numerical methods of solving differential equations. They extend the field as a function of depth by a small range step, given the field as a function of depth at the start of the step. The boundary conditions at the sea bed and the sea surface must be given.

Thus, the parabolic equation method can be used to calculate the acoustic field as a function of range and depth, given the starting field at the original range.

The main advantages of the parabolic equation method are:

a) It handles environments where sound speed varies with range and depth.

b) It is a wave-based method and hence predicts diffractive effects.

The main disadvantages of the parabolic equation method are:

a) It is less practical for higher frequencies than for lower. This is because run times of computer implementations tend to rise as the square of the frequency.

b) The derivation of the equation limits the solution to sound emitted at small angles from the horizontal.

4. FAST FIELD THEORY

The Fast Field Theory approach can be applied to environments where sound speed varies with depth but not with range. The ocean is modelled as being made up of a series of horizontal layers, each having a certain density.

The wave equation involving the acoustic field as a function of depth, range and time is Fourier transformed to give an equation involving acoustic field as a function of depth, range and frequency. The equation is then Hankel transformed to give an equation involving the acoustic field as a function of depth, horizontal wavenumber and frequency. This equation is known as the depth-separated wave equation.

The above procedure is performed on the wave equation with and without a source term. The field in each layer is then expressed as a sum of the equation with the source term and a linear combination of two equations without the source term, one representing an upward-going wave and the other representing a downward-going wave. The equation can be solved analytically if the source strength and the two combination coefficients are known.

Thus, the field in each layer is given as a sum of three terms with two unknown coefficients (from the linear combination of the two source-free equations). The field can thus be found if the boundary conditions at the surface and the sea bed are known.

The acoustic field as a function of range, depth and time can be calculated by performing inverse transforms on this solution. These transforms can be performed on a computer using the Fast Fourier Transform technique.

The solutions given by this approach would be exact solutions to the wave equation if no errors were incurred in the numerical implementations of the inverse tranforms. These errors can be reduced by a careful, experienced user but the process is complicated and requires considerable skill.

The advantage of using the Fast Field approach is that the only differences between the solutions it produces and exact solutions to the wave equation in the layered environment arise from the numerical implementations of the Fourier and Hankel transforms.

The disadvantages of the approach are:

a) It cannot be used for environments where the sound speed varies with both range and depth.

b) The procedure is not easily automated on a computer, and requires a knowledgable, experienced user.

5. NORMAL MODE THEORY

In its most simple form, normal mode theory is used to predict propagation through environments where sound speed varies only with depth. The approach can be extended to environments in which sound speed varies also with range.

Firstly, a separable form of the field is assumed:

$$U(r,z) = A(r) \times V(z).$$

This assumption yields a depth dependent wave equation and a range dependent wave equation. The depth dependent equation can be solved analytically if sound speed is modelled as varying in certain ways with depth. For instance, if the water column is split up into layers of constant sound speed, the depth equation solution is a sum of sine and cosine functions.

If the boundary conditions at the sea surface and sea bed are known, the depth dependent equation can be solved to give a series of vertical wavenumbers for which solutions exist. These wavenumbers, in turn, give a series of corresponding horizontal wavenumbers.

Thus, the field can be given by

$$U(r,z) = \sum_n An(r)Vn(z)$$

where Vn(z) is the nth function that satisfies the depth dependent wave equation and An(r) is the corresponding solution to the range-dependent equation.

If this form of the field is substituted into the Helmholtz equation, a differential equation linking An(r), Vn(z) and r is obtained. At this stage, use is made of the orthogonality of the depth dependent functions. This property can be mathematically expressed as

$$\int_0^\infty Vn(z)\ Vm(z)\ \rho(z)\ dz = 0 \qquad m \neq n$$

$$\int_0^\infty Vn(z)\ Vm(z)\ \rho(z)\ dz = 1 \qquad m = n$$

where ρ (z) is the density.

If the equation linking An(r), Vn(z), z and r is multiplied by Vm(r) and integrated from z = 0 to infinity, the solution found is

$$U(r,z) = \text{constant term} \times \sum_n Vn(z')\ Vn(z)\ H(Kn,r)$$

where z' = source depth
Kn = nth horizontal wavenumber
and H = zero order Hankel function of the first kind.

This approach neglects the contribution of sound with horizontal wavenumber outside the discrete spectrum of values, Kn. The contribution of this sound is generally negligible at long ranges, but can be important close to the source.

The approach can be extended to environments where the sound speed varies with range as well as depth by allowing the depth dependent functions to vary with range, i.e.

$$Vn(z) \longrightarrow Vn(z,r).$$

The resulting wave equation can be greatly simplified by assuming that changes of sound speed with range take place over 'large' distances. This reduces the situation to one very similar to the range-independent normal mode case. If this approximation cannot be realistically made, the resulting equation is very complicated.

The advantages of using a normal mode approach are:

a) The theory is a wave theory, and includes effects such as diffraction.

b) The procedure is easily automated on a computer.

The disadvantages of using a normal mode approach are:

a) It can predict unrealistically low sound levels close to the source.

b) A realistic inclusion of the effect of the bottom can be difficult to implement.