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AN ACOUSTICAL MODEL FOR TRANSMISSION LINE WOOFER SYSTEMS

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Abstract

A low frequency (plane wave) acoustical model for the transmission line type woofer system is presented by considering the combination of loudspeaker and pipe. Cases for both a uniform pipe and the commonly used tapered pipe are considered as are the quantitative effects of sound absorbing materials on wave propagation in the pipe.

1.0 Introduction

The Transmission Line system of obtaining extended low frequency reproduction from a moving coil loudspeaker was first documented by Bailey (1) in 1965. His paper described the use of a fibre - filled pipe, or transmission line, extending behind the drive unit to completely absorb the propagating acoustic wave (figure 1.1). However due to the difficulty of successfully absorbing low frequency energy, because of the large wavelengths involved, the length of the pipe and density of the filling material were arranged such that the pipe effectively acted as a low pass acoustic filter. Thus mid and high frequencies are subject to large attenuation in the pipe but low frequencies re-emerge from the open end with a phase such that reinforcement with the direct radiated sound from the woofer takes place.

This paper attempts to formulate a quantitative model for a such a combined pipe and driver system, considering both uniform and tapered pipes along with the effects of fibrous materials.

2.0 Background

Bailey's first description, and later re-examination (2), however do not include any quantitative analysis of the combined effect of loudspeaker and fibre - filled pipe. Indeed no attempt was made to examine the effect of a fibrous tangle on the propagation of an acoustic wave through a pipe, uniform or otherwise. A later paper by Bradbury (3) contained rigorous analyses of the effects of fibrous materials on plane wave propagation in a uniform pipe by considering the aerodynamic properties of the fibres. He produced an expression for the complex wavenumber resulting from a mixture of air and fibrous tangle and thus was able to derive the acoustic impedance at the end of a uniform pipe at low frequencies. However he confines his discussion to concentrate on the effects of the filling materials rather than formulating a complete model for a transmission line woofer system. To the authors knowledge the only documented work concentrating on such a model is that of Bullock and Hillman (4), who investigated a combined driver and uniform pipe system. Although a comparison is made with measured data, the experimental rig used consisted of a driver mounted on the end of a non-uniform pipe. Also the assumption is made that the radiation impedance of an open pipe can be ignored at low frequencies.

3.0 Theory

3.1 Uniform Transmission Line

At low frequencies we make the assumption that the pipe only supports plane wave propagation, so we can limit ourselves to a one-dimensional solution of the wave equation. Thus the expression for the pressure at the open end of the pipe, ie at x = L (see figure 1.1), at time t is given by :

$$pL = A \cdot \exp(-i\Omega t) + B \cdot \exp(i\Omega t) \qquad [1]$$

where A and B are constants representing the magnitudes of the incident and reflecting waves respectively and Ω is the angular frequency. Similarly the particle velocity at the same position is given by (as defined in Appendix A):

$$uL = \frac{1}{rc} \cdot A \cdot \exp(-i\Omega t) - B \cdot \exp(i\Omega t) \qquad [2]$$

where c is the velocity of sound in air of density r. Having these expressions enables us to derive the specific acoustic impedance at the open end of the pipe given by :

$$zL = pL/uL \qquad [3]$$

If we transform the pressure and velocity down the pipe we also have the expression for the specific acoustic impedance at the driver end of the pipe, ie at x = 0, as :

$$\frac{zo}{rc} = \frac{A \cdot \exp(ikL) + B \cdot \exp(-ikL)}{A \cdot \exp(ikL) - B \cdot \exp(-ikL)} \qquad [4]$$

at t = 0, where k is the wavenumber. By eliminating the constants A and B from equations [3] and [4] an expression can be derived for the specific acoustic impedance at x = 0 in terms of that at x = L ie :

$$\frac{zo}{rc} = \frac{zL/rc + i\cdot \tan kL}{1 + i(zL\cdot \tan kL)/rc} \qquad [5]$$

3.2 The Loudspeaker

Taking the loudspeaker governing electromechanical equations (5) we have :

$$V = I\cdot Zeb + Bl\cdot u \qquad [6]$$

and

$$F = -Bl\cdot I + Zmo\cdot u \qquad [7]$$

where V is the terminal voltage, I the driving current, Zeb the blocked (ie u = 0) electrical impedance, Bl the transduction coefficient (or force factor), F the net force on the diaphragm and Zmo the open circuit mechanical impedance not containing any of the acoustical elements. For a loudspeaker this latter quantity is simply a combination of the impedances due to diaphragm mass and suspension stiffness and damping. If we equate [7] to -Zr.u, where Zr is the total radiation impedance acting on the diaphragm, and combine [6] and [7], for a loudspeaker driven from a voltage source, we get the expression for the velocity :

$$u = \frac{Bl\cdot V}{(Zmo + Zr)Ze} \qquad [8]$$

where Ze is the total electrical impedance obtained by combining V and I in [6] and [7]. This is better expressed with Zr split into two components; that of the front radiation impedance, Zrf and the rear radiation impedance Zrr :

$$u = \frac{Bl\cdot V}{(Zmo + Zrf + Zrr)Ze} \qquad [9]$$

3.3 Effects of Fibrous Materials

Full analysis of the effects of fibrous materials introduced into the line on acoustic wave propagation are dealt with in reference [3] so no attempt is made here to reproduce that work. All that suffices is to say that the aerodynamic drag as a result of the fibres interacting with the movement of air results in a propagating wave which can be characterised by a complex wavenumber, $\mathbf{K}$, which is given by :

$$\mathbf{K} = (\alpha - i\cdot\beta)k \qquad [10]$$

where α and β are frequency variable parameters dependant on the diameter, density and packing density of the fibres. Full expressions for α and β are given in Appendix B.

It can be seen from the above expression that a wave, with propagation characterised in the form Q.exp(-ikx) + R.exp(-ikx), is modified in two distinct ways. Firstly, the speed of propagation is reduced by a factor of α, meaning that in practice wavelengths in the pipe are effectively reduced by the same factor, and secondly the wave decays exponentially with distance according to the magnitude of β. This new complex wavenumber can be inserted in equation [5] to derive the specific acoustic impedance at the loudspeaker end of the pipe. Thus the new expression for zo is :

$$\frac{zo}{r\mathbf{C}} = \frac{zL/r\mathbf{C} + i\tan\mathbf{K}L}{1 + izL/r\mathbf{C}\tan\mathbf{K}L} \qquad [11]$$

where **C** is the complex speed of sound given by $\Omega/\mathbf{K}$. Please note that Appendix B defines α and β as positive quantities, hence the minus sign in equation [10].

3.4 Combining Loudspeaker and Transmission Line

Since equation [9] gives the velocity of the loudspeaker diaphragm for a given input voltage we need to insert the appropriate quantities for Zmo, Zr and Ze to produce a full expression for a combination of loudspeaker and pipe. Since we are dealing only with low frequencies we can consider the radiation impedance on the front part of the loudspeaker diaphragm to be that of an unbaffled circular piston which can be approximated at low frequencies to :

$$\frac{Zrf}{rcSd} = \frac{1}{2}\{(k\cdot ad)^2 + i(0.6)\}k\cdot ad \qquad [12]$$

where Sd is the area of the loudspeaker diaphragm and ad its radius. The expressions for Zmo and Ze are given in appendix C. For the rear radiation impedance, this is equal to the acoustic impedance at the loudspeaker end of the tube, so we can obtain this from equation [11] by inserting an expression for zL similar to equation [12] and dividing by Sd. Thus :

$$Zrr = zo/Sd \qquad [13]$$

With this final expression defined we can use equation [9] to obtain the actual velocity of the loudspeaker diaphragm when loaded by a transmission line and this enables us to determine the particle velocity at the open end of the line, ie at $x = L$.

3.5 Tapered Transmission Line

Most practical applications of a transmission line woofer system use a pipe which reduces in cross sectional area progressively from loudspeaker end to open end. This means that the theory is somewhat different to that for a uniform pipe although it can be arrived at in a similar fashion. Figure 3.1 shows the basic arrangement. The main difference in deriving the theory for a tapered pipe rather than a uniform one is that wave propagation in the pipe obeys the form, given by a solution to the horn equation (5) at t = 0, as follows :

$$p = A\cdot\frac{\exp(-iKx)}{(xo + x)} + B\cdot\frac{\exp(iKx)}{(xo + x)} \qquad [14]$$

where A and B are constants as in equation (1) and x is the distance along the axis of the pipe. Therefore evaluating the particle velocity gives :

$$u = \frac{1}{rC}\cdot\left\{A\cdot\frac{\exp(-iKx)}{(xo + x)} - B\cdot\frac{\exp(iKx)}{(xo + x)}\right\}\cdot\left\{1 + \frac{i}{K(xo + x)}\right\} \qquad [15]$$

where xo is defined in figure 3.1. With these two expressions we are able to derive the specific acoustic impedance at the loudspeaker end of the pipe in a similar way to that in section 3.1. Thus for a tapered pipe we have :

$$\frac{zo}{rC} = \frac{zL/rC + i(zL/rCKxo + \tan KL)}{1 - \frac{zL\cdot\tan KL}{rCKxo} - \frac{zL\cdot\tan KL}{rCK(xo+L)} + i\left\{\frac{1}{K(xo+L)} + \frac{zL\cdot\tan KL}{rC} - \frac{zL\cdot\tan KL}{rCK^2xo(xo+L)}\right\}} \qquad [16]$$

So having equation [16] enables us to evaluate zo for a tapered transmission line and use the result in equation [9] to obtain the velocity of the loudspeaker diaphragm in a similar way to the method described in section 3.4.

4.0 Discussion

Firstly it is interesting to investigate some of the properties of the above expressions. Taking equation [5] for the specific acoustic impedance on the rear of a loudspeaker mounted at the end of an unfilled uniform tube and considering the case when the other end of the tube is sealed, ie $zL = \infty$, we see that $zo/rc = -i\cdot \cot kL$. This expression, being negative and purely imaginary, represents a compliance (inverse stiffness) presented to the rear of the loudspeaker and therefore gives us the form of a single driver in a sealed box. We can use this expression to evaluate the resonance frequencies of the pipe by equating zo to zero. Replacing the standard wavenumber, k, with the complex version for a filled pipe would give us different frequencies according to the values of α and β.

By studying equation [16] for the impedance on the rear of a loudspeaker mounted in a tapered tube we see that in the case when $xo = \infty$, ie the tube turns from tapered into uniform, we obtain equation [5] again.

We can consider an example of a transmission line woofer system by inserting values in the above equations to represent a suitable driver and pipe combination. Figure 4.1 shows the volume velocities (multiplied by frequency therefore proportional to sound pressure) of the diaphragm and at the open end of a 1.5m long by 150mm diameter unfilled uniform tube to which the loudspeaker is mounted. The parameters describing the loudspeaker are given in Appendix D. In this case the system behaves rather like an organ pipe with a series of harmonically related modes. Also on the graph is the diaphragm velocity for the same combination of driver and pipe but with the end closed as if it was a sealed cabinet system.

Placing some filling in the tube has three effects which are seen in figure 4.2. Not only are the resonant modes at lower frequencies than in the case of the unfilled tube but because of the frequency dependant nature of the parameters α and β the modes no longer occur at harmonically related frequencies. Also the presence of the filling materials provides damping for the system.

For the case of a tapered transmission line the results are very similar and these are seen in figure 4.3 for a tube of the same length containing the same type and packing density of filling. Changing the type and quantity of the filling material can influence the results significantly and figures 4.4 and 4.5 give two such alternative cases.

5.0 Conclusions and Recommendations

A model is presented to describe the performance of a loudspeaker and transmission line combination. Cases for a tapered pipe as well as a uniform pipe are presented. The effects of filling introduced into the pipe are examined in terms of the influence on the speed of propagation of sound in the tube and its rate of decay with distance. From the discussion it is shown that little significant difference exists between the low frequency performance of straight or slightly tapered pipes.

The model could be easily expanded to incorporate combinations of different diameter pipes, whether straight or tapered, and series combinations of different filling materials.

Since the effects of filling materials presented are only predictions, a method of measuring their properties is desirable. One way by which this could be done is with an impedance tube method, however, work is progressing on investigating the use of the loudspeaker electrical impedance to derive the material properties.

6.0 References

(1) Bailey, A R, "A Non-Resonant Loudspeaker Enclosure Design", Wireless World, October 1965.
(2) Bailey, A R, "The Transmission-Line Loudspeaker Enclosure", Wireless World, May 1972.

(3) Bradbury, L J S, "The Use of Fibrous Materials in Loudspeaker Enclosures", J Audio Eng Soc, Vol 24, April 1976.

(4) Bullock, R M and Hillman, P E, "A Transmission-Line Woofer Model" Audio Eng Soc Preprint No. 2384, November 1986.

(5) Kinsler, L E, Frey, A R, Coppens, A B, Sanders, J V, "Fundamentals of Acoustics" 3rd ed. Ch 14, Wiley 1982.

7.0 Appendices

A. The relationship between pressure and particle velocity is given by Euler's equation which, in one dimensional form, is as follows :

$$r.\frac{\delta u}{\delta t} = -\frac{\delta p}{\delta x}$$

B. For evaluation of the parameters α and β from equation [10] use is made of the following expressions :

$$\alpha = \left\{\frac{(1 + P/r)^2 + (\Omega P/D)^2}{1 + (\Omega P/D)^2}\right\}^{\frac{1}{4}}.\cos\Theta$$

$$\beta = \left\{\frac{(1 + P/r)^2 + (\Omega P/D)^2}{1 + (\Omega P/D)^2}\right\}^{\frac{1}{4}}.\sin\Theta$$

where P is the packing density of the fibrous tangle and D is the aerodynamic drag parameter given by :

$$D = \frac{27\mu}{d^2}.\left\{\frac{P}{rf}\right\}^n$$

and
$$\Theta = \frac{1}{2}\left\{\tan^{-1}(\Omega P/D) - \tan^{-1}\frac{(\Omega P/D)}{(1 + P/r)}\right\}$$

where rf is the density of the fibrous material, d is the diameter of the fibres and μ is the viscosity of air (0.0000185 kg/m·s). The value of 27 is said to be arrived at by experiment and n is given as 1.4.

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C. The open circuit mechanical impedance of a loudspeaker is given by :

$$Zmo = i(\Omega m - s/\Omega) + Rm$$

where m is the effective diaphragm mass, s the total suspension stiffness and Rm the suspension damping, and its total electrical impedance is :

$$Ze = Zeb + \frac{Bl^2}{(Zmo + Zr)}$$

D. In the examples in section 4.0 a loudspeaker with the following parameters was used :

Diaphragm mass = 15 g; suspension stiffness = 1250 N/m
suspension damping = 1.0 kg/s; Bl = 8 Tm; Zeb = 6.4 ohm

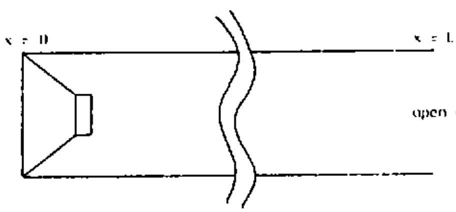

Figure 1.1 : Basic Transmission Line

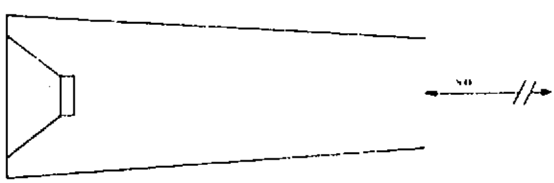

Figure 5.1 : Tapered Transmission Line
(xo is the distance from the open end to the imaginary apex of the tube)

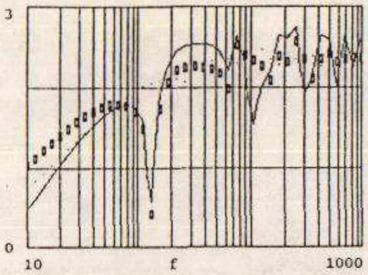

Figure 4.1

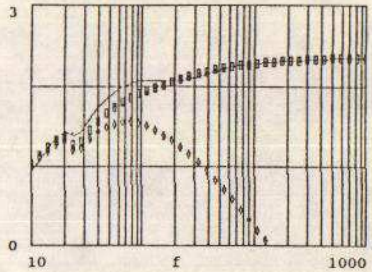

Figure 4.2

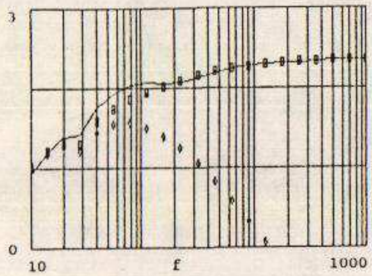

Figure 4.3

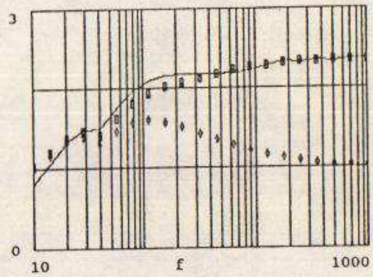

Figure 4.4

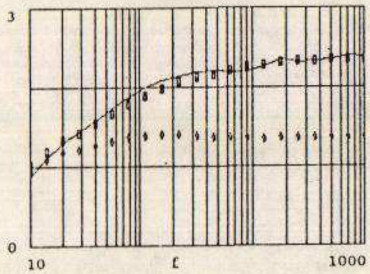

Figure 4.5

Figures 4.1 to 4.5 : Calculated particle velocities for transmission line system; □ = loudspeaker; ◊ = end of pipe; · = loudspeaker in equivalent sealed box; — = summed velocities of loudspeaker and pipe.
The filling material specifications are as follows :-
4.1 : P = 0
4.2 : P = 10 kg/cu.m; rf = 4000 kg/cu.m; d = 5 μm (glassfibre)
4.3 : as 4.2, 4.4 : P = 5; rf = 4000; d = 5 μm, 4.5 : P = 10; rf = 1000; d = 25μm (BAF wadding). The diameters of the open end and loudspeaker end of the tapered pipe were 150 and 300 mm respectively.