

## CAVITATION PROBLEMS IN SONAR

7.0

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### 1. INTRODUCTION

Until recently sonar has been used mainly in naval applications and by the fishing industry. However, during the last five years there has been a considerable growth in the use of sonar by the oil industry, especially in the exploitation phase of industry operation, namely pipeline route, drilling rig and production platform site surveys. One potential application of sonar in the exploration phase, not fully utilised, is the estimation of sea bed topography in very deep waters.

The purpose of this paper is to discuss the limitation imposed by acoustic cavitation on the maximum acoustic power radiated by high powered sonar arrays.

### 2. LIMITATION ON SONAR POWER

When operating high powered sonar arrays, it is essential to generate the maximum amount of acoustic power with minimum distortion. However, there are two factors which can limit the maximum acoustic power radiated by high powered sonar arrays. One factor, known as the finite amplitude effect, is due to the non-linearity of the water and the other factor is due to acoustic cavitation.

#### 2.1 Finite Amplitude Effects

The limitation imposed by the non-linearity of the propagation medium on the maximum amount of acoustic power radiated by a sonar array has only recently been recognised. The dependence of the output pressure on the acoustic intensity at the face of a transducer, obtained (1) for the case of a transducer resonant at 946.5 kHz when driven by a pulsed sinusoidal voltage, is given in Figure 1. The output pressure of the transducer, with the harmonics being filtered out, was measured by a hydrophone placed at 3.1 m along the transducer axis. It can be seen from Figure 1 that when the transducer input power is fairly small, the transducer input - output curve is linear. However, when the transmitted acoustic intensity becomes large, the transducer input - output curve begins to depart from the linear relationship. It can also be seen from Figure 1 that further increase in the transducer input power results only in a small increase in the transducer output acoustic power and saturation is said to be reached. The saturation is due to the formation of a pressure shock wave and the transformation of the initially sinusoidal pressure waveform into a saw-tooth pressure waveform. Consequently, when saturation is reached almost all the increase in the transducer input power is used in generating higher harmonics.

For the case of a plane sinusoidal pressure waveform, the critical distance  $X_c$  from the transducer at which a shock wave is formed is given (2) approximately by:-

$$X_c \approx 600\lambda/\sqrt{I} \quad (1)$$

where  $\lambda$  is the acoustic wavelength and  $I$  is the acoustic intensity (watt/cm<sup>2</sup>) at the transducer face.

## 2.2 Acoustic Cavitation

Sea water is said to be cavitating when gas bubbles which already exist in the sea water are caused to grow due to the presence of high intensity acoustic pressure. There are two types of acoustic cavitation; one type which is known as transient cavitation is characterised by the presence of bubbles having unstable radial motion, and the other type of acoustic cavitation, referred to as gaseous cavitation, is associated with the presence of stable bubbles.

When dealing with the problem of acoustic cavitation it is desirable to be able to predict the cavitation threshold, that is the amplitude of the applied acoustic pressure required for the onset of cavitation. Blake (3) was the first to develop an expression giving the transient cavitation threshold. Unfortunately, Blake's threshold pressure is not only independent of the applied acoustic frequency, but also much larger than the measured value.

It can be shown that when the amplitude of the applied acoustic pressure exceeds some value, the motion of bubbles with resonance frequencies half that of the applied acoustic frequency becomes unstable and subharmonic pressure waves will be generated. Since it has been found that the onset of the generation of subharmonic waves coincides with the onset of transient cavitation, it is justifiable to take the threshold for the onset of subharmonic pressure waves as the threshold for the onset of transient cavitation.

The non-linear differential equation governing the radial motion of a gas bubble excited by an acoustic pressure  $P_A \cos \omega t$  can be transformed into a simple differential equation with a time varying parameter. From the solution of the simplified differential equation for the case when the gas bubble is resonant at half the driving acoustic frequency, the threshold pressure  $P_t$  for the onset of subharmonic pressure waves is given by Safar (4):-

$$P_t \approx 6\gamma (1 + d/10)/Q \quad (2)$$

where  $\gamma$  is the air specific heats ratio,  $d$  is the depth (metre) of the bubble and  $Q$  is the bubble quality factor. It should be pointed out that a similar expression of the threshold pressure for the onset of subharmonic pressure waves has also been obtained by Eller and Flynn (5), and by

Nayfeh and Saric (6). However, the expression obtained by Eller and Flynn (5) and by Nayfeh and Saric (6) differs from that given by equation (2) in that their expression is independent of the air specific heats ratio  $\gamma$ .

Figure 2 shows the dependence of the threshold pressure for the onset of subharmonic pressure waves on the driving acoustic frequency for the case when  $\gamma = 1.4$  and bubble depth equals 1 m and 10 m. The values of the bubble quality factor  $Q$  used for calculating the threshold pressure  $P_t$  are those predicted by Devin (7).

It is worth pointing out that the dependence on depth of the threshold pressure  $P_t$  for the onset of subharmonic pressure waves is in agreement with that found experimentally by Rusby (8) for the onset of transient cavitation. The amplitude of the applied acoustic pressure required for the onset of a subharmonic pressure wave as predicted by equation (2) is consistent with that measured by Rusby (8) for the case of a driving acoustic frequency of 7 kHz at a depth of 1 m.

One likely factor which is responsible for the onset of gaseous cavitation is the growth of microbubbles by rectified diffusion. Rectified diffusion is a mechanism whereby air bubbles begin to grow in size when the applied acoustic pressure amplitude exceeds a certain value. It is well established that water contains microbubbles with size distribution function such that the number of very small bubbles is much larger than that of larger bubbles.

In the presence of acoustic pressure in water the microbubbles' size distribution will be a function of the duration, the amplitude and frequency of the applied acoustic pressure. Although the present theories of rectified diffusion (9, 10, 11) predict fairly accurately the threshold pressure for the onset of rectified diffusion, the predicted rates of growth are found (12) to be considerably smaller than the observed values. Therefore it is not possible to calculate the effect of rectified diffusion on the bubble size distribution using the existing theories for rectified diffusion.

### 3. DISCUSSION

Sonar array designers usually assume that the maximum acoustic power radiated by sonar arrays operating with high frequency are limited by finite amplitude effects, whereas in the case of low frequency sonar arrays used for geological purposes, the limitation is due to transient cavitation. However, it will be shown below that the maximum acoustic power radiated by geological sonar arrays is, in fact, limited by gaseous cavitation and not by transient cavitation.

Kikuchi and Shimizu (13) studied experimentally the effect of acoustic cavitation on the radiation resistance of a transducer radiating high intensity acoustic pressure at 28 kHz into water. From the experimental

results obtained by Kikuchi and Shimizu (13) it was shown (14) that a large number of stable bubbles with resonance frequencies much higher than 28 kHz were present in the water when the acoustic intensity at the face of the transducer exceeded 0.3 watts/cm<sup>2</sup>. The fractional volume of air per unit volume of water in the form of bubbles with resonance frequencies much higher than 28 kHz for various values of acoustic intensity is given in Table 1.

The effect of the presence of air bubbles in the near field of a sonar array on the pressure waveform radiated by the array has been investigated (14) theoretically. The second harmonic amplitude  $P_2$  generated by an initially sinusoidal pressure wave propagating through a layer of bubbles with resonance frequencies much higher than the driving acoustic frequency is given by (14):-

$$P_2 \approx \rho_0 \pi f c_0 \left[ \eta/A + 10^{-12} U / (1 + d/10)^2 \right] P_{10}^2 L \quad (3)$$

where  $f$  is the sonar array driving frequency,  $P_{10}$  is the average pressure amplitude radiated by the array in the near field,  $\eta/A$  is the water non-linearity parameter,  $L$  is the thickness of the bubbly layer and  $U$  is the fractional volume of air per unit volume of water, and it is given by:-

$$U = \int_0^R V_0 n(R_0) dR \quad (4)$$

where  $V_0$  is the air bubble equilibrium volume and  $n(R_0)$  is the air bubble's size distribution function defined as the number of bubbles per unit volume of water per unit radius interval for a given duration, amplitude and frequency of the applied acoustic pressure.

We can deduce from Table 1 and equation (3) that the effect of the presence of bubbles with resonance frequencies much higher than the driving acoustic frequency is to increase the non-linearity of the propagation medium within the array near field considerably.

As far as I am aware, there are only two experimental works dealing with the limitation by acoustic cavitation of the maximum acoustic power radiated by high powered sonar projectors. One experimental investigation was carried out by Rusby (8) who found that significant harmonic distortion occurred at acoustic intensity well below that required for the onset of transient cavitation. The other work which was conducted by Liddiard (15) involved a series of tests using operational sonar projectors resonant in the frequency range 14 to 31.6 kHz. A typical result obtained by Liddiard (15) is given in Figure 3 which shows the dependence of the source level on the input power of a sonar projector resonant at 15 kHz and radiating a pulse 3 ms long at a depth of 5 m. The source level was measured by a hydrophone placed well outside the projector near-field.

The saturation effect illustrated in Figure 3, which is similar to that shown in Figure 1 for the case of a pulsed high frequency acoustic pressure, confirms the conclusion reached (14) when dealing with the propagation of high intensity pressure waves in a bubbly medium, namely, the onset of gaseous cavitation makes the formation of shock waves possible even at relatively low acoustic frequencies.

#### 4. CONCLUSIONS

We may conclude that not only a large amount of acoustic power is wasted but also a considerable amount of harmonic distortion is generated by most low frequency sonar arrays currently used for geological purposes. This is because the maximum acoustic power radiated by the arrays is based on the threshold for the onset of transient cavitation. Another conclusion is that the onset of gaseous cavitation which limits the maximum acoustic power can be monitored continuously by measuring the level of harmonic distortion.

Finally, our present knowledge concerning the dependence of the onset of gaseous cavitation on acoustic frequency and pulse length is somewhat incomplete. Therefore, more experimental work is needed so that high powered sonar arrays are operated more efficiently.

#### 5. ACKNOWLEDGEMENTS

I wish to thank the Chairman and Directors of the British Petroleum Company for permission to publish this paper.

#### 6. REFERENCES

1. M. H. Safar, 1969 Memo No 391, Department of Electronic and Electrical Engineering, University of Birmingham, High intensity limitations to the exploitation of finite amplitude effects in underwater acoustics.
2. D. T. Blackstock, 1964 J. Acoust. Soc. Am. 36, 217 - 219. On plane, spherical and cylindrical sound waves of finite amplitude in lossless fluids.

3. F. G. Blake, Jr., 1949 Technical Memo No 12, Acoustic Research Laboratory, Harvard University, Cambridge, Mass., U.S.A. The onset of cavitation in liquids.
4. M. H. Safar, 1970 J. Phys. D: Applied Physics 3, L47 - L51. Measurement of viscosity by subharmonic pressure threshold for a bubble in a viscous liquid.
5. A. I. Eller and H. G. Flynn, 1969 J. Acoust. Soc. Am. 46, 722 - 727. Generation of subharmonics of order one-half by bubbles in a sound field.
6. A. H. Nayfeh and W. S. Saric, 1973 J. Sound and Vibration 30 (4) 445 - 453. Non-linear acoustic response of a spherical bubble.
7. C. Devin, Jr., 1959 J. Acoust. Soc. Am. 31, 1654 - 1667. Survey of thermal, radiation and viscous damping of pulsating air-bubbles in water.
8. J. S. M. Rusby, 1970 J. Sound Vib., 13 (3), 257 - 267. The onset of sound wave distortion and cavitation in water and sea water.
9. D. Y. Hsieh and M. S. Plesset, 1961 J. Acoust. Soc. Am. 33, 206 - 215. Theory of rectified diffusion of mass into gas bubbles.
10. M. H. Safar, 1968 J. Acoust. Soc. Am. 43, 1188 - 1189. Comment on papers concerning rectified diffusion of cavitation bubbles.
11. A. I. Eller and H. G. Flynn, 1965 J. Acoust. Soc. Am. 37, 493 - 503. Rectified diffusion during non-linear pulsations of cavitation bubbles.
12. A. I. Eller, 1969 J. Acoust. Soc. Am. 46, 1246 - 1250. Growth of bubbles by rectified diffusion.
13. Y. Kikuchi and H. Shimizu, 1959 J. Acoust. Soc. Am. 31, 1385 - 1386. On the variation of acoustic radiation resistance in water under ultrasonic cavitation.
14. M. H. Safar, 1973 Proceedings: Finite amplitude wave effect in fluids symposium, Copenhagen, 174 - 179. Propagation of acoustic waves of finite amplitude in water containing air-bubbles.
15. G. E. Liddiard, 1953 U.S. Navy Electron. Lab. Report No 376 Short pulse cavitation thresholds for projectors in open water.

TABLE 1

Fractional volume U of air per unit volume of water  
in the form of bubbles with resonance frequencies much higher than  
28 kHz for various values of acoustic intensity  
radiated by a transducer at 28 kHz

Acoustic Intensity $I(\text{W}/\text{cm}^2)$	Fractional Volume U ( $10^{-6}$ )
1/3	5.3
0.5	22.0
0.66	33.0

LIST OF FIGURES

- Figure 1      The effect of the water non-linearity on the acoustic pressure radiated by 40 mm diameter disc transducer resonant at 946.5 kHz. The straight line is obtained by extrapolating from the experimental data at low intensities.
- Figure 2      Threshold pressure for the onset of subharmonic pressure waves generated by bubbles with resonance frequencies half the driving acoustic frequency with  $\gamma = 1.4$ . Lower curve 1 m depth, Upper curve 10 m depth.
- Figure 3      The effect of the non-linearity caused by cavitation on the source level of a sonar projector resonant at 15 kHz when radiating 3 ms pulse at a depth of 5 m (14).



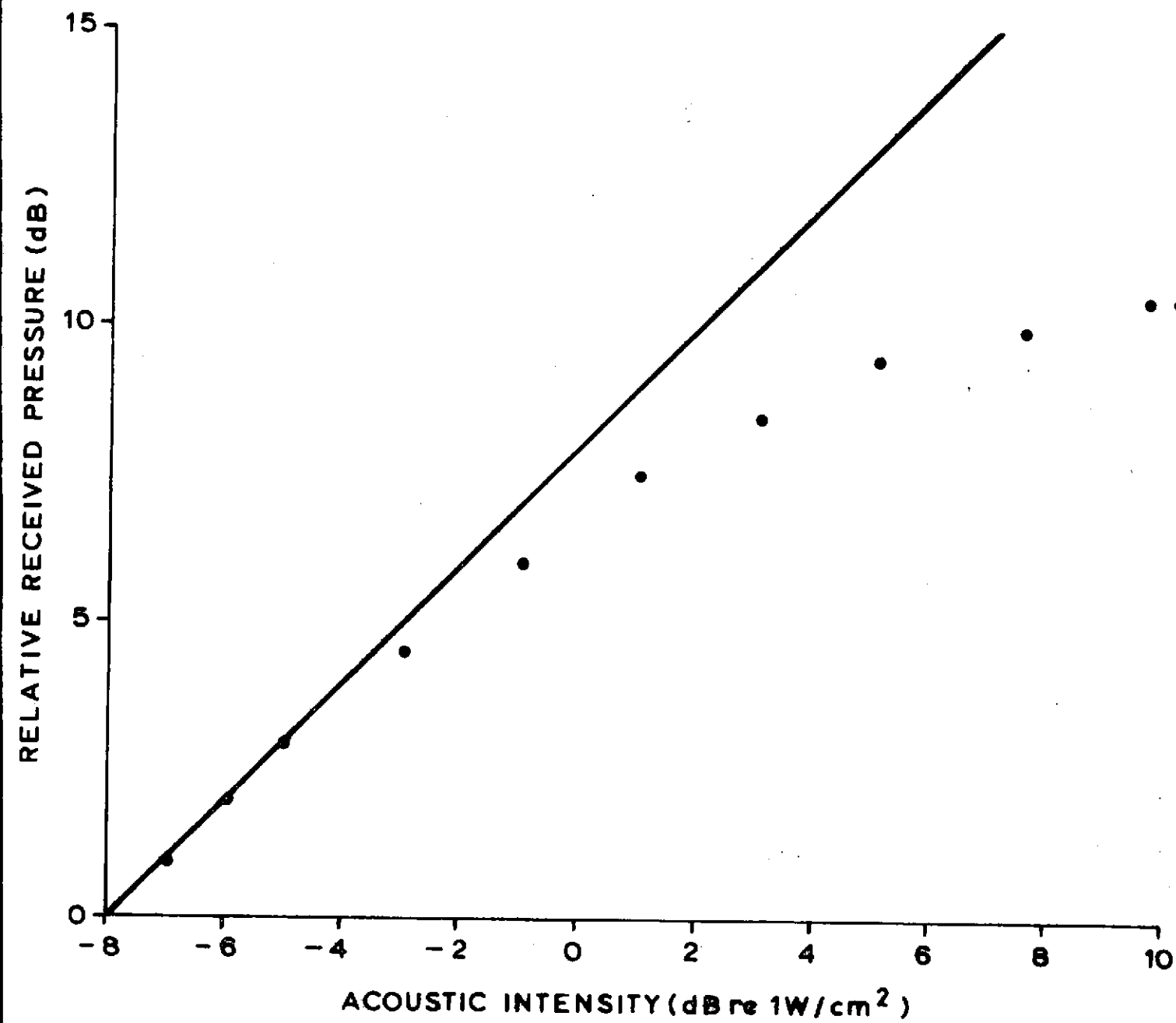
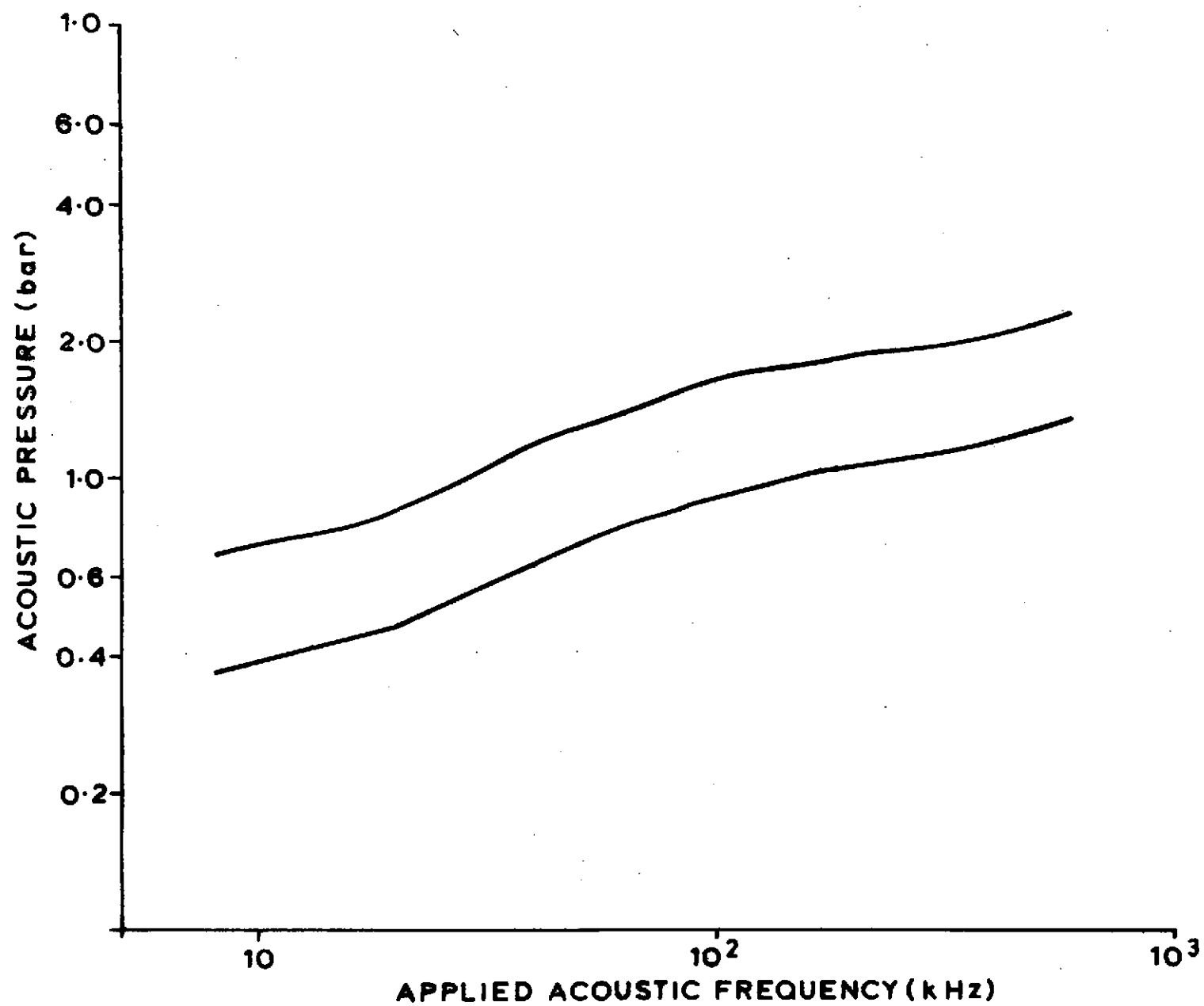


Figure 1.

Figure 2



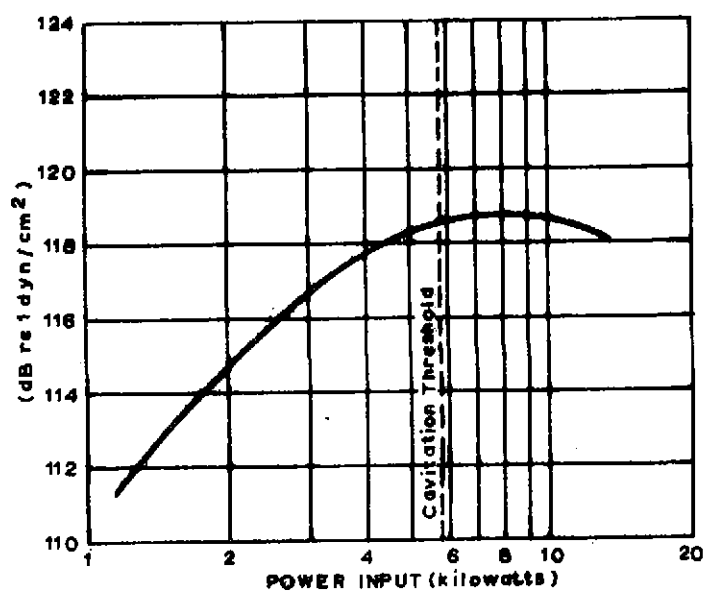


Figure 3

### DISCUSSION

There was some discussion whether the formula derived by the Author for the sub-harmonic threshold pressure (equation 2) could be considered more reliable than those derived by other authors (see references 5 and 6). Dr. Crum wondered whether it was correct to assume adiabatic conditions. The Author replied that only a rough estimate of the threshold was required; otherwise, he agreed that a more realistic equation of state would have been used.