Angular distribution of nonlinearly generated difference frequency sound.

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I. Introduction.

In course of the last 10 years there has been an increasing interest in the study of non-linear wave effects, such as for instance interactions between sound waves, and a number of reports concerning the distribution of difference frequency sound generated by nonlinear interaction of two coaxial primary waves have appeared [1-11]. In another paper presented at this meeting[12] we discuss the axial distribution of the difference frequency sound. In this paper attention will be focused particularly on the angular distribution of the difference frequency sound. In addition to discussing the results derived from different theoretical models, we shall present experimental results which demonstrate that for accurate comparisons with existing theories it is important to use small primary intensities and to measure sufficiently far from the interaction region.

II. Theory.

Analytical solutions to the far field of the difference frequency sound produced by interactions in a beam have been obtained by several authors[2,3,5,6]. We shall be concerned mainly with models applying to the case where the primary beams are radiated from a circular piston source. All the soulutions may be expressed in the form

$$P_-(r,\theta,t) = R(r)D(\theta)\ \sin(\omega_- t - k_- r + \psi(r,\theta)) \qquad (1)$$

where $\{r,\theta\}$ are spherical coordinates, $\omega = 2\pi f$ = angular frequency, $k = 2\pi/\lambda$ = wavenumber. We are here only interested in the pressure amplitude

$$P_-(r,\theta) = R(r)D(\theta) \qquad (2)$$

and expressions for the phase $\phi(r,\theta)$ will not be considered.

The wellknown result of Westervelt[2] for an axial array (line distribution of virtual sources) may be written as

$$R_A(r) = \frac{A+2}{8}\rho_o k_-^2(\pi a^2 L_A)U_a U_b \frac{e^{-\alpha_- r}}{r} \tag{3}$$

$$D_A(\theta) = (1 + (2L_A k_- \sin^2(\tfrac{1}{2}\theta))^2)^{-\frac{1}{2}}$$

where A is the nonlinearity parameter of the medimum (=B/A), ρ = medium density, a = source radius, α = attenuation coefficient, and U = primary velocity amplitude at the source. The suffix o refers to ambient values while a,b and − refers to the different frequency components. $L_A = (\alpha_a+\alpha_b-\alpha_-)^{-1}$ is the array length.

When interaction takes place only in the far field of the primary waves (Bessel array) Lauvstad et al.[5] obtained

$$R_B = R_A(r)$$

$$D_B = \begin{cases} D_A(\theta)\ \frac{2}{\pi}(\text{Arccos}(\mu) - \mu(1-\mu^2)^{\frac{1}{2}}) & \mu<1 \\ 0 & \mu\geq 1 \end{cases} \tag{4}$$

where $\mu = \frac{1}{4}k_- a \sin(\theta)$.

For a beam of plane but collimated primary waves (collimated array) Naze et al.[6] found

$$R_C = R_A(r)$$

$$D_C = D_A(\theta)\left|\frac{2J_1(k_- a\sin(\theta))}{k_- a \sin(\theta)}\right| \tag{5}$$

$$= D_A(\theta)D_D(\theta)$$

where D_D is the directivity which should exist if sound of difference frequency was radiated directly from the primary source.

We shall of course always have $D_A, D_B, D_C, D_D \leq 1$. From the expressions for the directivity factors we may at once draw some general conclusions. Since $D_B/D_A \leq 1$ and $D_C/D_A \leq 1$, the Bessel array and the collimated array must always give a directivity at least as strong as that of the axial array. Since also $D_C/D_D \leq 1$, the collimated array will never give a broader directivity than direct radiation from the source. For the collimated array we may now easily recognize two factors which determine the angular distribution: (1) an aperture factor D_D dependant only on the cross section of the array (k_-a), and (2) a length factor depending on the effective (nondimensional) array length $L_A k_-$ only. (The Bessel array has a corresponding aperture factor D_B/D_A which is not as easily recognized as D_D). Thus the distribution will tend towards that of an axial array when the array radius is reduced, and towards that of direct radiation when the array length is reduced.

Recently attention has been drawn toward the effects which appears when great primary intensities are used. At great intensities the primary waves are more strongly attenuated, an effect which may be attributed to nonlinear attenuation which adds to the linear one. From the discussion above one may easily arrive at a qualitative description of what one should expect to happen to the directivity when the primary intensities are increased to such a level that nonlinear attenuation becomes important. When attenuation is increased, the array length will be reduced and thus the distribution D_C will approach D_D which should, however, only be reached in the limit.

From the expression for D_C it will be noticed that the difference frequency sound will have a system of side-lobes with zeroes at the same angles as for direct radiation. A proof of the existence of such side-lobes is demonstrated in fig. 1, which shows the observed angular distribution $D(\Theta)$ of 3 MHz difference frequency sound ($a=6.5$ mm, $f_a=17.6$ MHz, $r=60$ cm). The first side-lobe is easily recognized, while the rest are embedded in noise (because of D_A). The existence of side lobes cannot be explained in terms of the side-lobes of the primary beam. These are also neglected in the model of the collimated array. The observation of side-lobes is thus qualitatively in accordance with the latter model.

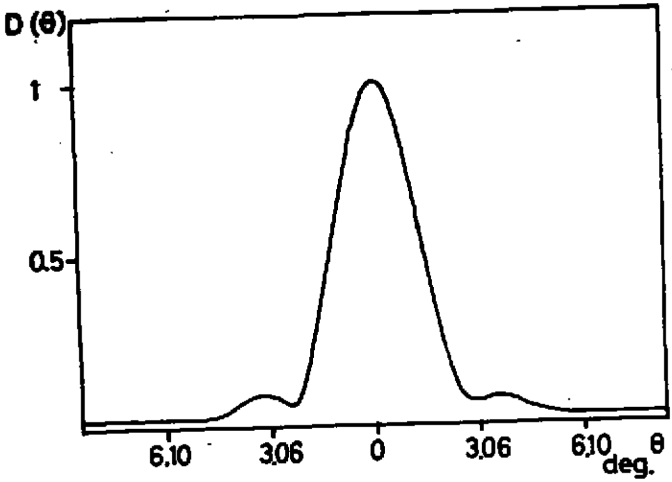

Fig.1. Angular distribution of 3 MHz difference frequency sound, which shows existance of the first side-lobe.

The amplitude relations between the side lobes and the main lobe should be different from those of direct radiated sound, because of D_A which attenuates the side-lobes more strongly. Hence we may notice that the aperture factor D_D, or rather k_-a, should determine the positions of the side-lobes, whereas the length factor D_A, or rather $L_A k_-$, determines the relative amplitudes of the side-lobes relative to those in a direct radiation pattern. When non-linear attenuation is important, reducing the array length L_A, we should expect the relative amplitudes of the side-lobes to increase, while they remain fixed in space. Accurate experimental investigations of these effects should, of course, be performed at a sufficient distance from the interaction region.

The above discussion is based upon idealized theoretical models. The real near-field of the primary waves may therefore still give different distributions for the difference frequency, if the models are far from experimental conditions. This point will be touched upon in the next section (see also [12]).

III. Experiment.

The experimental configuration in the experiments was basically the same as earlier[8]. The two primary waves with frequencies f_a = 17.68 MHz and f_b = 16.68 MHz are radiated from a plane, circular quartz source with an effective radius of 6.5 mm into a tank of (tap) water 2 m long. The difference frequency sound is measured with a probe whose effective diameter is 1.5 mm. Because of the resonant conditions of the source and the probe system, and the desire to use small primary amplitudes, the frequencies could not normally be changed much from these values. The calibration of the equipment is discussed briefly in [12].

Use of the primary frequencies 17.6 and 16.6 MHz leads to the ka values 480 and 454 respectively, and near field lengths ($L=a^2/\lambda$) of L_a=0.50m and L_b=0.47m. Since the array length is L_A=0.069 m the interaction will take place well inside the near-fields of the primary waves. This may seem to make discussions of our results rather complicated since, until recently, the near field has been poorly understood. The question has therefore been raised[6] how the complicated fluctuations of phase and amplitude inside the near field will affect the generated sound. Recent numerical computations of the near field[13] has brought some light to these problems. The computations show that in a region near the source ($<L/2$) and within the distance a from the axis, the amplitude and phase does not deviate much from those of a plane wave,[12] except in a small region at and very near the axis (paraxial region). Energy is still radiated away from this region, which leads to a slight spatial tapering of the "plane wave region". If, however, the array length L_A is much less than the length of the primary near-fields $L_{a,b}$ it seems reasonable to expect that the

model of plane collimated primary waves should apply. The Bessel array model should be appropriate when $L_A > L_{a,b}$. For both models one must have $ka \gg 1$ for the primary waves.

IV. Results.

The effect of increased primary amplitudes on directivity is demonstrated in fig. 2. The observed half-pressure angles $\theta_{\frac{1}{2}}$ are presented versus the primary pressure (in atmospheres) when $P_a = P_b$, and also versus the Reynolds number

$$Re = (Re_a Re_b)^{\frac{1}{2}}, \quad Re_{a,b} = \frac{P_{a,b} k_{a,b}}{2 c_o^2 \rho_o \alpha_{a,b}} \qquad (6)$$

The latter is a more fundamental quantity when one is concerned with the nonlinear attenuation of the primary waves. The observations are taken at a distance 40 cm from the source. Also the half-pressure angle θ_D for direct radiation and θ_C calculated from the collimated array model are shown. Fig. 2 shows clearly how the half-pressure angle increases with the primary intensity. From a relatively stable region below $Re = 0.05$ the angle increases with intensity, most rapidly between $Re = 0.1$ and 0.15, and thereafter more slowly. The latter deceleration may perhaps be affected by the existance of still more complex forms of nonlinear attenuation[12,14]. Anyway it is clear that we are still far from reducing the effective length of the array to zero at these intensities, since $\theta_{\frac{1}{2}}$ is far from reaching θ_D. It is also evident that if comparisons are to be made with the present theories, observations should be made in the stable region at low intensities, and that this sets an upper limit to the primary intensities which should be used.

In fig. 2 the correspondence between the low intensity observations of $\theta_{\frac{1}{2}}$ and the calculated θ_C is not too good (note the expanded scale). Now it turns out that measurements of the angular distribution at different distances from the source show systematic variations.

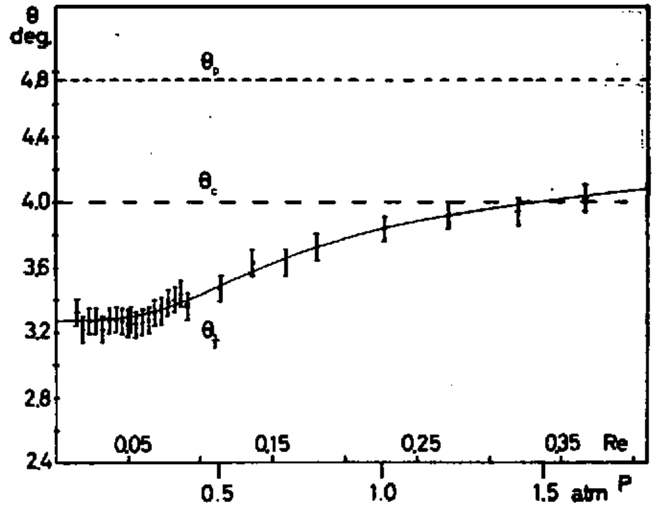

Fig.2.Variation of the half-pressure angle with primary intensity. The latter is presented both in terms of the Reynolds number and the pressure in atmospheres. Also shown is the corresponding angle θ_C calculated from the collimated array model, and direct radiation θ_D. Note the expanded scale for θ.

Fig. 3a shows the result of such observations of $\theta_{\frac{1}{2}}$ with primary intensities in the low intensity region. From large values near the source $\theta_{\frac{1}{2}}$ reaches a minimum at some distance from the source,and thereafter increases toward a limit in the far field. These effects are shown more clearly by fig. 3b, which indicates what really happens in the y-z plane. Here the observed half-pressure angles of fig. 3a are plotted as real lateral distances y against the axial distance z (note the different scales). It is to be kept in mind that the sound is generated from a volume of a certain extent and not by a well localized source. Inside this interaction volume the generated sound seems to behave like a beam, reaching its final directivity only well outside this region, and then as if it was radiated from somewhere near the end of the array. Evidently the directivity

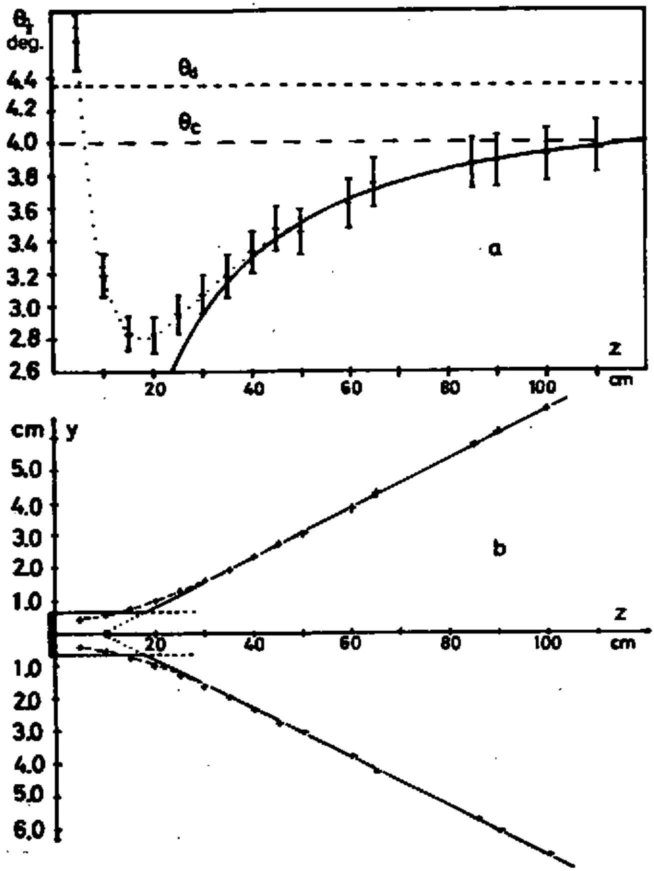

Fig. 3a Observed variation of $\theta_{\frac{1}{2}}$ with distance from the primary source. Also shown is the relation between $\theta_{\frac{1}{2}}$ and θ_d giving best fit to observations, as well as the angles θ_d and θ_c.

Fig. 3b The observed variation of $\theta_{\frac{1}{2}}$ versus Z in the y-z plane.

should thus be referred to some point characterizing an effective termination of the array, instead of, as is usually done, to the center of the primary source. This effective origin can, of course, not be rigorously fixed, but if observations are made well outside the interaction region its position should remain reasonably stable. If we assume its position to be at some distance d in front of the source (on the axis), the relation between the half-pressure angle θ_d referred to this origin and $\theta_{\frac{1}{2}}$ referred (as earlier) to the primary source should be

$$\theta_{\frac{1}{2}} = \theta_d\left(1 - \frac{d}{z}\right) \qquad (7)$$

provided the angles are small. Evidently $\theta_{\frac{1}{2}}$ and θ_d should coincide in the very far field, and θ_d is thus the value to be compared to the theoretically calculated half-pressure angle.

Near the source the variations in $\theta_{\frac{1}{2}}$ may be attributed to the beam properties of the generated sound. A simple model to describe this could be $\tan \theta_{\frac{1}{2}} = \frac{a}{z}$, but comparisons with observations show that this gives values of $\theta_{\frac{1}{2}}$ which are somewhat too large. This is also indicated in fig. 3b.

From the observed values of $\theta_{\frac{1}{2}}$ the best fit of eq.(7) as plotted in fig. 3a, was obtained with d = 9.6 cm and θ_d = 4.35 deg., the latter with an accuracy of 5%. The calculated half-pressure angle θ_C is 4.0 degrees. According to fig. 2 the finite primary intensities used may give a 2% correction to the θ_d-value, leaving a discrepancy between θ_d and θ_C of about 7%. This indicates that the model of collimated plane primary waves really gives a good description of the angular distribution of the difference frequency sound. The small deviation we do observe may be explained by a small tapering of the assumed plane wave region that will be apparent from numerical computations of the primary field [12,13], and experimental errors.

These results and their interpretations demonstrate the necessity of observing not only far from the primary source, but also sufficiently far from the interaction region, if results are to be properly interpreted.

The observed variation of the halfpressure angle with distance may also be of some interest to possible practical applications of nonlinear interaction, indicating that in a certain region a sharper resolution can be obtained by such an array than would be expected from present theoretical predictions for the far field. However, the full understanding of these variations for other parameter values must await further theoretical and experimental work.

V. Conclusions.

The results presented here demonstrate the importance of taking such effects as primary intensities and observation distance into account when comparisons are to be made with existing theories. In several earlier papers these factors have only been partly accounted for, thus introducing the possibility of misleading results and wrong conclusions. Our experimental results show, however, that the theory does provide a good description of the angular and axial distribution of the generated difference frequency sound, even with regard to absolute amplitudes[12]. For greater primary intensities, however, where higher order interactions becomes important, the second order approximation fails to describe the distribution adequately. Provided the primary intensities are not too great, it has proved possible to modify the equations to give a quantitatively correct description for the distribution on the axis, and within the near field. A similar modification yet remains to be done for the angular distribution.

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