

## PROPAGATION OF SOUND FROM A POINT SOURCE INTO THE ACOUSTIC SHADOW PRODUCED BY WIND SPEED AND TEMPERATURE GRADIENTS CLOSE TO THE GROUND

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### 1. INTRODUCTION

For the majority of meteorological conditions there will always be regions within a 10km radius of a source which are in the acoustic shadow. Very often three of the four quadrants are completely in shadow.

The acoustic shadow is defined here as that zone which cannot be entered by acoustic rays which are characteristics of the eikonal equation. It is delineated in one plane by the ground and the path of the limiting ray after that ray has experienced a grazing incidence on the ground. The acoustic shadow is produced by temperature and/or wind gradients close to the ground such that there is an effective decrease in sound speed with height.

It has been appreciated in the last 30 years that sound energy can penetrate the shadow zone and very recently that there are commonly occurring meteorological conditions which can produce weak shadows; in these propagation can occur over distances of up to 2km with relatively small attenuation.

Pridmore-Brown<sup>[1]</sup> presented a solution for the field in a shadow produced by temperature and wind gradients. Pierce<sup>[2]</sup> re-analysed the problem under the assumption of a near constant negative sound speed gradient, and presented a residue series solution which permitted computation of levels inside the shadow zone. Daigle<sup>[3]</sup>, Don<sup>[4]</sup> and Raspet<sup>[5]</sup> have used Pierce's treatment to produce sound level predictions. Don and Raspet have also adapted the residue method to allow prediction of waveshapes. Berry and Daigle<sup>[6]</sup> have made improvements in Pierce's residue formulation which give accurate solutions at all location, including the insonified zone provided the sound speed gradient is constant. All of the above authors have noted that their predictions show peak pressures considerably less than those measured.

In this paper the methods of Pierce and of Berry and Daigle are compared. In addition a novel procedure for obtaining an improved average sound speed gradient close to the ground is described. The use of this gradient in both models is shown to produce estimates of peak pressure in closer agreement with measurements.

### 2. THE RESIDUE MODELS

Pierce's and Berry and Daigle's models have been found to give identical results inside the shadow zone. Only close to the shadow boundary or where shadows are very weak does Berry and Daigle's procedure give more accurate results.

Pierce's residue series for the pressure amplitude at a distance  $r$  from a source of strength  $\hat{S}$  at frequency  $f$  is

$$\hat{p} = \hat{S} \frac{e^{i\pi/4}}{r^2} \sqrt{\frac{2\pi}{k_0 r}} \exp(i k_0 r) \sum_n \exp\left[\frac{i r r_n}{2 k_0 r^2}\right] z_{resn} \quad (1)$$

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$$\text{where } \varrho = \left( \frac{R}{2k_0^2} \right)^{1/3}$$

$R$  is the limiting ray radius,  $R = c(0)/g$ ,  $c(0)$  is the sound speed close to the ground and  $g$  is the sound speed gradient;

$$k_0 = 2\pi f/c(0) ;$$

$$r_n = b_n e^{-\frac{12\pi}{3}}, \quad b_n \text{ are roots which are obtained by an iterative procedure described below;}$$

$Z_{res_n}$  are the residues.

Equation (1) is strictly valid only when  $r > \sqrt{2}z_0 + \sqrt{2}Rz$  where  $z_0$  is the source height and  $z$  is the receiver height. This is when the observer is in the shadow zone and when  $R$  is large. In addition Pierce has assumed that most of the contributions from the Fourier-Bessel integral used to derive (1) are for  $k$  in the vicinity of  $k_0$ . Tests have confirmed that this is the case provided  $R$  is sufficiently large.

Berry and Daigle's residue series for the pressure amplitude is obtained without any approximations!

$$\hat{p} = \hat{s} \frac{\pi}{\varrho^2} e^{\frac{i\pi}{6}} \sum_n H_0'(k_n r) Z_{res_n} \quad (2)$$

$$\text{where } k_n = k_0 \sqrt{1 + \frac{b_n}{k_0^2 \varrho^2}} e^{-\frac{2\pi i}{3}}$$

and  $H_0'$  is a Hankel function of the first kind and zero order. If the argument of the Hankel function,  $k_n r$ , as well as  $k_0 \varrho$  are large, equation (2) reverts to Pierce's approximate equation (1). The residues for the two series are also approximately equal (see below).

### 2.1 Determination of the Roots $b_n$

The roots  $b_n$  are obtained by solving the equation below iteratively using the Newton-Raphson procedure.

$$F = A_1'(b_n) + q e^{i\pi/3} A_1(b_n) = 0 \quad (4)$$

The procedure described by Cramond<sup>[7]</sup> was used with the following improvements:

- (a) the first and second derivatives were obtained explicitly rather than by approximate methods,
- (b) the iteration was stopped when  $F/F'$  and  $F$  were less than tolerance values which were set up in order to guarantee sufficient accuracy.

At the lower frequencies when  $R$  is large, as many as twenty terms are needed in the residue calculation. As a consequence the arguments of the Airy function and its derivative can become large. Care has to be exercised in evaluating these functions by use of the appropriate asymptotic expansions.

### 2.2 The Residues $Z_{res}$

Pierce gives two forms of  $Z_{res}$  corresponding to 'hard' and 'soft' ground conditions. The 'hard' condition is relevant for most practical cases and will cover ground flow resistivity,  $\sigma$ , values greater than 30K Rayls/m.

$$Z_{res} = \frac{\ell w_1 \left[ r_n - \frac{z_0}{\ell} \right] w_1 \left[ r_n - \frac{z}{\ell} \right]}{w_1^2 (r_n) (r_n - q^2)} \quad (3)$$

$$\text{where } w_1(\eta) = 2\pi^{\frac{1}{2}} e^{\frac{12\pi}{3}} \text{Ai} \left[ \eta e^{\frac{12\pi}{3}} \right]$$

$$q = \frac{1k_0 \rho c(o)\ell}{Z_s}$$

$$\frac{Z_s}{\rho_0 c} = \left\{ 1 + 0.05106 \left[ \frac{z}{f} \right]^{0.75} \right\} + 1 \left\{ 0.07682 \left[ \frac{z}{f} \right]^{0.73} \right\}$$

$Z_s$  is the ground impedance.

Berry and Daigle's  $Z_{res}$  is applicable for all ground impedance values.

$$Z_{res} = \frac{\ell \text{Ai} \left[ b_n - \frac{z_0}{\ell} e^{\frac{12\pi}{3}} \right] \text{Ai} \left[ b_n - \frac{z}{\ell} e^{\frac{12\pi}{3}} \right]}{\frac{2\pi\ell}{e^3} \left\{ \text{Ai}'^2(b_n) - b_n \text{Ai}^2(b_n) \right\}} \quad (4)$$

Examination of (4) shows that it is no more precise than (3) under the conditions for (3) to be valid and it involves more computation because of the Airy derivative in the denominator.

More detailed descriptions of the two models, including the procedures for computing the pressures in equations (1) and (2), exactly or using approximations, are given in reference (8).

### 3. MEASUREMENT OF SOURCE STRENGTH $\hat{S}$

The pressure waveform was measured at 25m from a 1kg explosive charge. A Bruel and Kjaer  $\frac{1}{4}$ " microphone with an ONOSOKI digital analyser was used to capture the waveform. (Figure 1). The signal was first downsampled using a three stage FIR filter/decimator from the initial sample rate of 256kHz down to 8kHz. The shadow zone calculations only require low sample rates since the shadow transfer functions behave effectively as low pass filters with cut-off frequencies generally below 1kHz.

The signal was then further processed to remove dc and to pad it with zeros so that the spreading in time produced by the shadow effect does not give aliasing errors.

The source strength amplitude  $\hat{S}$  was simply obtained by multiplying the above pressure by

$25/\exp(25ik_0)$ , to give the equivalent pressure amplitude 1m from the source. It is assumed that all non linear effects have become negligible at the 25m distance, and that because of the non-linear behaviour in the near source region, the effects of ground reflection on the above source strength value may be neglected.

### 4. DETERMINATION OF THE AVERAGE SOUND SPEED GRADIENT, $g$

The most serious error in the prediction procedures described stems from the inaccuracies in the measurement of sound speed gradient  $g$  close to the ground. In general this gradient is rarely constant and may vary considerably over the crucial first few metres above ground. Assuming that the above models are approximately valid when  $g$  is variable we can only obtain reliable predictions if a representative average gradient value can be obtained between the ground and the receiver height (typically 1.5 to 2m).

A novel two stage procedure has been developed for determination of these  $g$  values from the average radius of curvature of the limiting ray,  $R_{AV}$ , which is very sensitive to the precise meteorology in the atmospheric surface layer. The first stage is the computation of the micrometeorological wind temperature profiles based on the meteorological measurements at 2 and 10m heights. The Monin Obukhov similarity theory for the atmospheric surface layer has been used to give the precise profile. The theory has been rigorously tested<sup>[9]</sup> and gives very reliable profiles under all categories of meteorological conditions. The implementation of the procedure for profile calculation is described in reference<sup>[8]</sup>.

The second stage of the procedure employs a specially developed precision ray tracing program which can use the above micrometeorological data. The ray tracing may be restricted to a single plane without any loss of accuracy because of the proximity of the shadow boundary to the source. The limiting ray is found by launching rays from the source at very small angular intervals. The ray tracer is set up to generate curvatures over height changes as small as 0.1m. The average radius of curvature over the receiver height is obtained by averaging the above ray piece curvatures each weighted according to its length.

### 5. COMPARISON OF MEASURED AND PREDICTED WAVEFORMS AND ATTENUATIONS

Measured and predicted waveforms from either model at 500m from a 1kg blast source for  $g=0.152$  over a ground with flow resistivity 120K Rayls/m are shown in Figure 2. The predicted waveform's positive peak pressure is very close to the measured value but its waveshape is in error. The attenuation curve predicted for the above case is shown in Figure 3 together with the predicted attenuations for flow resistivities of 30 and 300K Rayls/m. The shadow zone attenuation is clearly not very sensitive to ground impedance.

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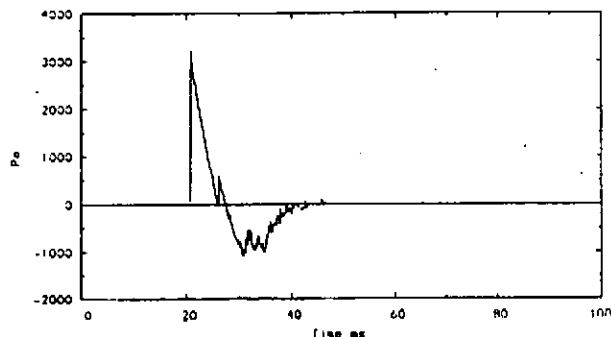


Figure 1. Measured Waveform at 25m

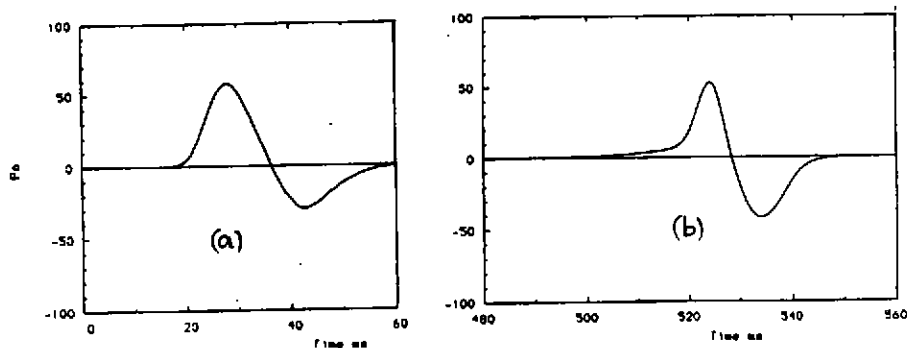


Figure 2. (a) Measured and (b) Predicted Waveforms at 500m for  $z_0 = 2m$ ,  $z = 1.5m$ ,  $\sigma = 120kr/m$

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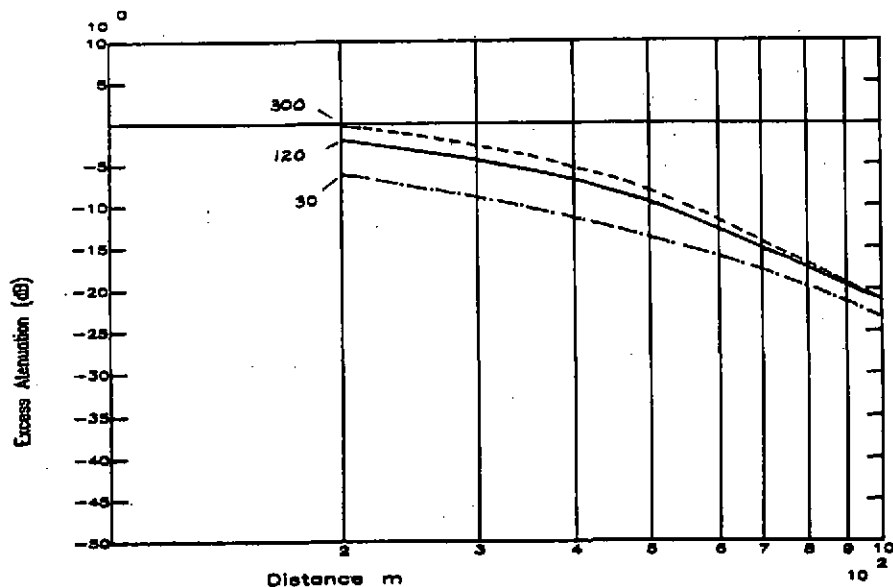


Figure 3. Predicted Excess Attenuation. Parameter is flow resistivity  $\sigma$  (kR/m).  $z_0 = 2\text{m}$ ,  $z = 1.5\text{m}$ .