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Measurement of the Oblique Incidence Absorption of
Porous Materials with a Waveguide.

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Summary

Improvements in the acoustic waveguide have been made which enable accurate measurement of impedance as a function of angle of incidence. Predicted impedances of a mineral wool absorbent are compared with the waveguide measurements.

1 Introduction

The variable sectioned waveguide has been briefly described in an earlier publication¹. A section through the guide is shown in fig.1. The first cross mode is excited by energy entering the guide at orifices AA'. This mode is equivalent to a plane wave of wavelength λ travelling down the guide and striking the sample B at an angle given by $\sin\theta = \lambda/2b$. In order to change the angle of incidence at a given frequency the wall separation, b must be changed. A new sample is used for each angle of incidence, great care being taken to ensure that each sample has the same physical constitution.

The incident and reflected modes form a pseudo-standing mode which is detected by the two probes CC' running along the guide walls which are each coupled to a microphone. By performing a vectorial subtraction of the two microphone outputs it is possible to eliminate the effect of unwanted plane waves in the guide. The precision difference amplifier which performs the subtraction has been improved so that plane wave effects can be removed at a given point provided their amplitude is less than half that of the first cross mode. Unfortunately, at certain angles of incidence (usually between 30° and 50° at 1 kHz) the amplitude of plane waves present far exceeds that of the first cross mode.

In order to reduce these plane waves electrical phase and amplitude controls were added to the inputs of the amplifiers feeding the horns coupled to pipes AA'. At the angles where the plane waves predominated it was found that the electrical controls had little effect on the relative phases and amplitudes of A and A'. This was because the horns and their pipes were being loaded by the impedance offered by the guide at A and A'. Variable length sections were added to the pipes so that they could be tuned. The monitor microphones, DD', were used to indicate when the source amplitudes and phases were correct.

2. Examples of Modal Reflection

The reflection coefficient measured above will only correspond to that obtained for an infinite plane wave incident on an infinite sample if the m th incident guide mode is reflected into the m th mode and there is no scattering into other modes. In order to show that this is so we must know the absorbing mechanism of the sample. The mechanisms or models used for describing absorbent behaviour have been discussed in an earlier publication.²

2.1 Fluid (or Hard Porous)

This is the most versatile model since it can be applied to a large range of practical materials. A longitudinal wave is assumed to propagate in the fluid which has a velocity less than 344 ms⁻¹ and is rapidly attenuated.

For simplicity we shall initially consider the fluid absorbent to be infinitely thick. The velocity potentials in the air and fluid are respectively

$$\phi = \phi_m^i \psi_m e^{ik_m z} + \sum_n \phi_{mn}^r \psi_n e^{-ik_n z}, \quad \phi^f = \sum_n \phi_{mn}^f \psi_n e^{ik_n^f z}$$

where z is the distance from the air-fluid interface.

The ϕ_n are the modal amplitudes and the k_n their wave numbers given by $k_n^{(f)} = \sqrt{(k^{(f)2} - n^2 \pi^2 / b^2)} = k^{(f)} \cos \theta_n^{(f)}$.

θ_n is the angle of incidence of the n th mode and θ_n^f is the corresponding angle of refraction. The propagation constant of the fluid is $ik^{(f)}$. The potential variation across the guide is given by $\psi_n = \begin{cases} \cos(\frac{n\pi x}{b}) & n \text{ even} \\ \sin(\frac{n\pi x}{b}) & n \text{ odd} \end{cases}$

where x is the distance from the guide axis. The suffixes 'i' and 'r' refer to incident and reflected potentials and 'f' refers to the fluid potential. Note that all symbols having the suffix 'f' may be complex.

Applying the conditions of continuity of pressure and normal particle velocity at the air-fluid interface gives

$$\begin{aligned} \phi_m^i \psi_m + \sum_n \phi_{mn}^r \psi_n &= m_f \sum_n \phi_{mn}^f \psi_n \\ k_m \phi_m^i \psi_m - \sum_n k_n \phi_{mn}^r \psi_n &= \sum_n k_n^f \phi_{mn}^f \psi_n \end{aligned}$$

where $m_f = \rho_f / \rho$, ρ_f and ρ being the fluid and air densities respectively. Multiplying each equation by ψ_m and integrating between $x = +b/2$ and $-b/2$ we obtain two equations with only m th mode terms. The m th mode reflection coefficient R_{mm} is obtained from these equations:

$$R_{mm} = \frac{\phi_{mm}^r}{\phi_{mm}^i} = \frac{z \cos \theta_m - 1}{z \cos \theta_m + 1}, \quad \text{where } z = m_f k / (k^f \cos \theta_m^f)$$

which is the normal impedance of a fluid surface of infinite extent.

The above proof is easily extended to the case of a fluid layer. Here we assume two wave systems to propagate in the layer

$$\phi^f = \sum_n (\phi_{mn}^{fi} \psi_n e^{ik_n^f z} + \phi_{mn}^{fr} \psi_n e^{-ik_n^f z})$$

Knowing the relation between ϕ_{mn}^{fi} and ϕ_{mn}^{fr} which is obtained by applying the condition of zero particle velocity normal to the

rigid backing, we can repeat the above process. The scattered components again have zero amplitude and the n th mode reflection coefficient corresponds to that of a layer of infinite extent.

2.2 Solid Termination (or Effective Solid)

In practical absorbent materials there is usually a shear wave in addition to the longitudinal fluid wave(s)! (A structure-borne longitudinal wave may also be present, which is coupled to the main fluid wave in the pores. The reasoning above will apply to this wave). The boundary conditions at $z=0$ must now include "zero stress tangential to the sample surface" in addition to the conditions applied in 2.1. The proof that there is no scattering at the air-solid interface is otherwise similar to that for the fluid. This analysis is easily extended to the case of a rigidly backed layer, where there is again no scattering.

3. Predicted and Measured Impedance of a Mineral Wool Layer

The first measurements of impedance with the waveguide were carried out on a flexible polyurethane foam.¹ There was a considerable error in the impedance measured at any given angle because of the effect of contact with the guide wall. Nevertheless, a definite increase in the real part with angle was noted. A theoretical prediction of the foam's impedance could not be attempted because of the unknown effect of the frame.

Mineral wool absorbents are influenced much less than foams by the guide walls. In addition a mineral wool having short fibres can be shown to approximate to the fluid model. Figure 2 shows the predicted impedance of a mineral wool having a flow resistance of $20,000 \text{ kg} \cdot \text{m}^{-3} \cdot \text{S}$. The real part of the impedance shows little variation with angle until the thickness becomes quite small. The imaginary parts show virtually no variation with angle. The measured curves for Stillite of nominal density $80 \text{ kg} \cdot \text{m}^{-3}$ are shown in figure 3. These agree well with the calculated curves. Many mineral wool absorbents have long fibres lined up in one direction. This usually gives rise to a higher flow resistance normal to the surface than parallel to it. Pyett³ has modified the fluid model to take the effect of this anisotropy into account. Using flow resistances of 50,000 and 20,000 the Pyett model gives a real part variation with angle substantially the same as figure 2. This appears surprising since we would expect increased external reaction. The explanation lies in the increased surface reflection from this type of absorbent due to its high normal flow resistance. Waveguide measurements on an absorbent of this type (Rocksil) will be reported at a later date.

References

1. Ford, R.D., Lord, P and West M. Acoustic waveguide having a variable section. *J.Acoust.Soc.Amer.* 47, 1, 12-13. 1970.
2. Ford, R.D., Landau, B.V., and West, M. Reflection of Plane Oblique Air Waves from Absorbents. *J.Acoust.Soc.Amer.* 44, 2, 531-536 1968.
3. Pyett, J.S. The Acoustic Impedance of a Porous Layer at Oblique Incidence. *Acustica* 3,6, 365-408 1953.

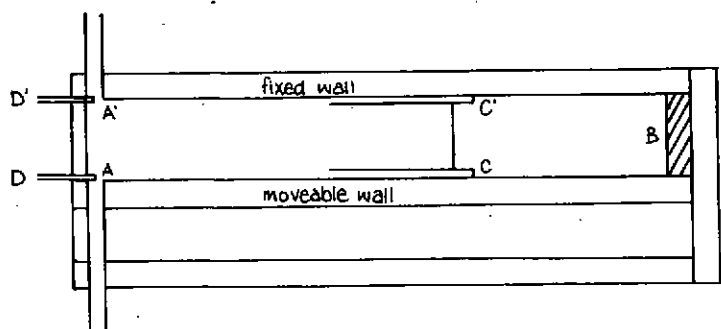


Fig 1 Section through the Acoustic Waveguide

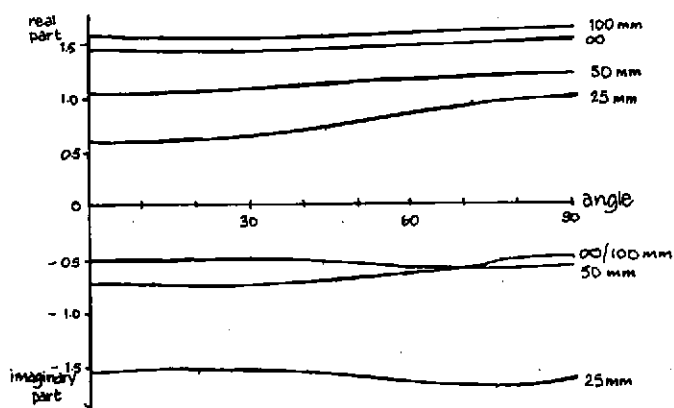


Fig 2 Predicted impedance ratio of a Mineral Wool Flow resistance 20000 kgm^{-3}

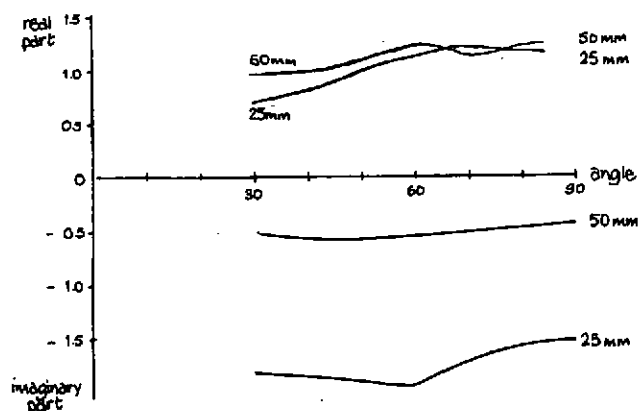


Fig 3 Measured impedance ratio of Stillite nominal density 80 kgm^{-3}